Q How to actually solve to get complexity?
Ans Few Few & ways :-

1) Plug & Chug

OR

2) Master's theorem.

OR.

3) Akra Bazzai {1996} :- ~~solve~~ ~~easy~~

** Akra Bazzi :-

formula :- $T(x) = \Theta\left(x^p + x^p \int_1^x \frac{g(u)}{u^{p+1}} \, du\right)$

words : Time complexity = $T(x)$

$g(u)$:- this g funt^n is basically a time complexity
" " it-self. & we know that in time complexity
like constants & more dominating terms
are ignored.
we know that this going to be the simplest
form for the function because all the
less dominating terms and other things
will be ~~so~~ removed.

P :-

$$a_1 b_1^P + a_2 b_2^P + \cdots = 1$$

i.e $\sum_{i=1}^{k} a_i + b_i^P = 1$ //

$$T(N) = T(N/2) + C$$

Q In constants why do we write O(1)?

Ans Because we don't really care about constants for eg :- we can write O(C) or O(2k+C) all of these are constant, ∴ we can ignore the constants

∴ Basically O((2k+C) × 1)

∴ Anything multiplied by anything, you can ~~ing~~ ignore the ~~constant~~ constants.

So, Hence anything in constants can be written as O(1).

Eg: 1) $T(N) = 2T\left(\frac{N}{2}\right) + (N-1)$

$a_1 = 2$

$b_1 = \frac{1}{2}$

$g(x) = xN - 1$

$$\therefore \cancel{2} \times \left(\frac{1}{\cancel{2}}\right)^P = 1$$

$$\therefore P = 1.$$

Once you've found the p then ~~&~~ substitute it into the Akra - Bazzi' formula :-

$$T(n) = \Theta\left(x^1 + x^1 \int_1^x \frac{u-1}{u^2}\, du\right)$$

$$= \Theta\left(x + x \int \frac{1}{u} - \frac{1}{u^2}\, du\right)$$

$$= \Theta\left(x + x\left[\int_1^x \frac{du}{u} - \int_1^x \frac{du}{u^2}\right]\right) \qquad = u^{-2} \quad \frac{u^{-1}}{-1} = \frac{-1}{u}$$

$$= \Theta\left(x + x\left[[\log u] + \left[\frac{1}{u}\right]\right]_1^x\right)$$

put ↑.

$$= \Theta\left(x + x\left[\log x + \frac{1}{x} - 1\right]\right)$$

$$= \Theta\left(\cancel{x} + x \log x + \frac{1}{\cancel{x}} - \cancel{x}\right)$$

$$= \Theta(x \log x + 1)$$

$\therefore \Theta(x \log x)$. // Time complexity.

So, for array of size N :- Merge sort complexity $= O(N \log N)$.

2) $T(N) = 2T\left(\frac{N}{2}\right) + \frac{8}{9}T\left(\frac{3}{4}N\right) + N^2$

$\therefore \quad \cancel{2} \times \left(\frac{1}{\cancel{2}}\right)^p + \frac{\cancel{8}^2}{\cancel{9}_3} \times \left(\frac{\cancel{3}^1}{\cancel{4}_1}\right)^p = 1$

$\cancel{1 + \frac{2}{3}}$ so, put $p = 2$

$\cancel{2}^1 \times \frac{1}{\cancel{4}_2} + \frac{\cancel{8}^1}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{16}_2} = 1$

$\therefore \quad \frac{1}{2} + \frac{1}{2} = 1$

$\therefore \quad p = 2$ //.

Subsidute.

$$T(x) = \Theta\left(x^2 + x^2 \int_1^x \frac{u^2}{u^3}\,du\right)$$

$= \Theta(x^2 + x^2 \log x)$ {ignore the less dominating term}

$= \Theta(x^2 \log x)$

If you're unable to find value of P:

Q. $T(x) = 3T\left(\frac{x}{3}\right) + 4T\left(\frac{x}{4}\right) + x^2$

① $P = 1$ Case 1

$= 3 \times \left(\frac{1}{3}\right)^1 + 4 \times \left(\frac{1}{4}\right)^1 = 1$

$\therefore 1 + 1 = 1$

$2 \neq 1$ $\because 2 > 1$ This means

what? I need to increase the denominator

② $P = 2$ Case 2

$3 \times \frac{1}{9} + 4 \times \frac{1}{16} = 1$

$\frac{1}{3} + \frac{1}{4} = \frac{7}{12} < 1$

Hence, P is less than 2.

So, P actually lies in the range 1 & 2

Note:

When $P <$ power of $(g(n))$ than you your ans $= g(n)$.

Here, $g(n) = x^2$

$p < 2$ {i.e power of $g(n)$}

Hence, ans $= O(g(n))$.

$$T(x) = \Theta\left(x^p + x^p \int_1^x \frac{u^2}{u^{p+1}}\, du\right)$$

$$= \Theta\left(x^p + x^p \int_1^x u^{1-p}\, du\right)$$

$$= \Theta\left(x^p + x^2\right)$$

$\because$ $p < 2$

$\therefore$ x^p will become less dominating term x^p Hence, ignore.

$\therefore$ $\Theta(x^2)$ //

Hence proved.