Handout 9 Exercise Questions

Question 1

An investor can invest in two assets, X and Y. Asset X has the expected return of 0.06 and the variance of 0.0004, and asset Y has the expected return of 0.08 and the variance of 0.0025. The correlation coefficient of the rate of return of the two assets, ρ, is 0.5. The investor is assumed to have the following expected utility function:

$$E[U] = E[r_i] - \alpha Var(r_i)$$

where α is a positive constant.

a) Determine, as a function of α, the portfolio that maximizes the expected utility.
b) Show that, as α increases, the investor selects an increasing proportion of Asset X and explain why the relationship holds.

Question 2

An investor's preferences can be modelled by the utility function:

$$U(w) = 3w - w^2, w > 0$$

a) Determine the range of values over which this utility function can be applied.
b) Explain how the investor's holdings of risky assets will change as his wealth decreases.
c) Which of the following investments will the rational investor choose?

Investment X		Investment Y		Investment Z	
outcome	probability	outcome	probability	outcome	probability
0.1	0.3	0	0.3	0.2	0.45
0.3	0.4	0.2	0.2	0.3	0.1
0.5	0.3	0.9	0.5	0.4	0.45

Question 3

An investor's utility function is $U(w) = \sqrt{w}$. He has an initial wealth of \$140,000 and faces a potential loss of \$100,000 with 1%.

a) Calculate the maximum premium that he is willing to pay to insure against the loss.
b) Suppose that an insurer has an initial wealth of \$100 million and a utility function of $U(w) = w$. Calculate the minimum premium the insurer would require in order to offer the coverage.

Question 4

An investor's preference can be modelled by the marginal utility function:

$$U'(w) = \frac{1}{w}$$

a) Determine the range of wealth, *w*, over which this utility function can be applied.
b) Explain how the investor's holdings of risky assets will change as his wealth decreases.
c) With no other wealth, the investor will gain either 2 with 75% or 1 with 25%. Determine the level of a fixed gain that makes her indifferent from the uncertain gain.

Question 5

Individuals face potential loss of their wealth. Use the listed information regarding individuals and loss to answer the following questions.

- Individual's initial wealth: 100,000
- Loss amount: 50,000
- Probability of loss: 10%
- Utility function: $u(w) = \sqrt{w}$

a) What is the risk premium that an individual is willing to pay to obtain full coverage insurance?
b) Find the optimal insurance coverage for a risk-averse individual if premium rate (premium per unit coverage) is 0.11. Show all the steps of your calculation to earn full credit.
c) Derive expressions for the absolute risk aversion and relative risk aversion measures for the utility function, $\sqrt{w}$, and explain what they indicate about the individual's desire to hold risky assets.

Question 6

An investor has a quadratic utility function: $U(w)=w-10^{-5}w^2$. Her income is uncertain and will be either \$40,000 with 75% or \$30,000 with 25%.

a) Determine the minimum level of a fixed income that makes her indifferent from the uncertain income.
b) Which of the following statement is true for a risk-averse individual? Prove your answer.
   (i) Decreasing relative risk aversion implies decreasing absolute risk aversion.
   (ii) Decreasing absolute risk aversion implies decreasing relative risk aversion.