Handout 8 Exercise Questions

Question 1

It is given that $1 + i_t$ follows an independent lognormal distribution with $\mu = 0.04$ and $\sigma^2 = 0.02$ for $t = 1, 2, 3$, and with $\mu = 0.03$ and $\sigma^2 = 0.03$ for $t = 4, 5$. Furthermore, i_t's are independent. Find the mean and variance of (a) the FV at $t = 5$ of 1 invested at $t = 0$ and (b) the PV of 1 payable at $t = 5$

Question 2

Given that $1 + i_t$ follows an independent lognormal distribution with $\mu = 0.05$ and $\sigma^2 = 0.005$ for $t = 1, 2, \ldots$, find the 95% confidence interval of the PV of 3 payable at the end of 4 years.

Question 3

An individual purchases 100,000 nominal of a bond on 1 January 2003 which is redeemable at 105% in 4 years time and pays coupons of 4% per annum at the end of each year. The investment manager wishes to invest the coupon payments on deposit until the bond is redeemed. It is assumed that the rate of interest at which the coupon payments can be invested is a random variable and the rate of interest in any one year is independent of that in any other years. Calculate the mean value of the total accumulated investment on 31 December 2006 if the annual effective rate of interest has an expected value of 5.5% in 2004, 6% in 2005 and 4.5% in 2006.

Question 4

A lump sum of 10,000 will be invested at time 0 for 5 years at a constant annual rate of interest rate i. $(1+i)$ has a lognormal distribution with mean 1.05 and variance 0.007. What is the probability that the investment will be accumulated to more than 15,000 in 5 years' time?

Question 5

A fund manager is interested in the probability that current $700,000 will be sufficient to pay out $1 million at the end of 6 years. She assumes that annual growth factors (i.e., $1+i$) for each future year are lognormal random variables where the mean and standard deviation of the annual rate of return are 6% and 20%, respectively. Calculate the probability under the assumption that the growth rates for each year are independent.

Question 6

$10,000, $20,000, and $15,000 are invested at the beginning of the first year, the second year, and the third year, respectively. The investment yields interest at the end of each year, and interest is always reinvested. The annual effective rate of interest in each of the 3 years is expected to be:

- During the first year, one of 4%, 6%, or 8% with equal probability.
- During the second year, either 8% with probability 0.75 or 4% with probability 0.25.
- During the third year, either 8% with probability 0.6 or 4% probability 0.4.

The rate of interest applicable in any one year is independent of the rate applicable in any other year.

(a) Calculate the expected accumulated amount of the investment at the end of three years.
(b) Calculate the standard deviation of the accumulated amount of the investment at the end of three years.
(c) Calculate the probability that the accumulated amount of the investment is more than $51,000 at the end of three years.

Question 7

You are thinking of investing $10,000 in a fund and holding for 15 years. The yield on the investment in any year is expected to be 4% with probability 0.2, 7% with probability 0.6 and 10% with probability 0.2, and is independent of the yield in any other year.

(a) Calculate the mean accumulated value at the end of 15 years.

(b) Calculate the standard deviation of the accumulated value at the end of 15 years.

(c) Without carrying out any further calculations:

    (i) explain how your answers in parts (a) and (b) are changed (if at all) if the yields had been 6%, 7% and 8% instead of 4%, 7%, and 10% per annum, respectively.

    (ii) discuss whether *time diversification effect* under the assumption of independent yields reduces the risk of the investment if it is held for 30 years instead of 15 years.

(d) Calculate the probability that the accumulated value will be less than 15,500 at the end of 5 years.