Spectral Element Libraries in Fortran (SELF)

Barotropic, Geostrophic Circulation

Steady State Solver

Reference Manual

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Contents

1 Equations and Discretization

The dynamics of the large scale oceanic circulation on earth are largely contained within the hydrostatic primitive equations. In these equations, the vertical momentum balance is approximated by hyrdostatic balance - an approximation that is justified by the use of the "thin shell" approximation, where the fluid depth is far smaller than the lateral length scales of motion. For a constant density fluid, the hydrostatic primitive equations can be reduced to the shallow water equations, so long as the effects of thin boundary layers are parameterized. For sufficiently slow moving flows, the effects of gravity waves can be approximated as instantaneous adjustments of the fluid surface that act to keep the fluid transport divergence free; this is known as the rigid lid assumption. Steady state, linear flows subject to all of the previous assumptions are now governed by two equations :

$$f\hat{z} \times \vec{u} = \nabla P + \frac{\vec{\tau}}{H} - \frac{C_d \vec{u}}{H} \tag{1.1a}$$

$$\nabla \cdot (H\vec{u}) = 0. \tag{1.1b}$$

In Eqs. (1.1), f is the coriolis parameter, $\vec{u} = u\hat{x} + v\hat{y}$ is the lateral velocity field, P is the barotropic fluid pressure, $\vec{\tau}$ is the stress imposed on the fluid at the surface, $C_d\vec{u}$ is a parameterization of the stress felt at the sea-floor, C_d is a drag coefficient, and H is the fluid depth.

Eq. (1.1b) is satisfied exactly if the transport is written in terms of a stream function

$$H\vec{u} = \hat{z} \times \nabla\Psi. \tag{1.2}$$

Further, the equation set (1.1) can be reduced to a single equation for the stream function by taking the curl of (1.1a) and substituting in (1.2). Doing so gives

$$\nabla \cdot \left(\frac{C_d \nabla\Psi}{H^2} \right) + \nabla\Psi \times \nabla \left(\frac{f}{H} \right) = \nabla \times \left(\frac{\vec{\tau}}{H} \right) \tag{1.3}$$

1.1 Nodal Continuous Galerkin Discretization

A Spectral Element Primer

A.1 Vector Spaces

A.2 Spectral Approximations and Error Analysis

B Differential Geometry