

## MH4511 Sampling & Survey

Tutorial 9 Solution

AY2025/26 Semester 1

### Problem 9.1 (Solution)

a) Using Lahiri's method:

$f_i$	$s_i$	$M_{f_i}$ : Class size	Decision
3	25	14	Reject psu 3
7	23	33	Include psu 7
2	43	50	Include psu 2
6	30	17	Reject psu 6
9	18	54	Include psu 9
2	52	50	Reject psu 2
5	39	62	Include psu 5
10	28	29	Include psu10, STOP
4	9	20	

b) Using cumulative-size method:

Class Index	Number of Students	Cumulative Range	Decision
1	23	1 – 23	23; include psu 1
2	50	24 – 73	
3	14	74 – 87	
4	20	88 – 107	98; include psu 4
5	62	108 – 169	145; include psu 5
6	17	170 – 186	
7	33	187 – 219	196; include psu 7
8	27	220 – 246	
9	54	247 – 300	287; include psu 9
10	29	301 – 329	
total	329		

Problem 9.2 (Solution)

Sampled Unit ( $i$ )	$M_i$	$\psi_i$	$y_{ij}$	$\bar{y}_i$	$\hat{t}_i = M_i \times \bar{y}_i$	$\frac{\hat{t}_i}{\psi_i}$
14	65	65/807	3, 0, 0, 4	1.75	113.75	1412.25
23	25	25/807	2, 1, 2, 0	1.25	31.25	1008.75
9	48	48/807	0, 0, 1, 0	0.25	12.00	201.75
14	65	65/807	2, 0, 1, 0	0.75	48.75	605.25
16	2	2/807	2, 0	1.00	2.00	807.00
6	62	62/807	0, 2, 2, 5	2.25	139.50	1815.75
14	65	65/807	1, 0, 0, 3	1.00	65.00	807.00
19	62	62/807	4, 1, 0, 0	1.25	77.50	1008.75
21	61	61/807	2, 2, 3, 1	2.00	122.00	1614.00
11	41	41/807	2, 5, 12, 3	5.50	225.50	4438.50
Average						1371.90
Std Dev						1179.47

$$\hat{t}_\psi = \frac{1}{n} \sum_{i=1}^n \frac{\hat{t}_i}{\psi_i} = \frac{1}{10} \left[ \frac{113.75}{65/807} + \dots + \frac{225.50}{41/807} \right] = 1371.90$$

$$SE(\hat{t}_\psi) = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^n \left[ \frac{\hat{t}_i}{\psi_i} - \hat{t}_\psi \right]^2} = \frac{1}{\sqrt{n}} \left[ \text{Stdev} \left( \frac{\hat{t}_i}{\psi_i} \right) \right] = \frac{1179.47}{\sqrt{10}} = 372.98$$

Problem 9.3 (Solution)

Sampled Unit ( $i$ )	$M_i$	$\psi_i$	$y_{ij}$	$\bar{y}_i$	$\hat{t}_i = M_i \times \bar{y}_i$	$\frac{\hat{t}_i}{\psi_i}$
13	65	65/800	3, 8, 5, 4	5.0	325	4000
22	26	26/800	2, 2, 2, 4	2.5	65	2000
9	48	48/800	10, 6, 1, 5	5.5	264	4400
13	65	65/800	2, 5, 1, 7	3.0	195	2400
16	4	4/800	2, 3, 5, 8	4.5	18	3600
6	62	62/800	9, 2, 2, 5	4.5	279	3600
Average						3333.33
Std Dev						935.24

- a) Primary sampling units (psu): the 20 operational units of the plant.  
 Secondary sampling units (ssu): all the employees of the manufacturing plant.
- b)

$$\hat{t}_\psi = \frac{1}{n} \sum_{i=1}^n \frac{\hat{t}_i}{\psi_i} = \frac{1}{6} [4000 + \dots + 3600] = 3333.33$$

$$SE(\hat{t}_\psi) = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^n \left[ \frac{\hat{t}_i}{\psi_i} - \hat{t}_\psi \right]^2} = \frac{1}{\sqrt{n}} \left[ \text{Stdev} \left( \frac{\hat{t}_i}{\psi_i} \right) \right] = \frac{935.24}{\sqrt{6}} = 381.81$$