Tutorial 4

Question 1

Consider a MLR model with two predictors as follows,

Y	X_1	X_2
y_1	x_{11}	x_{12}
y_2	x_{21}	x_{22}
$\vdots$	$\vdots$	$\vdots$
y_n	x_{n1}	x_{n2}

Please derive the detailed form for $\mathbf{X'X}$, $\mathbf{X'Y}$ and $\mathbf{Y'Y}$.

Question 2

Consider a multiple linear regression (MLR) model with one response (Y) and two predictors $(X_1$ and $X_2)$, $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$, where $\epsilon \sim N(0, \sigma^2)$. Given that

$$\mathbf{X'X} = \begin{pmatrix} 25 & 1315 & 506 \\ 1315 & 76323.42 & 26353.3 \\ 506 & 26353.3 & 10460 \end{pmatrix} \quad \mathbf{X'Y} = \begin{pmatrix} 235.6 \\ 11821.432 \\ 4831.86 \end{pmatrix}$$

$$(\mathbf{X'X})^{-1} = \begin{pmatrix} 2.779 & -0.0112 & -0.106 \\ -0.0112 & 0.146 \times 10^{-3} & 0.175 \times 10^{-3} \\ -0.106 & 0.175 \times 10^{-3} & 0.479 \times 10^{-2} \end{pmatrix}$$

$$\mathbf{Y'Y} = 2784.11$$

Calculate the values of S_{yy}, SSE, $\widehat{\beta}$ and s^2. Why?

Question 3

Write down the $\mathbf{X}$ matrix and β vector for each of the following (transformed) regression model.

- $y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i1} x_{i2} + \epsilon_i, \quad i = 1, \cdots, 5;$

- $\sqrt{y}_i = \beta_0 + \beta_1 x_{i1} + \beta_2 \log x_{i2} + \epsilon_i, \quad i = 1, \cdots, 5.$