

# Solution to T8

## Question 1

Consider a two-way classification model as follows,

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}.$$

For each cell, we calculate the sample size  $n_i$ , sample mean  $\bar{y}_i$  and the SS  $\sum (y_{ij} - \bar{y}_i)^2$ . Put them into  $(n_i, \bar{y}_i, \sum (y_{ij} - \bar{y}_i)^2)$ , and then form the following table.

A	B	1	2	3
1		(4,35.25,1076.75)	(4,52.25,604.75)	(4,71.5,581)
2		(4,31.5,345)	(4,50.75,726.75)	(4,61.75,160.75)

Please derive the ANOVA table.

**Solution:** First it is easily seen that

$$\bar{y}_{...} = \frac{4 \times (35.25 + 52.25 + 71.5 + 31.5 + 50.75 + 61.75)}{24} = 50.5,$$

$$SSE = 1076.75 + 604.75 + 581 + 345 + 726.75 + 160.75 = 3495,$$

$$y_{1..} = 4 \times (35.25 + 52.25 + 71.5) = 636,$$

$$y_{2..} = 4 \times (31.5 + 50.75 + 61.75) = 576.$$

It follows that

$$c.f. = 2 \times 3 \times 4 \times (50.5)^2 = 61206, \quad SS_A = \sum_{i=1}^2 \frac{y_{i..}^2}{3 \times 4} - c.f. = 61356 - 61206 = 150.$$

Similarly,

$$y_{.1.} = 4 \times (31.5 + 35.25) = 267, \quad y_{.2.} = 4 \times (52.25 + 50.75) = 412,$$

$$y_{.3} = 4 \times (71.5 + 61.75) = 533.$$

It follows that

$$SS_B = \frac{\sum_{j=1}^3 y_{.j}^2}{2 \times 4} - c.f. = 65640.25 - 61206 = 4434.25.$$

Moreover,

$$\sum_{i=1}^2 \sum_{j=1}^3 y_{ij}^2 = (4 \times 35.25)^2 + (4 \times 52.25)^2 + (4 \times 71.5)^2 + (4 \times 31.5)^2 + (4 \times 50.75)^2 + (4 \times 61.75)^2.$$

Therefore

$$\begin{aligned} SS_{AB} &= \sum_{i=1}^2 \sum_{j=1}^3 y_{ij}^2 / 4 - SS_A - SS_B - c.f. \\ &= 65863 - 150 - 4434.25 - 61206 = 72.75. \end{aligned}$$

Based on the above we obtain the ANOVA table

Source	SS	df	MS	F
Main effect of A	150	1	$MS_A=150$	$F_A = 0.77$
Main effect of B	4434.25	2	$MS_B=2217$	$F_B = 11.41$
Interaction	72.75	2	$MS_{AB}=36.37$	$F_{AB} = 0.18$
Error	3495	18	MSE=194.17	
Total	81521	23		

## Question 2

Construct a test to check

$$H_0 : \alpha_1 = \alpha_2 = 0.$$

Conclusion about the above results.

**Solution:** From the above ANOVA table we see that the p-value is

$$P(F_{2,18} > 0.18) = 0.8533.$$

This implies that we have strong evidence that there are no interactions between two factors. Thus, testing for main effects is then meaningful. We find the p-value of the main effect of A is

$$P(F_{1,18} > 0.77) = 0.3918.$$

Therefore we can not reject  $H_0$ .