

DATE: January 31, 2007

TO: CheolHee Park

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SUBJECT: Rideshare Web Application Proposal

Introduction

Rideshare is a web application which aims to connect drivers and passengers who seek to participate in carpools. It will do so in an efficient manner, minimizing travel time and distance, as well as taking into account the preferences of customers.

These days, fuel costs are pressuring people to leave their cars at home and seek alternatives. One of the best choices is cooperative transportation, or carpooling. The carpooling alternative is one which is widely recognized as a good solution for alleviating traffic congestion [5][6][9]. Currently, there is very little being done to assist people in organizing carpools. Websites which are dedicated to carpooling do little more than offer their users a bulletin-board for posting their ride offers and requests. Users are left to search for and organize carpools themselves [7][8]. We want to develop a web service to fill this void, and automatically match users to one another based on compatibility and all-around efficiency.

Our application will help people coordinate rideshare programs and plan trips efficiently and effectively. People who share rides regularly will significantly reduce their ecological footprint and save on travel expenses. In addition, our solution will help people find rides which they would otherwise not be able to find without exhaustive effort.

Our team of four has a diverse background. Each team member brings to the table a set of skills which will be critical for the successful development of Rideshare. Patrick and Derrick are both receiving their BS in Computer Engineering, and have extensive experience in web development and the Linux/Apache/MySQL/PHP (LAMP) platform. Additionally, Derrick brings Google Maps experience to the table and Patrick has experience implementing evolutionary algorithms.

Garrett and Yoni - both inexperienced in LAMP development - have worked on projects in PERL, a scripting language similar to PHP. Yoni is receiving his degrees in Computer Engineering and Mathematics and has experience in artificial neural networks, evolutionary algorithms, and has been involved with the Rideshare optimization problem for the past year. Garrett is currently earning his BS in Electrical Engineering, with an emphasis on computer design.

In the following sections, we discuss the underlying computational problem involved in developing the rideshare paradigm. We present our approach for solving this problem efficiently and effectively, while providing a friendly and intuitive interface to the user. We then lay out the development framework and task division that we believe will ensure the timely and effective completion of the Rideshare application. We further break down the process of developing the application in terms of the tools and resources which we will need. Finally, we introduce a schedule for completing the tasks, and divide the work into tangible modules which each team member will implement. We conclude with a summary of our main points, and a positive outlook toward the development of Rideshare.

The Rideshare Challenge

Our web application must solve the problem of taking a dynamic, or changing, set of ride offers and requests, and finding efficient routing solutions. In other words, we must maximize the number of rideshares, while adhering to the user preferences, and route and time constraints. We must also provide a user-friendly environment for managing profiles, entering rideshare requests, and viewing results. This environment must be simple and intuitive in order to be effective.

In order to solve the problem of matching drivers and riders, we must understand the fundamental problem with which we are dealing. In academia and industry, this problem is typed as an optimization problem. We must take the set of drivers (D) and riders (R), and find routes between their starting positions and destinations such that we maximize the number of riders served, while taking into account user preferences and constraints, route length, and the user-specified time-windows.

A similar problem, which is called the Vehicle Routing Problem, is stated as finding minimal routes for a certain number of vehicles (k) to visit a certain number of customers (n), where the vehicles start at a central location called a 'depot.' Our problem, though similar, is different in that we allow for some customers to not be matched in the case that their criteria is not met, or if they are too far out of the way to route a vehicle to them. In addition, our problem has several constraints, which can be summarized as follows:

Constraints:

- Each route must originate at the drivers origin and terminate at their destination.
- Each vehicle's schedule must adhere to the driver and riders' time window constraints.
- Users will be required to specify definite departure and/or arrival times.
- The total number of riders in a vehicle at any given time cannot exceed that vehicle's capacity.
- User preferences, such as gender, ratings, and age, must be met.

Objectives:

- Maximize the number of riders matched.
- Maximize the total efficiency of the routes, in terms of time and distance, on a global level.

This problem is formally known as the Capacitated Multi Depot Vehicle Routing Problem with Time Windows (CMDVRPTW). It is a variation on several well-known problems, which have been shown to be NP-Complete, meaning that the time it takes to solve them grows exponentially relative to the number of inputs. Timeliness of results is important, since an effective result which doesn't complete in a timely manner is useless. This exponential growth means that, for a relatively small set of inputs, algorithms that solve this type of problem can potentially take longer to complete than the history of the universe. This last requirement necessitates the use of an algorithm which does not run in exponential-time, rather completes in some bounded time-frame tolerable by humans. [1][2][3][4]

Thus, our problem cannot be practically solved directly, i.e. the optimal solution would take too long to find. Fortunately, several methods have been developed for finding approximately

optimal solutions. The field of Operations Research deals directly with this problem. We will use several of the approaches used by academics for optimizing similar problems, and tailor their methods to our variation of the problem (CMDVRPTW). [1][2][3][4]

One of the most important aspects of our project will be providing an intuitive and user-friendly interface to allow the integration of the service into people's daily lives. If the interface is confusing, tedious, or annoying, users will choose not to use it. We must ensure that each user is able to use the tools which we provide them with to create rideshares. Several map-based interface technologies have recently emerged, which can facilitate an intuitive interaction between Rideshare and users.

Our Approach to Implementing Rideshare

Many of the current ride matching services act only as a forum for riders and drivers to post offers or requests [7][8]. We will eliminate the need for users to search through forum posts by matching them according to preferences. RideShare will provide a transparent interface which users can submit their desired itinerary and preferences for a trip in the hopes to be matched with a driver with a similar itinerary.

In order to implement the application, we plan on building a web-based interface framework with a database on the backend. Multiple packages are available at each level: the hosting platform (Windows, Linux), the web server (Apache, IIS, Tomcat, lighttpd), the database server (MySQL, PostgreSQL, Oracle), and the web toolkit (Ruby on Rails, PHP, Perl, ASP). We have past experience with the combination (Linux, Apache, MySQL, PHP) codenamed LAMP, and as such already have a server with it available [25].

On this server, we will install the rideshare application, which will be comprised of four main modules: Control, User Interface, Optimizer, and Database, as shown in Figure 1.

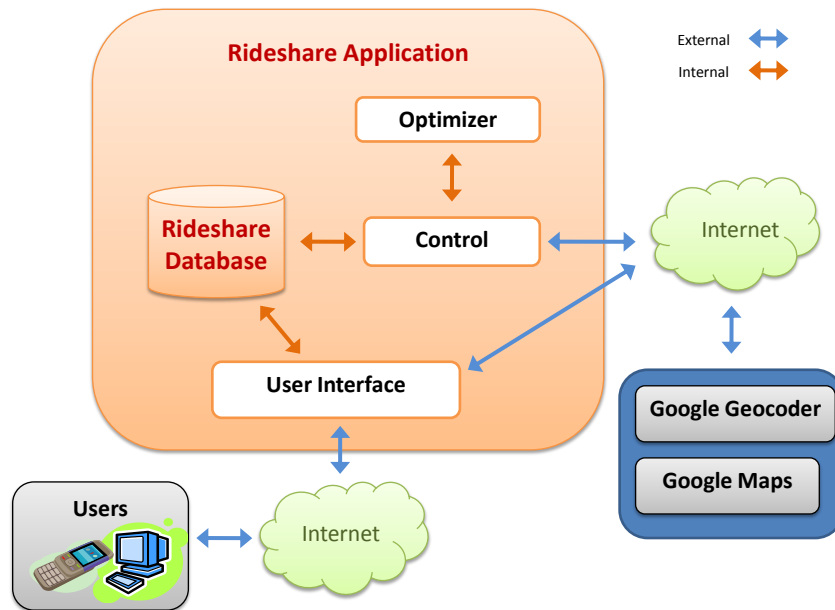


Figure 1: Rideshare Block Diagram

User Interface

My Rides **My Profile** **New Ride** **My Friends** **Main Page**

New Ride

☐ Riding ☒ Driving

Origin/destination

origin

destination

Time window

date

earliest departure

latest arrival

Confirm

Advanced Options ▾

Gender preference

Age preference

Rating

Driver Options

Maximum Passengers

Figure 2: Sample User Interface

Our approach for creating an intuitive and effective user-environment is to use a map-based interface, commonly known as a map "mashup." We plan on using the Google Maps Application Programming Interface (API) as the foundation for our interface. Users will have the option of finding and clicking their starting and ending location on a Google Map, inputting the addresses on a form, or selecting previous locations marked by icons on the map. The website will be a mix of map-based and form-based interactions, with an emphasis on minimizing forms. Figure 2 shows an example of what a ride entry page might look like.

Optimizer Module

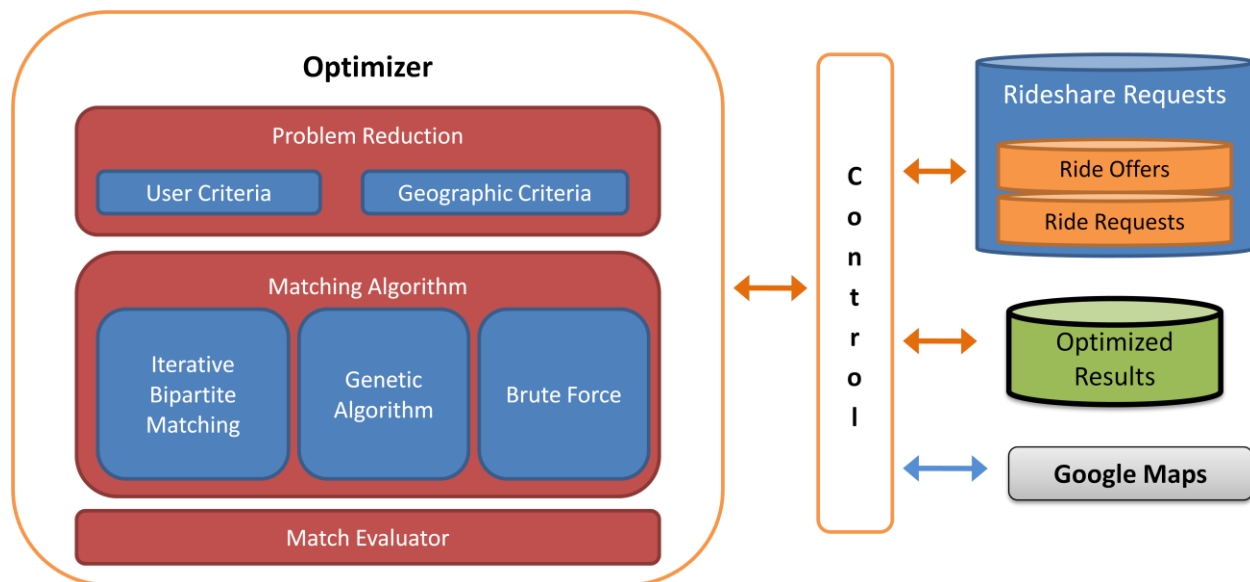


Figure 3: Optimizer Block Diagram

After users have submitted requests, the Optimizer (Figure 3) will match drivers with riders according to their desired origin and destination, available time windows, and preferences. These preferences will most likely include user ratings, gender, and age.

In order to find a maximal matching, we first need to reduce the search space of the problem at hand. Search space reduction is the process of eliminating possibilities which are clearly invalid, and thus reducing the amount of work necessary for finding matches. The Problem Reduction component of the Optimizer module will reduce the number of possible solutions by excluding matches between a driver and riders that are not on their route or do not share similar preferences. Using this reduced set, the optimizer algorithms will then sift through possible solutions looking for the most efficient routes. The Optimizer module will be the crux of the RideShare project, so a significant amount of the design process will be dedicated to researching and implementing usable solutions.

Optimization Algorithms

We will implement a variety of algorithms proven to be capable of approximating NP-complete problems effectively and efficiently. We will implement our solutions using three algorithmic approaches: simple-iterative (commonly referred to as brute-force),

iterative bi-partite solver, and a genetic algorithm. We feel that implementing these algorithms and analyzing their results is necessary to provide users with routes that are efficient and adhere to user preferences. In addition, we will develop a real-time visualization of these algorithms to help us visually understand and analyze the results produced by our algorithms.

The *brute force* method will simply iterate over all possible combinations of rides, i.e. the "naive" approach to finding the optimal solution. This method is meant to be used for comparing the other implementations, and for presenting the complexity of the problem. It will most likely not actually be used by the application for optimizing results of more than a very small number of users.

Another method for solving the problem relies on the fact that our routing problem can be modeled as a *graph*, which is a mathematical construct. A *graph* is a set of vertices (nodes) and edges (lines connecting the nodes). A *bipartite graph* is a graph in which the vertices of the graph can be divided into two disjoint sets, where no two vertices in the same set have an edge between them. In our case, the set of drivers and riders can be viewed as a bipartite graph, since we would never match a driver with another driver (i.e. no edges within the set of drivers). Viewing our problem as a *maximal matching problem* on a bipartite graph allows us to employ several algorithms which are dedicated to this, and similar, problems. Since bipartite graph matching allows only one rider per driver, we will perform bipartite matching *iteratively*, adding one passenger per driver at a time. For detailed information on graph theory and maximal matching, please refer to Appendix B: Graph Theory.

The *genetic algorithm* takes its approach from nature, mimicking the process of natural selection based on fitness. The genetic algorithm takes a set of randomly generated solutions and ranks them according to a fitness function (also known as a scoring function). It then takes the best few candidates and mutates them (usually some small permutation), then ranks those results and mutates the best of those, etc. Some implementations have it also occasionally pull in a completely new random set to avoid dead-end paths on the road to the best solution. [26]

Control Module

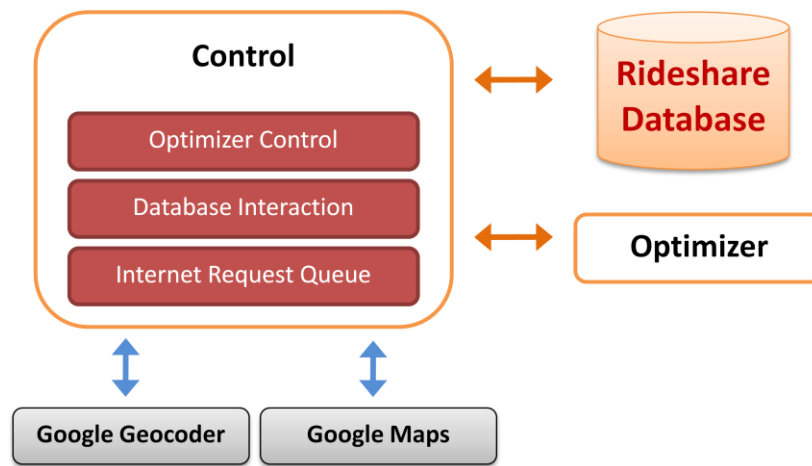


Figure 4: Control Block Diagram

The Controller module (Figure 4) of Rideshare will work in concert with the Optimizer module, acting as an interface to the database, which holds all ride requests and offers. We have restricted access to the database through the Controller in order to reduce code duplication in all the modules. For example, if all components of the Optimizer module contained separate code blocks that all had the same database functionality, maintaining the code could be problematic if changes were made to the database. Defining access in one module reduces the complexity of code maintenance and provides a uniform API for developers interfacing with the database.

Control will also be the interface for all modules to use when querying routing data on Google Maps. This is due to the fact that we see Google Maps queries as a potential bottle-neck in our application. Several modules may wish to query Google at the same time, and some priority must be established. The Control module will perform the role of arbitration in those cases.

Results

After drivers and riders have been matched, they will be notified via e-mail or text message, depending on their communication preferences. The routes generated will be available on a Google Maps canvas and an itinerary will be created for users to download. Once results are ready, a user will be able to review their possible matches, and confirm or deny a Rideshare. In

order for a Rideshare to be established, both parties must confirm the match. This is referred to as a "handshaking policy."

Rideshare Developer Activities and Responsibilities

Please see the Gantt chart in Appendix A for detailed task division and milestone dates.

In order to create a development environment, we have obtained and set up a server, and installed the various programs and tools we will need on it. The UT Communications Society loaned us a computer, as facilitated by Patrick and Derrick. After that, Derrick installed LAMP. Since then, Patrick and Garrett began constructing the website's front end in PHP. Garrett and Yoni do not have as much experience in PHP as Patrick or Derrick, so as work on the user interface continues, Patrick will help them gain expertise in the language.

During this time, Derrick began installing a version control and content management system (CMS). We decided on an open source CMS called Trac, because it was designed to be used with the Subversion (SVN) revision control system and provides a Wiki and Bug Tracking system [24]. Simultaneously, Yoni has been researching the problem definition and several algorithms and approaches for solving it [10].

During the coming week, we will determine the interfaces between the User Interface, Database, Controller, and Optimizer modules. These interfaces represent how the different modules "talk" to one another. Once they are established, the tasks of developing each module can be performed in parallel. All team members will be involved in the process of determining module interfaces.

The first features of the User Interface module will be those related to the database support for user profile management. These will be developed by Patrick and Garrett. Yoni and Patrick will work on support for ride request entry and accessing optimization results. Simultaneously, Derrick will be working on Google Map querying in the Controller.

Once these tasks are complete, we all will work on the front end of the user interface: Derrick and Yoni will collaborate on the map-based toolkit, and Patrick and Garrett will work on the site map and web page templates. By the end of February, we should have most of the web application functioning and ready for testing.

Throughout the first stages of the project, we will still be developing our approach for the optimization problem. Once most of the back end is ready, we will begin writing our optimization functions, with Derrick writing the Control module. Yoni will write a function to reduce the "search space" by eliminating obviously incompatible matches.

The actual optimization algorithms will be developed by all team members, as it is a significant portion of the total work. Garrett and Patrick will work on the genetic algorithm, and Yoni and Derrick on the brute force and bipartite matching algorithms.

Throughout the development process, each team member will test their functions on a local, module-level. Garrett will be in charge of writing a testing framework for the web application as a whole. Towards the end of February, we hope to have tested much of the functionality of the website, and we will use March to develop and test the optimization algorithms.

Once these are done, we will have all the basic functionality in place, so we can test the program in its entirety. The rest of semester we will work on the advanced features, as well as refining features we have already implemented. The last few weeks of April, we can work on supplemental documentation, usability, and visual design.

We have divided and timed these tasks according to the complexity, amount of effort, skills, and points of interest of each team member. Each major milestone has an "owner." As the semester unfolds, we will tweak this task division, with each milestone owner reporting on the progress made, and restructuring the task division as necessary to complete everything on time.

Tools and Resources

Project Resources

Dr. Constantine Caramanis is our faculty sponsor. He received his A.B. from Harvard University and eventually earned his M.S. and Ph.D. in Electrical Engineering at the Massachusetts Institute of Technology [11]. He wrote his Ph.D. on adaptable optimization. He is currently a member of the Wireless Networking and Communications Group (WNCG). His research interests include optimization, including combinatorial optimization problems like our own.

CheolHee Park is our technical TA. He attended Chung-Ang University in Seoul, South Korea where he received his B.S. and M.S. in Electrical Engineering, with honors. He is currently a student of the WNCG here at UT, working on his Ph.D. His research areas include wireless communications and networking. Also, he is skilled in algorithm analysis and he is familiar with C++.

We also have access to several academic papers and websites to reference. Google Scholar and IEEE Xplore are both search engines designed specifically for searching academic papers [12][13]. These will prove particularly useful for algorithm research, though if we need to look for more specific papers, we can browse through The VRP Web, which is devoted to the vehicle routing problem [4]. To a lesser extent, we can use Wikipedia to search for general information on any algorithms or tools we may use [15].

We will be working with programming languages and software which we may need help with. For PHP, we can reference their official website, which references useful documentation and frequently asked questions [16]. MySQL developer's page has even more information along similar lines, as it is an open source database [17]. If we need help with the Google Maps API, we will go to the Google Code site, where all they have published the program interface. In addition, the UT Library offers several online resources, including programming manuals.

Project Materials

The web server we are using was loaned to us by the UT Communication Society [19]. Luckily, it was loaned to us as a dedicated server, which means we were able to completely change its software down to the operating system, and it is available to us on a 24/7 schedule. It is an older computer, with only a 500MHz Pentium 3 processor, 512KB of L2 cache and 128MB of RAM. The server is connected to the internet through a 10/100 cable to the UT network in ENS, and from there to the internet on a T3 line, which is very fast. We are able to access the server through a secure connection from any computer with internet access. Fortunately, all the software (PHP, MySQL, Ubuntu, Apache, and Trac) we are likely to use is freely available and open source [16][17][20][21][22]. We will also be using a free software service as we interface through the Google Maps API [23].

We will take several measures to control costs. Primarily, the Communications Society loaning us a server has drastically cut costs. Additionally, we have limited the scope of the problem to what we feel can be completed by the April demo date. We will focus on completing the features set out in this document and will not add anything extra until we have completed them. The only costs that could be incurred are our time and the running time power costs of the server, which is defrayed by the university. To get a feel of our time budgeting, please reference the Gantt chart in Appendix A. Please see Table 1 below for further clarity on the dollar cost of our project.

Component	Cost
Server Hardware	Free
Server Upkeep	Free
Server Software	Free

Table 1: Rideshare Cost

Rideshare Milestones

1. Obtain Server (Jan 18)
2. Install LAMP and Trac (Jan 25)
 - a. Install and setup Ubuntu Server, MySQL, PHP, Apache2
 - b. Configure Subversion and Trac. Create accounts and setup development infrastructure
3. Jan 31: Proposal Due
4. Overall design of interfaces between the main modules (February 1st, all team members)
 - a. Database
 - b. Controller + Optimizer
 - c. User Interface
5. Framework (February 8)

Need functionality for creating and accessing:

 - a. User profile management (Patrick and Garrett)
 - i. personal info
 - ii. blacklist (i.e. users who shouldn't be considered for matching)
 - iii. Ride requests – Riders
 - iv. Ride requests – Drivers
 - b. Database tables for interfaces (Yoni and Patrick)
 - i. Ride Requests
 - ii. Results
 - c. Google Maps queries, format results from Google Maps for optimizers. (Derrick)
 - d. Controller to database communication link (Yoni and Patrick)
6. Design Review Due (February 14)
7. Establish our algorithmic approach. (All Members)
8. Finish Backend for UI/Optimization (February 29)
 - a. User Notification (email, text, rss) (Derrick)
9. Front end for UI (February 29)
 - a. Google maps visualization: (February 22, Yoni and Derrick)
 - b. All map-based interactivity (February 29, Yoni and Derrick)

- c. Site map and page templates (February 29, Garrett and Patrick)
- 10. Intellectual Property Report due (Feb 28)
- 11. Pre-Demo Performance milestone with TA (Feb 28)
- 12. Testing
 - a. Generating test-cases (February 29, Garrett)
 - b. Algorithm performance measures, visualization. (March 21, Garrett)
- 13. Optimizer (March 21)
 - a. Control Module - sets up the problem, calls the optimization algorithm.
 - b. Function which sets up the problem for optimization (i.e. which requests are going into the optimizer)
 - c. Optimization algorithms:
 - i. Genetic algorithm (March 14, Yoni, Garrett, Patrick)
 - ii. Brute Force - for comparative purposes (March 7, Yoni)
 - iii. Bipartite Graph Matching (allows only one rider per driver) (March 14, Derrick)
 - iv. Iterative Bipartite Graph Matching (March 21, Derrick)
- 14. Extras (no due date, optional features)
 - a. User profile management
 - i. Friends
 - ii. history of user requests/routes
 - b. Facebook Integration
 - c. RSS Feeds
- 15. Progress Report Due (March 20)
- 16. Final Project Demo Milestone with TA (April 1)
- 17. Refinement & Testing (April)
- 18. Oral Final Presentations I (April 29)
- 19. Final Report Due (May 1)
- 20. Oral Final Presentation II (May 1)

Conclusion

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Appendix A: Rideshare Development Schedule

Appendix B: Graph Theory

Some understanding of elementary graph theory is necessary in order to solve our optimization problem. We include this section to assist the reader in understanding the optimization problem at hand, and thus gain an understanding of the optimizer module's required functionalities.

Mathematical definitions are marked with bullets.

Definition of a graph:

- A *graph* is defined as a set of *vertices* $V=\{v_1, v_2, \dots, v_n\}$ and a set of *edges* $E=\{e_1, e_2, \dots, e_m\}$, for some positive integers m and n .
- The vertices can be thought of as dots, and the edges as lines connecting those dots, as shown in Figure A.1

Our optimization problem can be viewed as a graph theory problem. In our case, the drivers (D) and riders (R) can be viewed as vertices, where two vertices are connected by an edge if they can be partners in a rideshare.

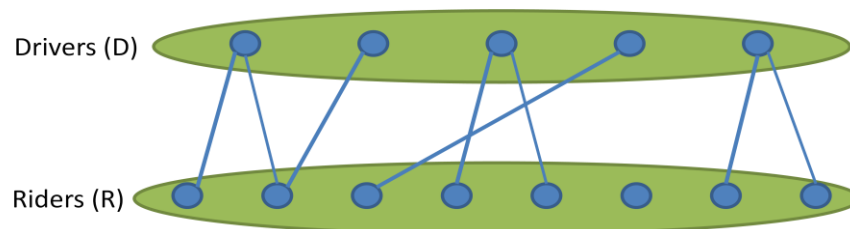


Figure A.1: Bipartite graph of drivers and riders

- A *matching* is a set of vertices and edges. For a matching M , two vertices are said to be matched if M contains an edge which connects them.

Our optimization problem can be viewed as equivalent to the maximal matching problem. The maximal matching problem is the problem of finding a matching within a graph which includes the maximum number of vertices. Translated to rideshare, this means finding the most matches between drivers and riders, and thus providing rideshares for as many people as possible.

In particular, our optimization problem can be viewed as equivalent to a bipartite graph optimization problem.

- A *bipartite graph* is a graph in which the vertices of the graph can be divided into two disjoint sets, where no two vertices in the same set have an edge between them.

In our case, we have two sets of vertices, D (drivers) and R (riders), which can never have rideshares within the sets, i.e. no two drivers will be in a rideshare together, and the same holds for riders. This relationship can be seen in figure 7.3.1, where no edges are visible within the sets D and R.