Solution for Question 5

(a)

The DMT (in years) is

$$= \frac{\sum tC_t v^t}{\sum C_t v^t} = \frac{6(Ia)_{\overline{10}|4.2\%} + 10 \times 100v^{10}}{6a_{\overline{10}|4.2\%} + 100v^{10}} = \frac{6 \times 41.451 + 1000 \times 0.66271}{6 \times 8.03074 + 100 \times 0.66271} = 7.963$$

(b)

The effective duration is calculated by 7.963/1.042 = 7.642, suggesting that the value of the bond would change, on average, by 7.642% when interest rate changes by 1%. That is, the measure indicates the interest rate risk to which the investor is exposed if the price of the bond falls before it is sold.

Solution for Question 6

(a)

PV of the liabilities in million is:

$$a_{\overline{40}|4\%} = \frac{1 - 1.04^{-40}}{0.04} = 19.7928$$

Since

$$(Ia)_{\overline{40}|} = \frac{\ddot{a}_{\overline{40}|} - 40 \times 1.04^{-40}}{0.04} = 306.32008$$

The duration is

$$\frac{(Ia)_{\overline{40}|}}{PV} = \frac{306.32008}{19.7928} = 15.48 \text{ years}$$

(b)

Since the PV(assets)=PV(liabilities) and the half of PV(assets) is invested to each bond, the PV of each bond is:

$$19.7928 \times 0.5 = 9.896387\ m$$

PV(5yr bond) of 100 nominal

$$10\left(\frac{1 - 1.04^{-5}}{0.04}\right) + \frac{100}{1.04^5} = 126.71093$$

PV(10yr bond) of 100 nominal

$$5\left(\frac{1 - 1.04^{-10}}{0.04}\right) + \frac{100}{1.04^{10}} = 108.1109$$

So the \$ nominal of the 5-year bond is

$$\frac{9{,}896{,}387}{126.7109} \times 100 = 7{,}810{,}208$$

The \$ nominal of the 10-year bond is

$$\frac{9{,}896{,}387}{108.1109} \times 100 = 9{,}153{,}922$$

(c)
Numerator of duration of the assets is defined by:

$$\left(10(Ia)_{\overline{5}|} + 5 \times 100v^5\right) \times 78{,}102.076 + \left(5(Ia)_{\overline{10}|} + 10 \times 100v^{10}\right) \times 91{,}539.219$$

$$= \left(10 \times 13.0065 + \frac{500}{1.04^5}\right) \times 78{,}102.076 + \left(5 \times 41.9922 + \frac{1000}{1.04^{10}}\right) \times 91{,}539.219$$

$$= 42.2554m + 810.603m$$

$$= 123.3157m$$

Duration is

$$\frac{123.3157m}{19.7928m} = 6.23 \text{ years}$$