Real Time Series analysis and modelling

Aid machine learning with dynamical insight

Hailiang Du
Department of Mathematical Sciences, Durham University
hailiang.du@durham.ac.uk
Data Science Institute, London School of Economics and Political Science
h.l.du@lse.ac.uk

All theorems are true, All models are wrong. All data are inaccurate. What are we to do?

The aim of this course is to teach you how to deal with real data, to increase your scepticism regarding reliable modelling in practice, and to expand the tool box you carry to include nonlinear techniques, both deterministic and stochastic with the aid of dynamical insight.

In short: to get you to think before you compute (and perhaps afterwards too.)

Lecture9

Summary, discussion and criticise aspects across the board

What did you take away from the past two days?

- A more general, geometric, view of the analysis of time series.

- An expanded toolkit for analysing data from nonlinear systems, nonlinear models of actual systems, actual observations and forecasts.

- A focus on information content and model inadequacy.

- A better understanding of
  - deterministic nonlinear dynamical system, its descriptive statistics, and relevance to real-world modelling.
  - the implications nonlinearity holds for statistical analysis, model building and forecasting of actual systems.
  - the interpretation and evaluation of probabilistic forecasts
  - the different beliefs as to what a "model" is and should be.

be more vigilant!

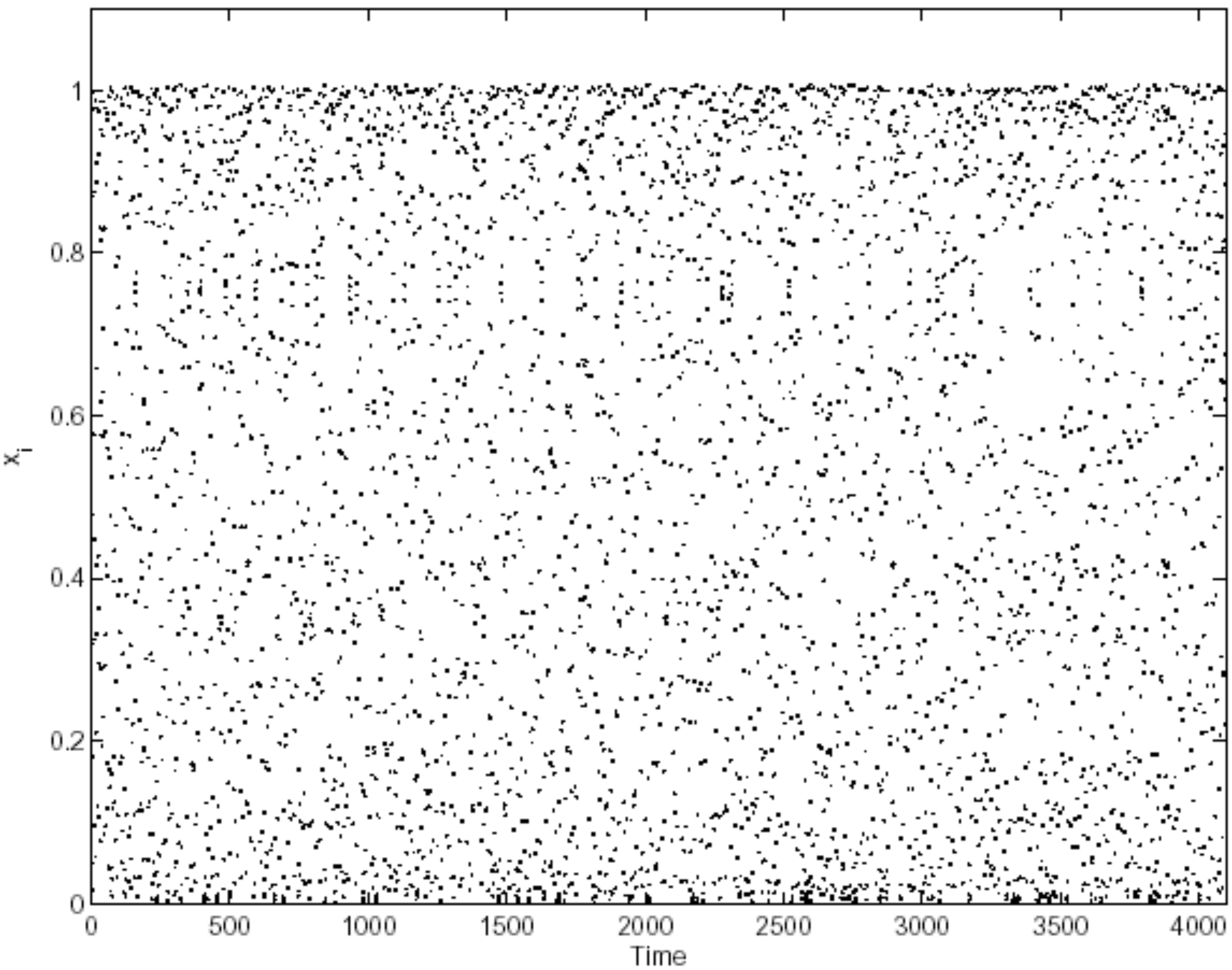
Defining Chaos

Deterministic dynamics

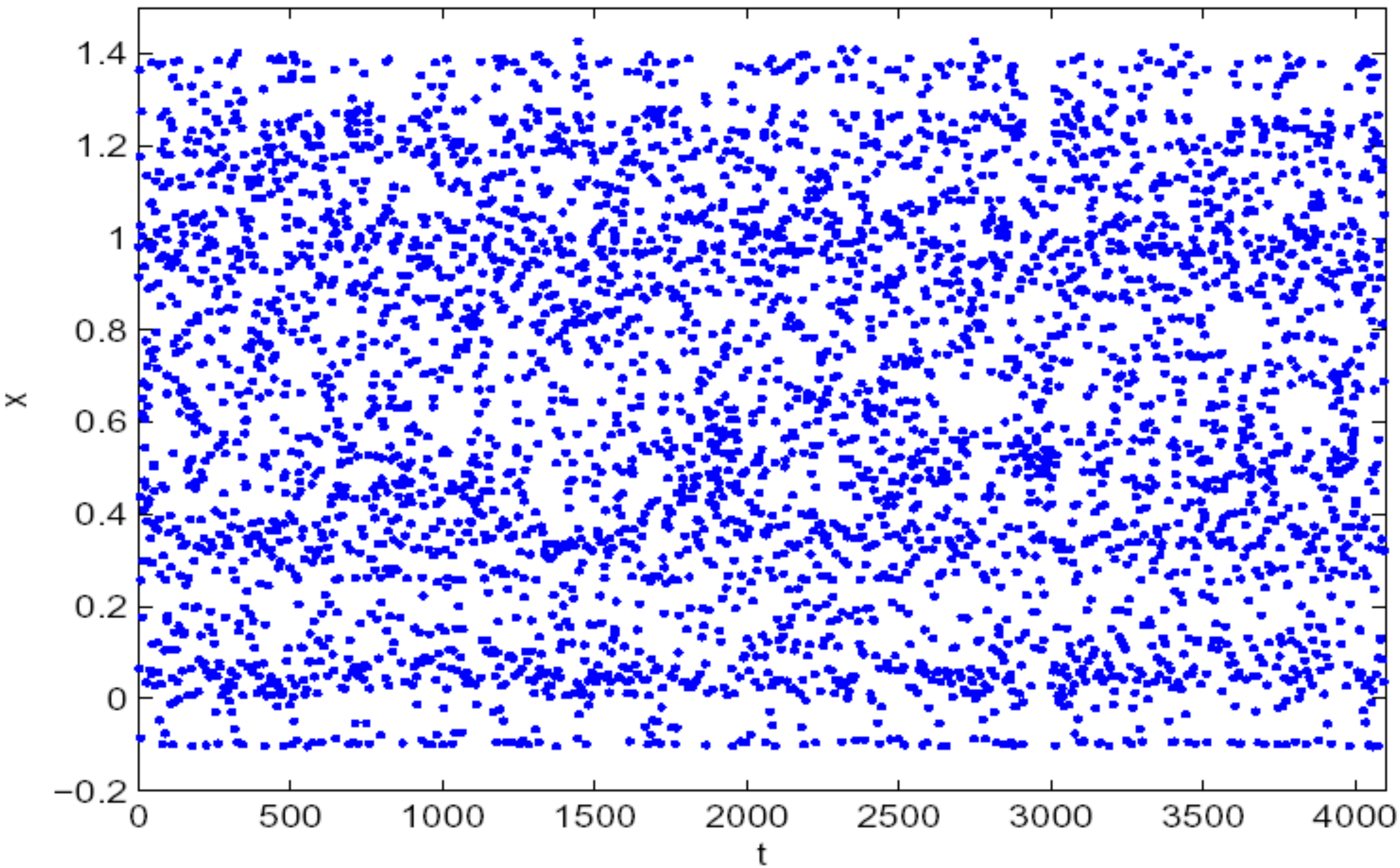
Sensitive Dependence on Initial Condition

Recurrent Dynamics

Chaotic systems, look randomly, in fact are all deterministic. If we know the system and the initial condition, the future is certain.



Random?



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Random?

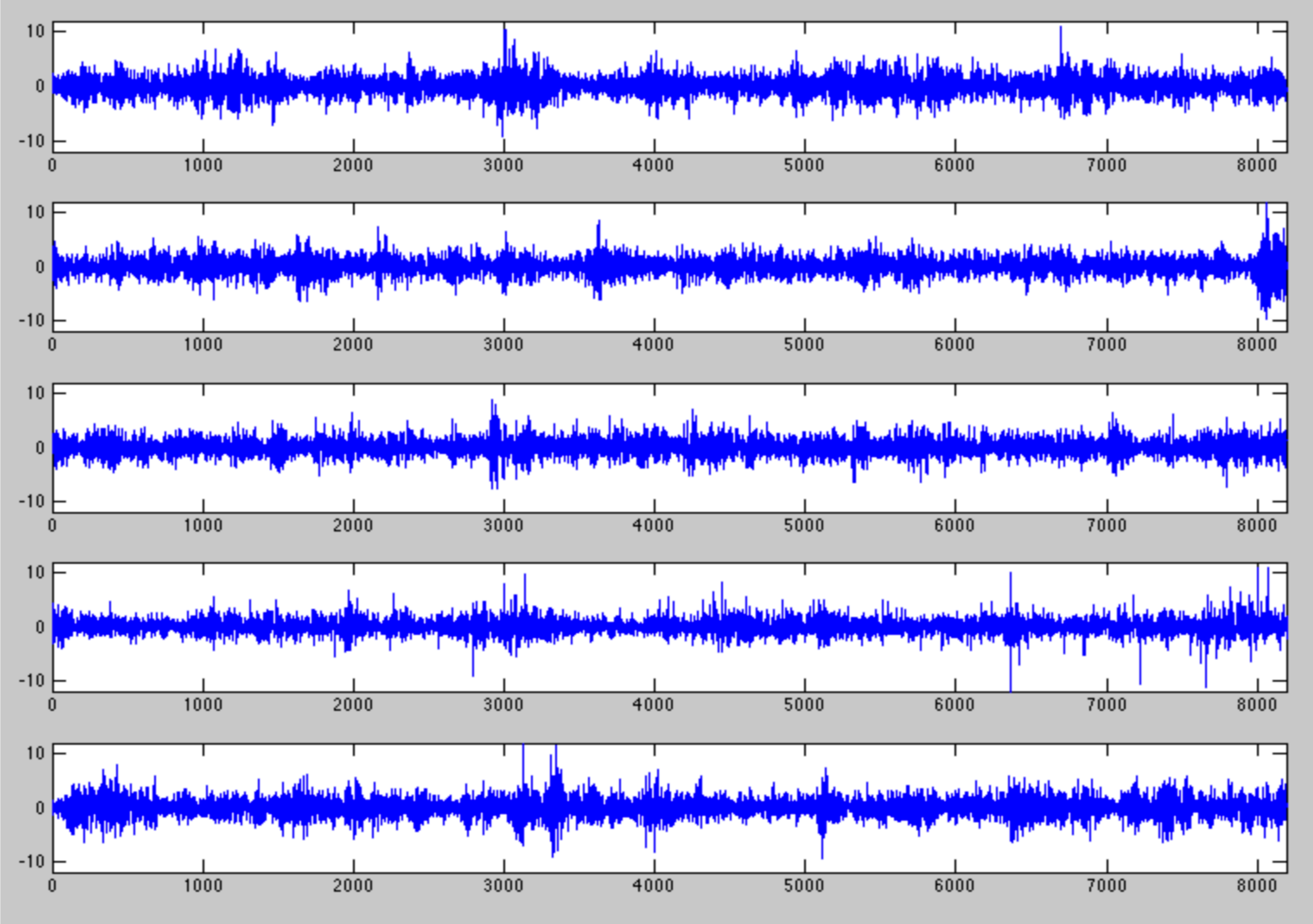
Classical example of Chaos: Double Pendulum

“Random”?

“Unpredictable”?

Aperiodic/Nonperiodic

Which one is different?



Example 4.2. To illustrate the application of TAR models in finance, we consider the daily log returns, in percentages and including dividends, of IBM stock from July 3, 1962 to December 31, 1999 for 9442 observations. Figure 4.2 shows the time plot of the series. The volatility seems to be larger in recent years. If GARCH

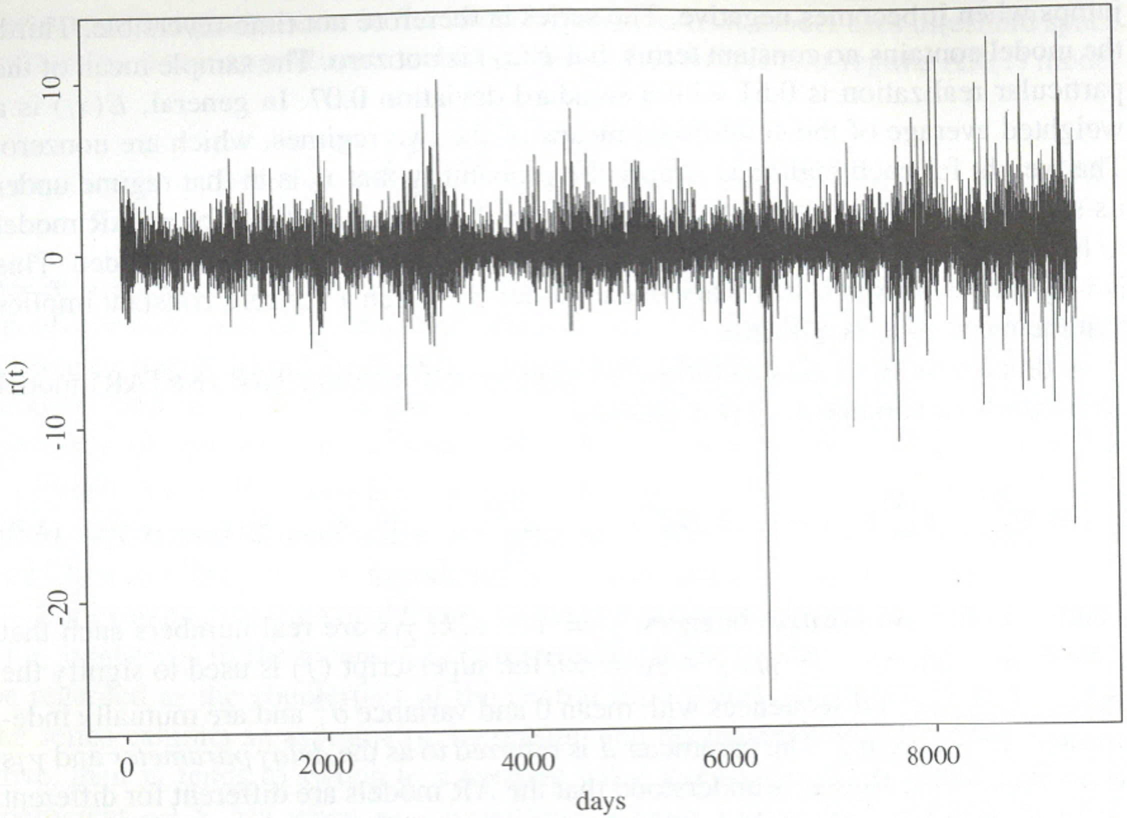


Figure 4.2. Time plot of the daily log returns for IBM stock from July 3, 1962 to December 31, 1999.

Analysis of Financial Time Series by Ruey S. Tsay, Wiley, 2002

$$r_t = 0.043 - 0.022r_{t-2} + a_t, \quad a_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = 0.002 + 0.097a_{t-1}^2 + 0.954\sigma_{t-1}^2$$
$$+ (0.056 - 0.051a_{t-1}^2 - 0.067\sigma_{t-1}^2)I(a_{t-1} > 0),$$

where $I(a_{t-1}) = 1$ if $a_{t-1} > 0$ and it is zero otherwise. Because the estimate 0.002 of the volatility equation is insignificant at the 5% level, we further refine the model to

Is it possible to prove the residuals are IID?

$$r_t = 0.043 - 0.022r_{t-2} + a_t, \quad a_t = \sigma_t \epsilon_t$$

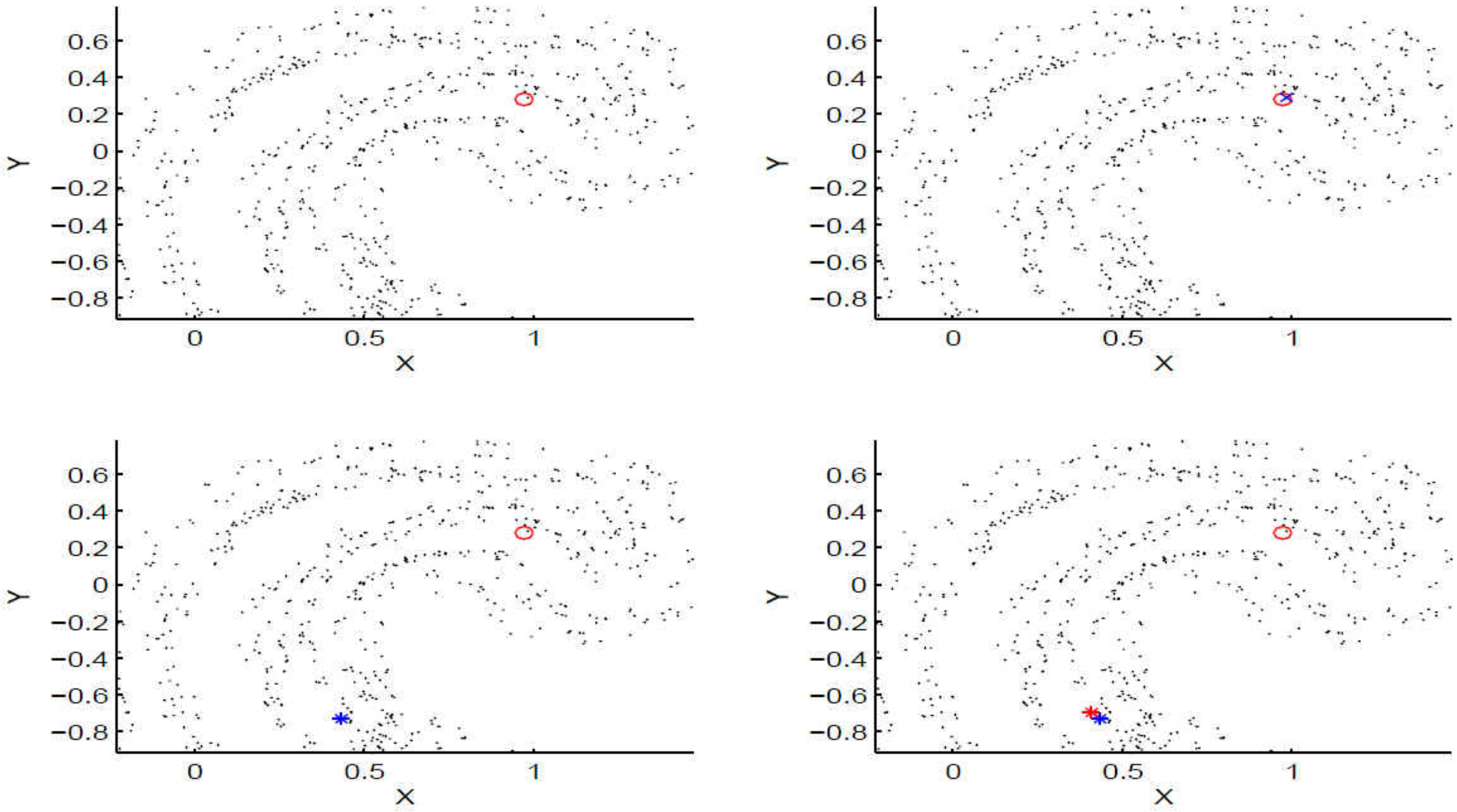
$$\sigma_t^2 = 0.098a_{t-1}^2 + 0.954\sigma_{t-1}^2$$
$$+ (0.060 - 0.052a_{t-1}^2 - 0.069\sigma_{t-1}^2)I(a_{t-1} > 0), \tag{4.11}$$

where the standard errors of the two parameters in the mean equation are 0.013 and 0.010, respectively, and those of the TAR-GARCH(1, 1) model are 0.003, 0.004, 0.005, 0.004, and 0.009. All of the estimates are statistically significant at the 5% level. The unconditional mean for r_t of model (4.11) is 0.042, which is very close to the sample mean of r_t. Residual analysis based on the Ljung–Box statistics finds no significant serial correlations or conditional heteroscedasticity in the standardized residuals. The AR coefficient in the mean equation is small, indicating that, as expected, the daily log returns of IBM stock are essentially serially uncorrelated. However, the volatility model of the returns shows strong dependence in the innovational process $\{a_t\}$ and evidence of asymmetry in the conditional variance. Rewriting the TAR-GARCH(1, 1) equation as

Are the residuals really random??

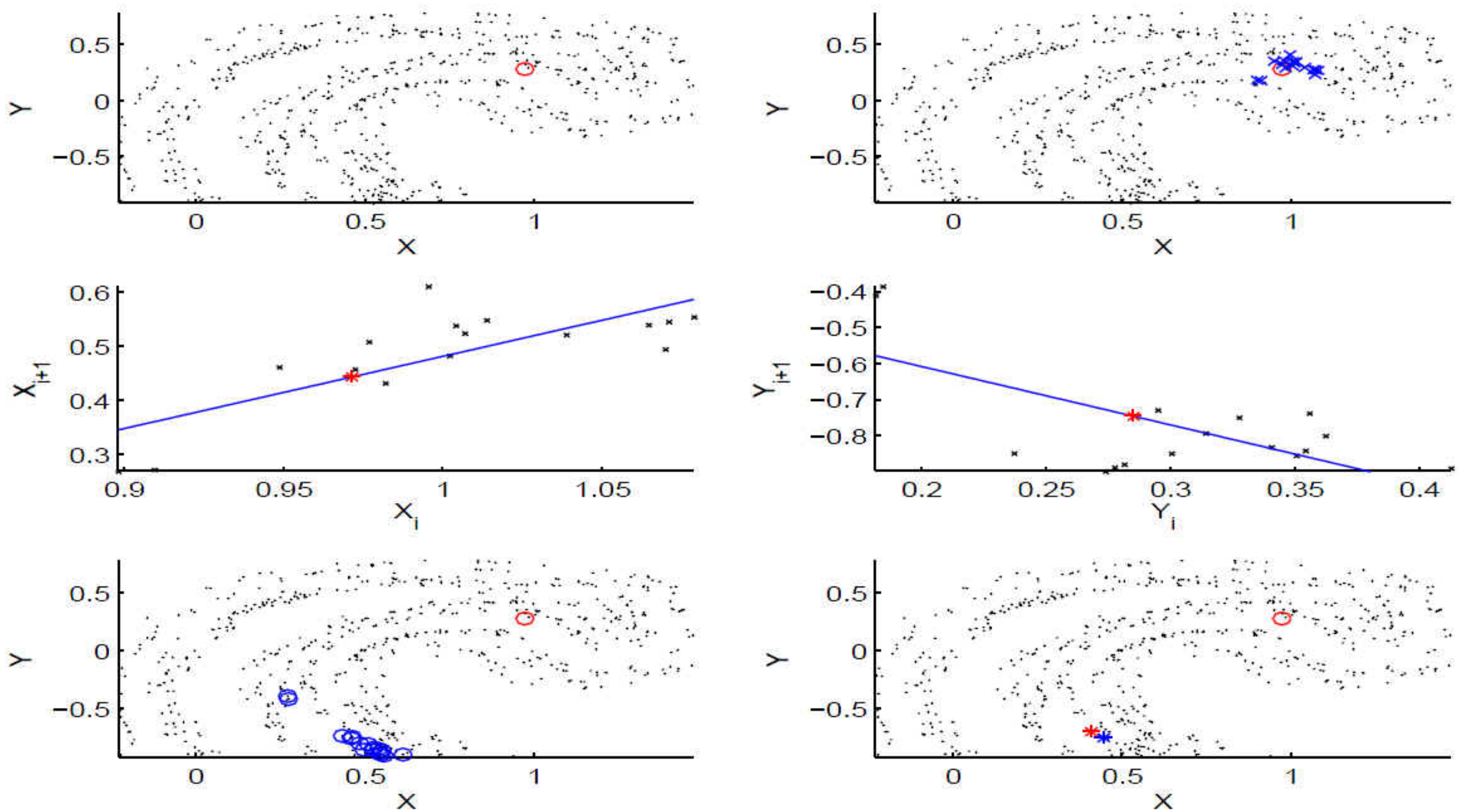
$$\sigma_t^2 = \begin{cases} 0.098a_{t-1}^2 + 0.954\sigma_{t-1}^2 & \text{if } a_{t-1} \le 0 \\ 0.060 + 0.046a_{t-1}^2 + 0.885\sigma_{t-1}^2 & \text{if } a_{t-1} > 0, \end{cases} \tag{4.12}$$

Local Analogue model

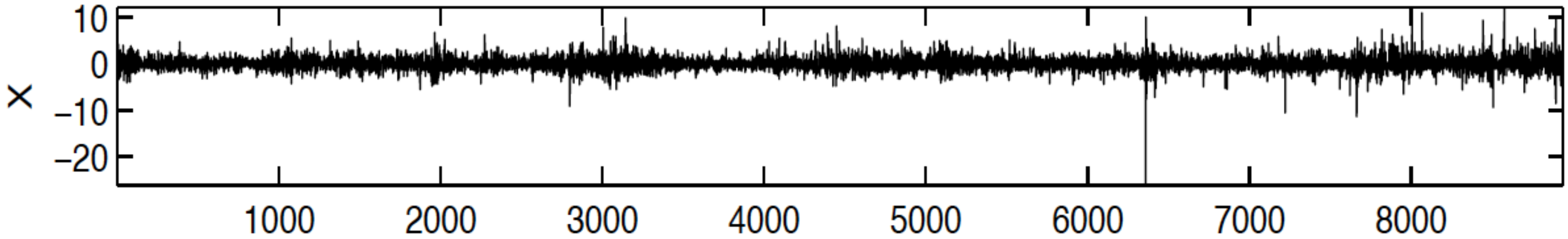


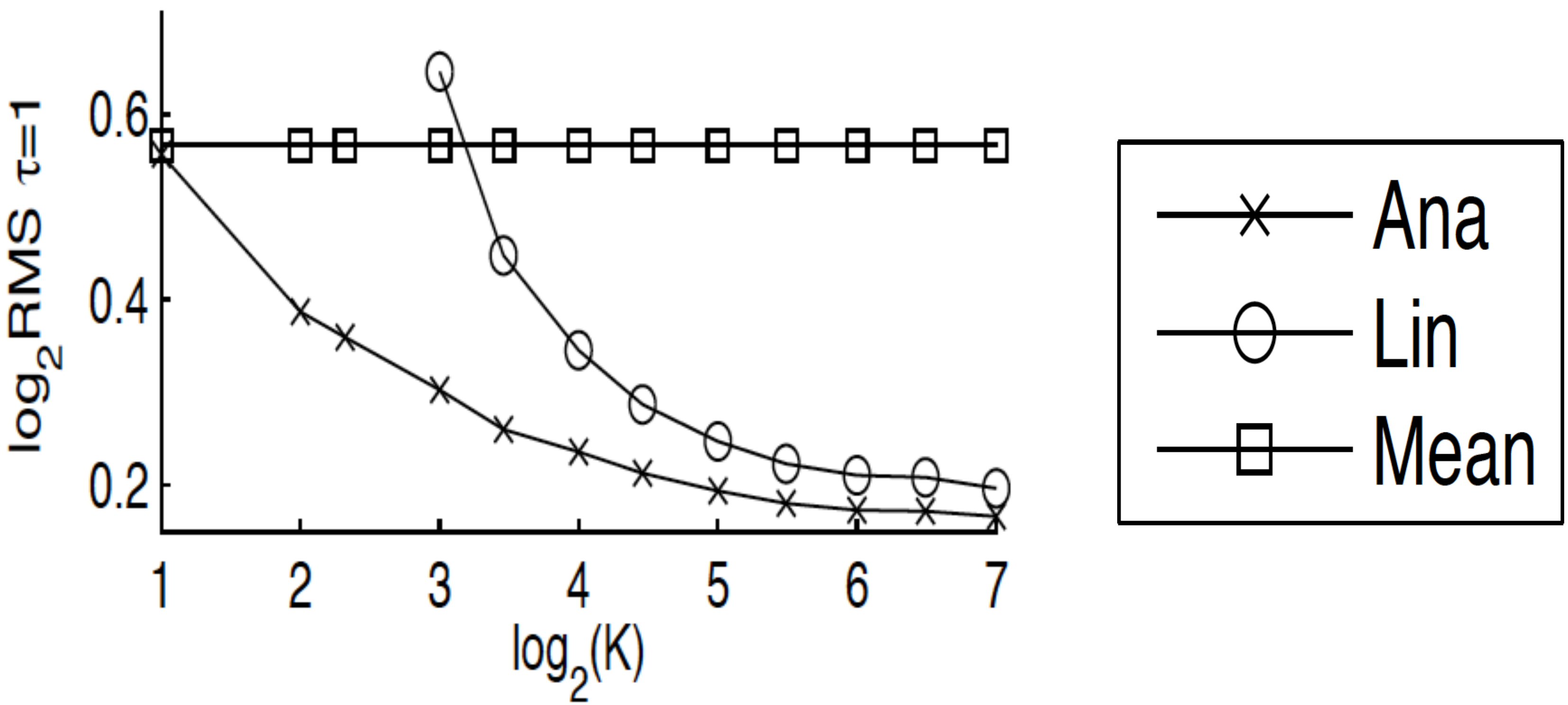
Local linear model

Given k points within a neighbourhood, a local linear predictor aims at the linear map with the smallest mean square error when interpolating the future observation.

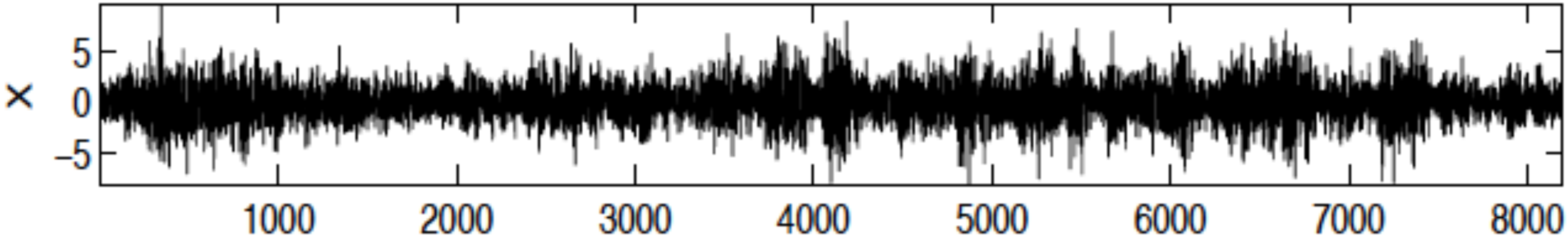


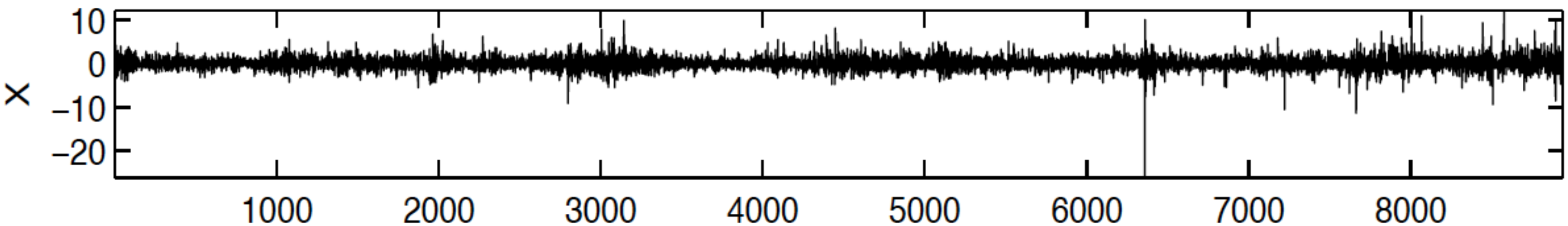
Modeling IBM Data

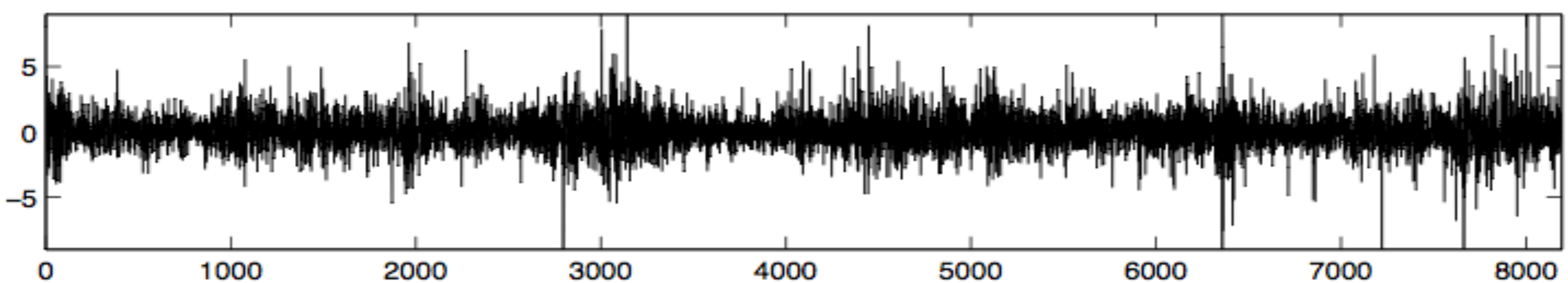




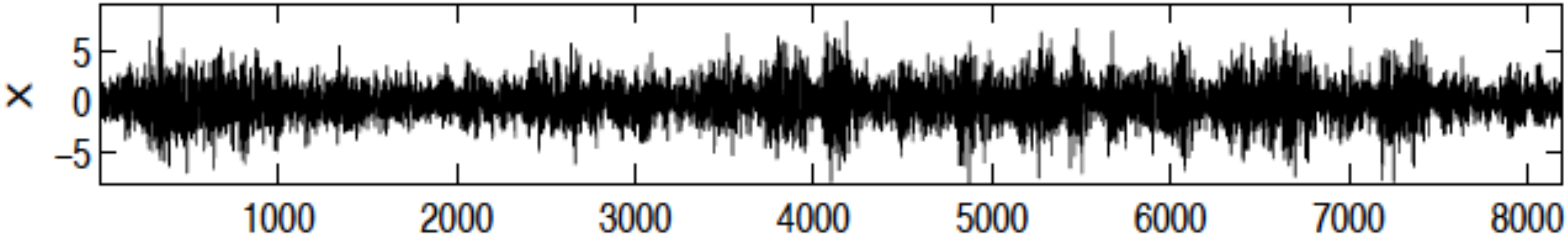
Modeling Data from T-GARCH

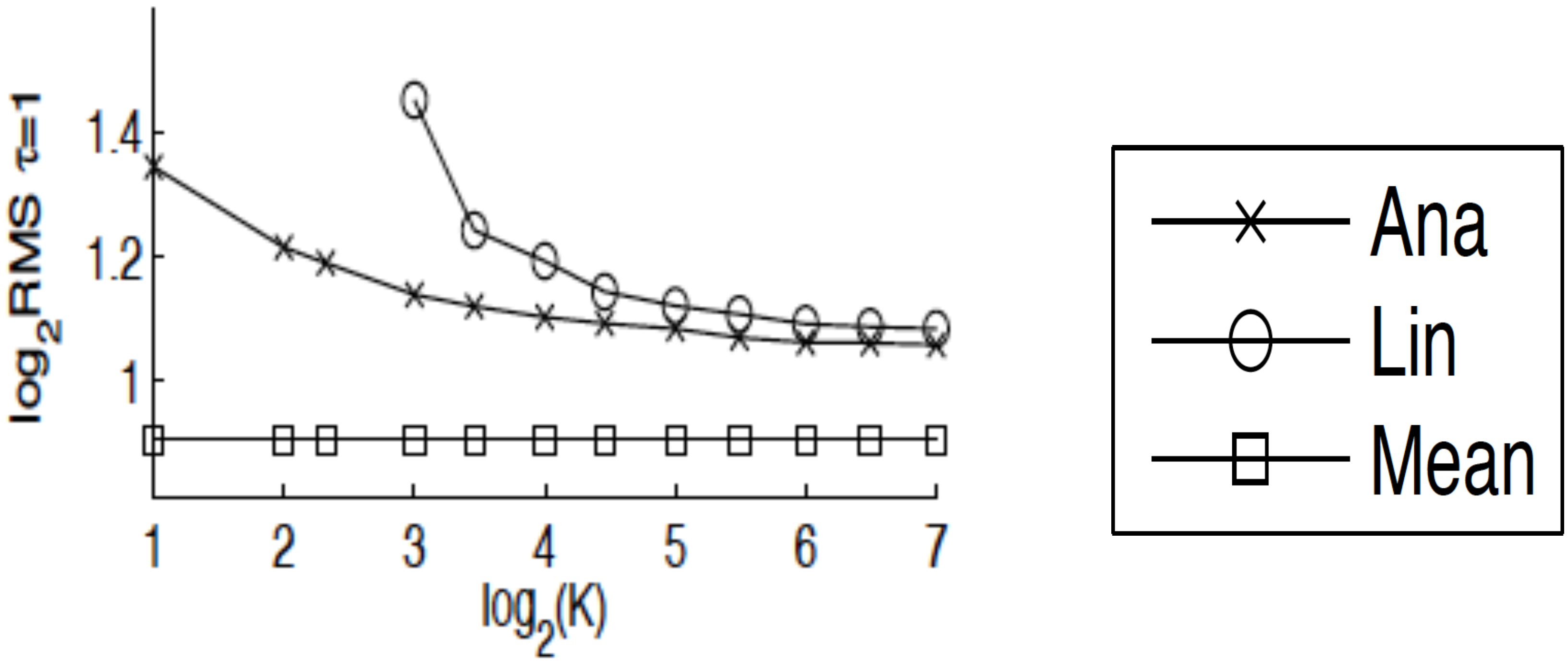






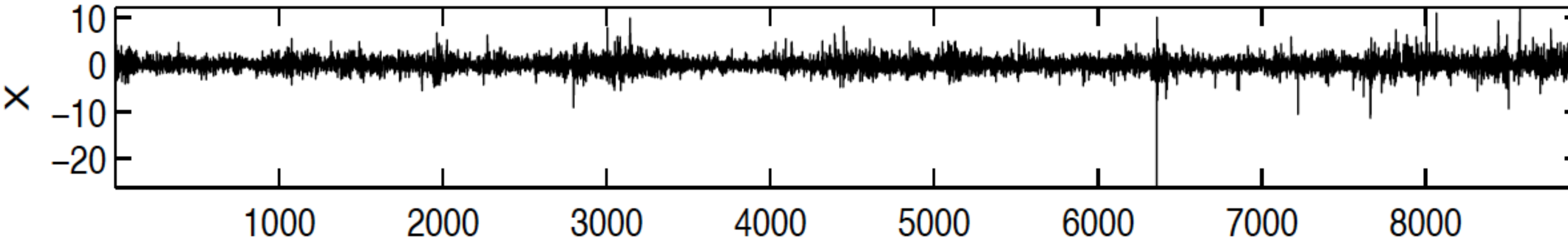
Modeling Data from T-GARCH

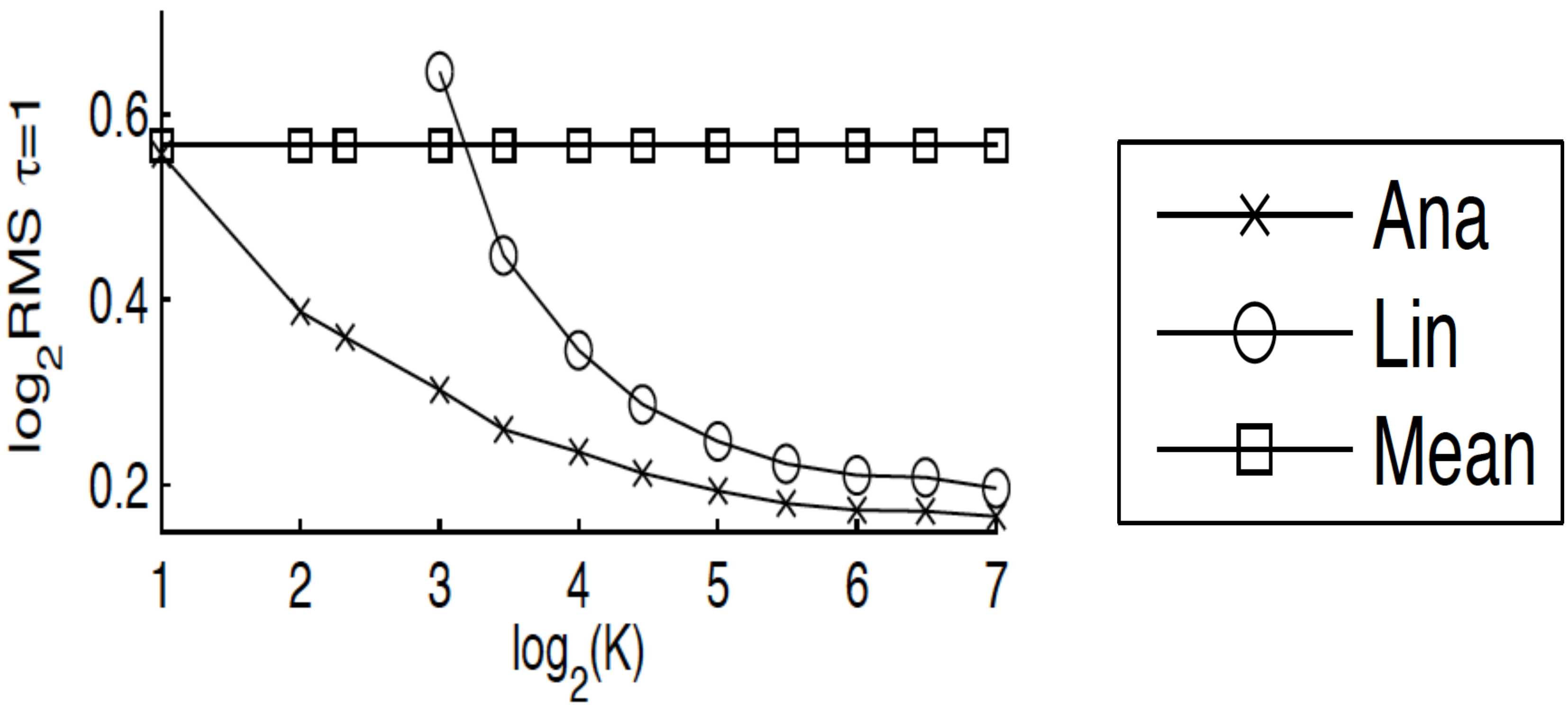




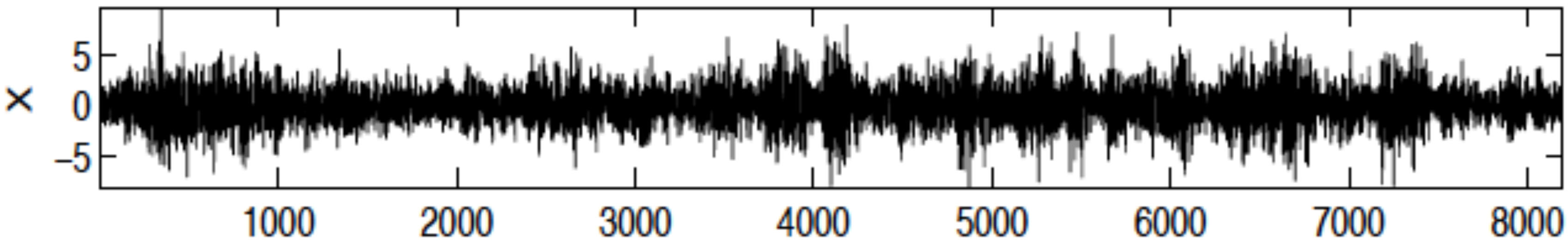
What does this tell us???

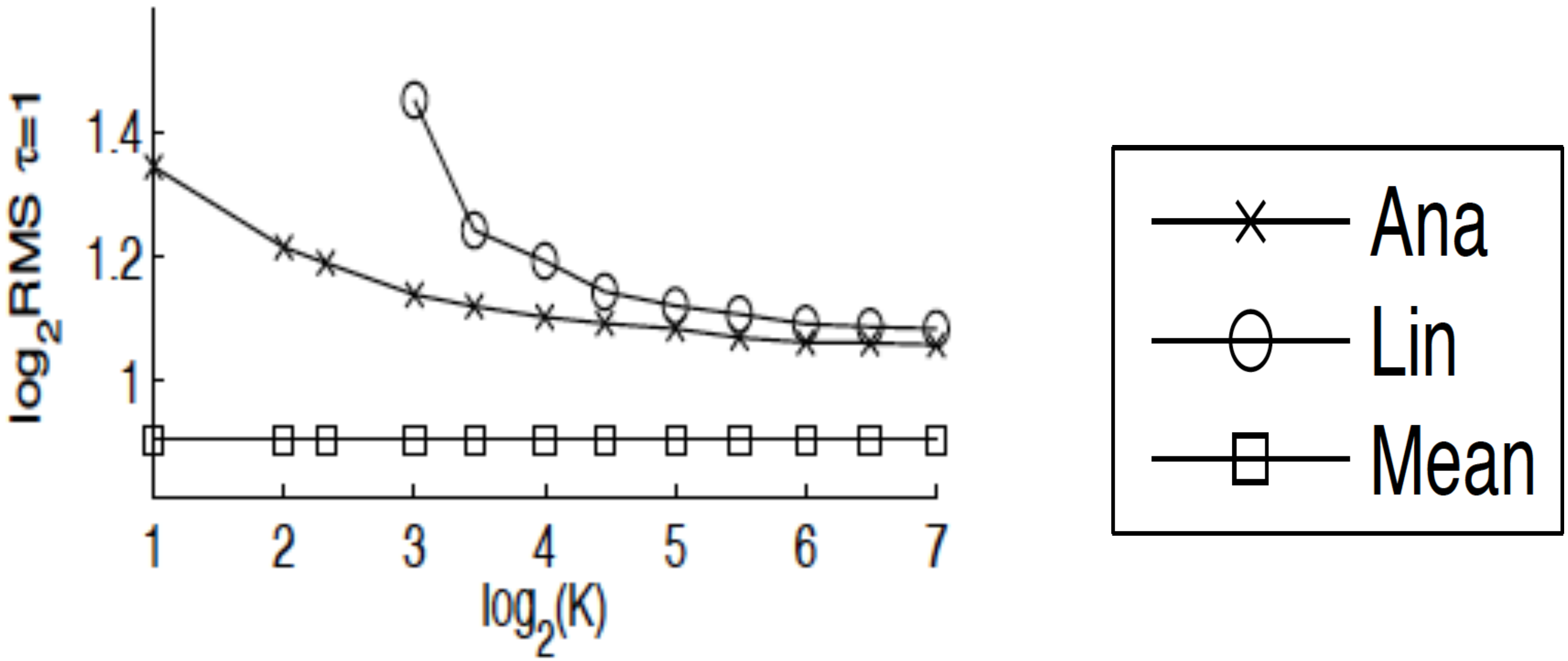
Modeling IBM Data





Modeling Data from T-GARCH





Are the residuals truly IID?

Information lost using T-GARCH model

$$r_t = 0.043 - 0.022 r_{t-2} + a_t, \quad a_t = \sigma_t \epsilon_t$$

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$$+ (0.056 - 0.051 a_{t-1}^2 - 0.067 \sigma_{t-1}^2) I(a_{t-1} > 0),$$

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$$\sigma_t^2 = 0.098 a_{t-1}^2 + 0.954 \sigma_{t-1}^2$$
$$+ (0.06$$

Recall: Boosting???

where the standard errors of

0.010, respectively, and those of the TAR-GARCH(1, 1) model are 0.003, 0.004, 0.005, 0.004, and 0.009. All of the estimates are statistically significant at the 5% level. The unconditional mean for r_t of model (4.11) is 0.042, which is very close to the sample mean of r_t. Residual analysis based on the Ljung–Box statistics finds no significant serial correlations or conditional heteroscedasticity in the standardized residuals. The AR coefficient in the mean equation is small, indicating that, as expected, the daily log returns of IBM stock are essentially serially uncorrelated. However, the volatility model of the returns shows strong dependence in the innovational process $\{a_t\}$ and evidence of asymmetry in the conditional variance. Rewriting the TAR-GARCH(1, 1) equation as

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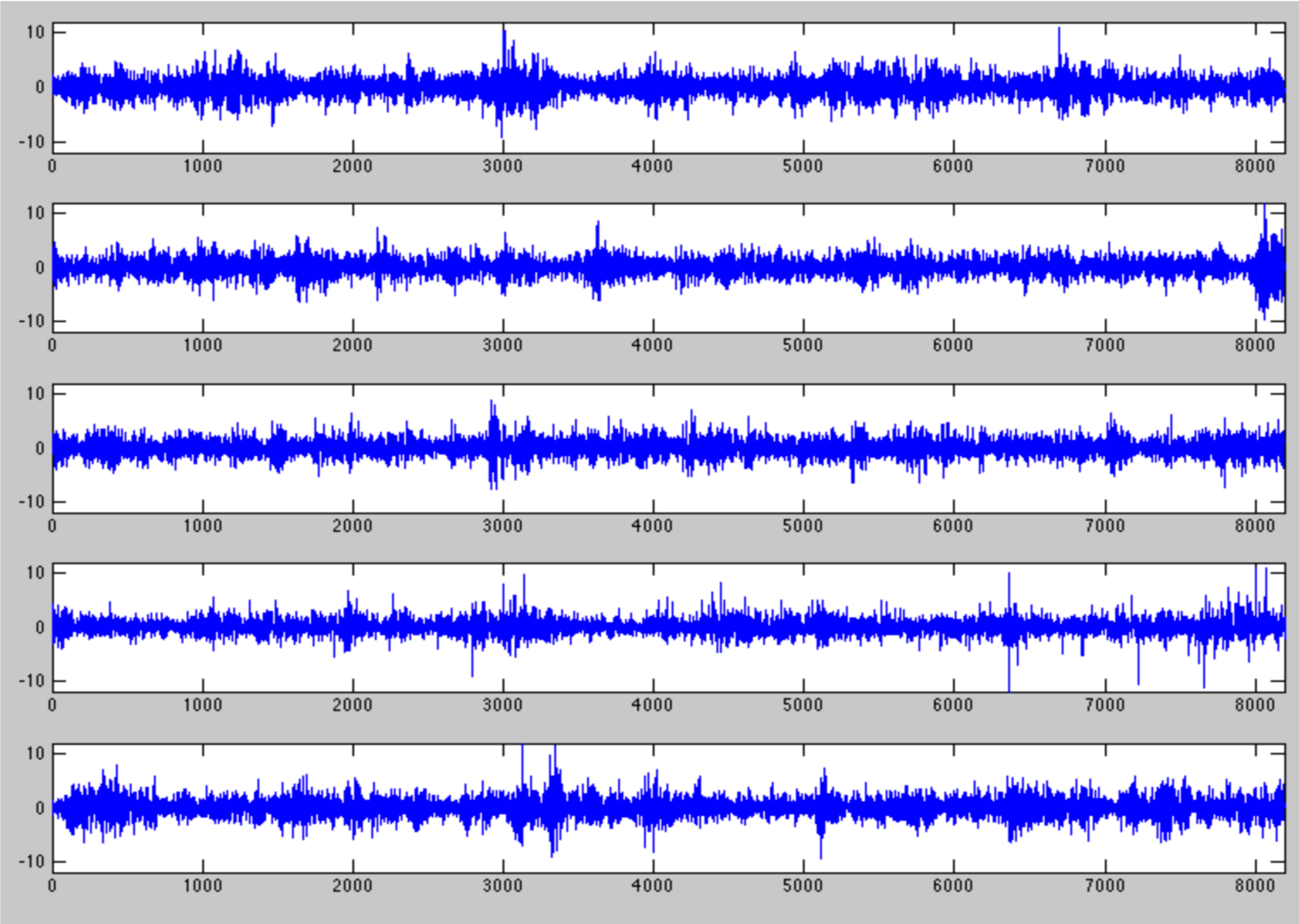
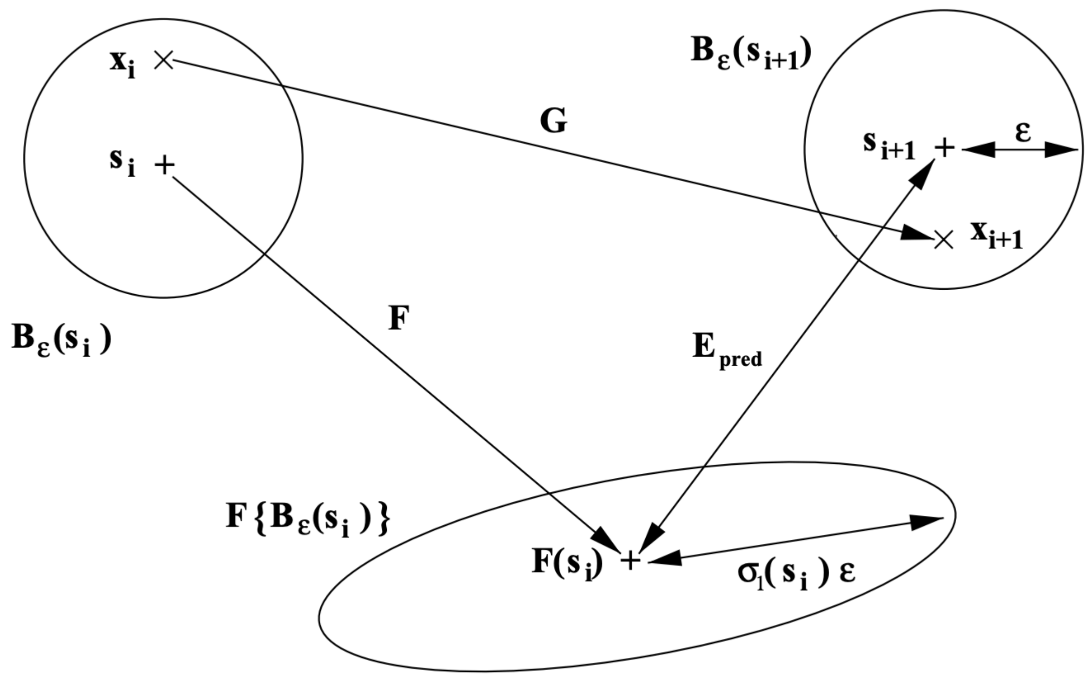
An example of Surrogate



Warning: Stay away from Financial Market

model discrepancy recognition

Consistency test



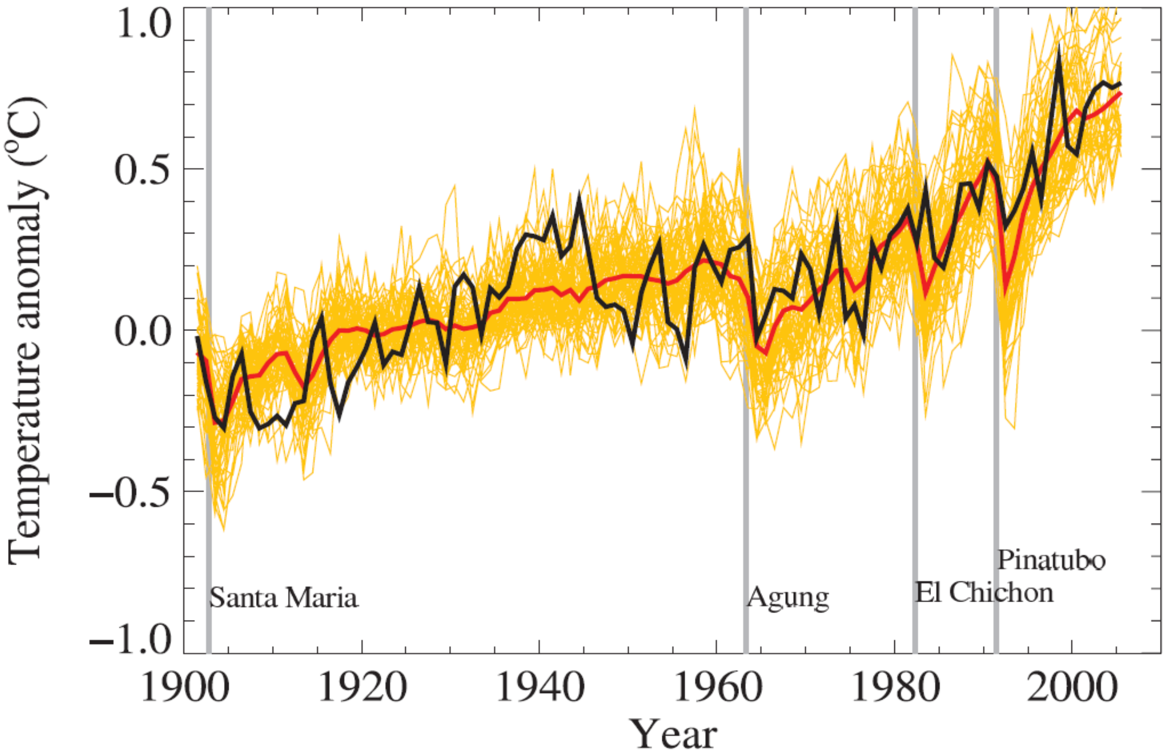
No model trajectory exists which is consistent with the observational uncertainty

McSharry, P.E. and Smith, L.A. (2004) 'Consistent nonlinear dynamics: identifying model inadequacy', Physica D, 192: 1-22.

Climate in Practice: In-sample examples.

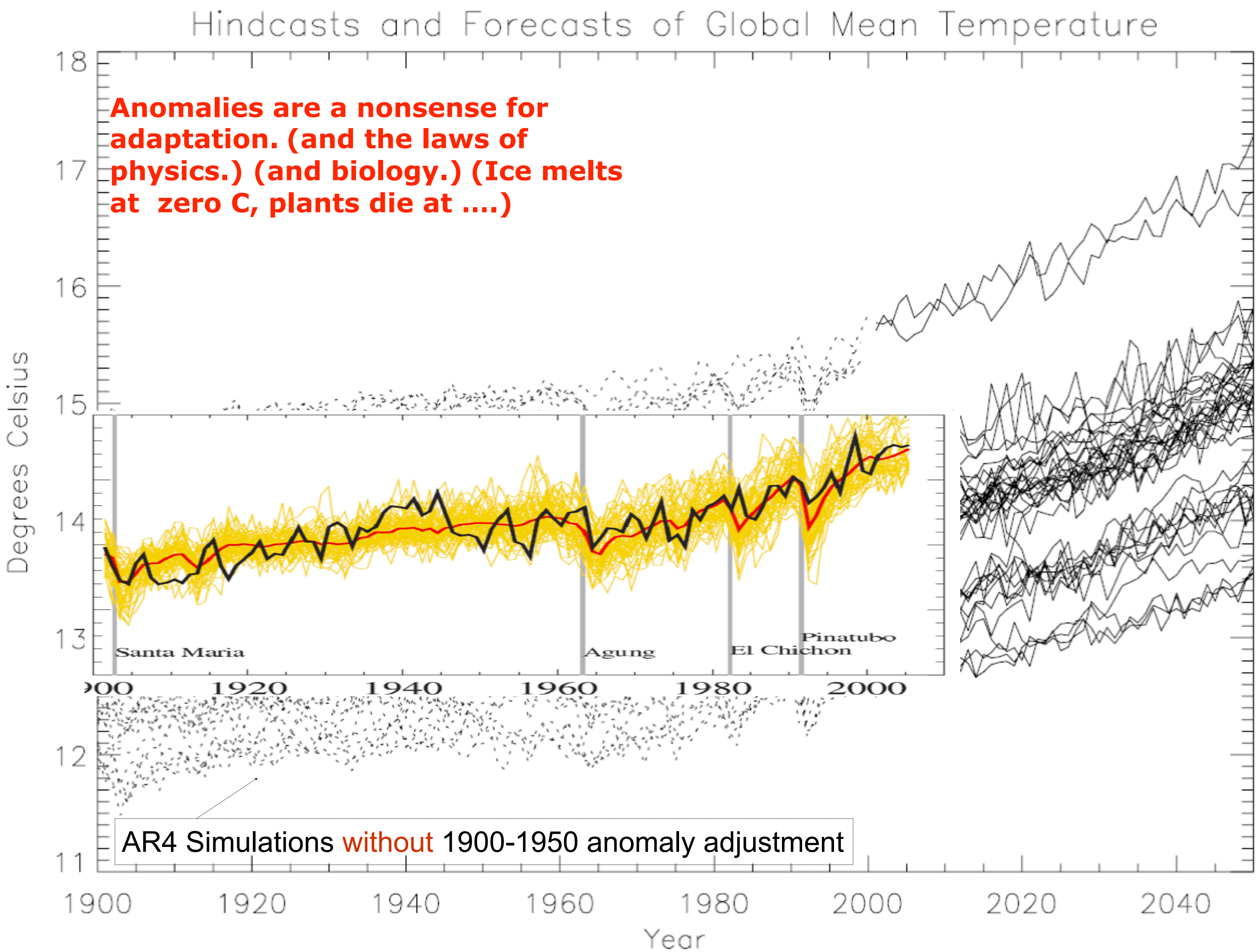
This graph tends to leave the impression GCMs do rather well.

FAQ 8.1, Figure 1. *Global mean near-surface temperatures over the 20th century from observations (black) and as obtained from 58 simulations produced by 14 different climate models driven by both natural and human-caused factors that influence climate (yellow). The mean of all these runs is also shown (thick red line). Temperature anomalies are shown relative to the 1901 to 1950 mean. Vertical grey lines indicate the timing of major volcanic eruptions. (Figure adapted from Chapter 9, Figure 9.5. Refer to corresponding caption for further details.)*



Climate in Practice: In-sample examples.

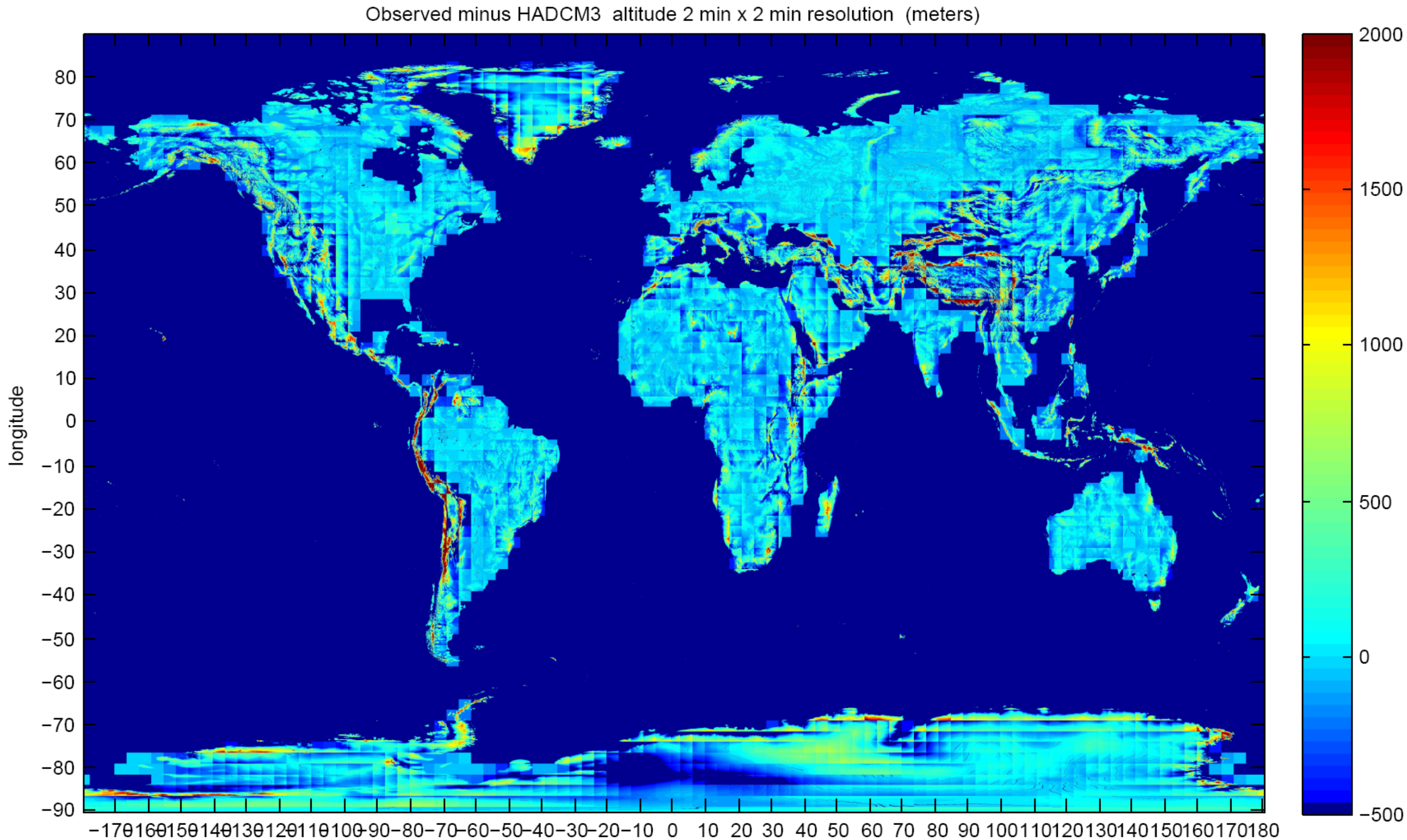
While systematic errors are larger than the observed effect



Moving to anomaly space requires letting go of the "Laws of Physics".
Note model anomalies are not exchangeable

Climate in Practice

Common, nontrivial, systematic errors



Missing Mountains: The fine structure shows what is missing.
Each AR4 model suffers from related inadequacies

Climate in Practice: In-sample examples.

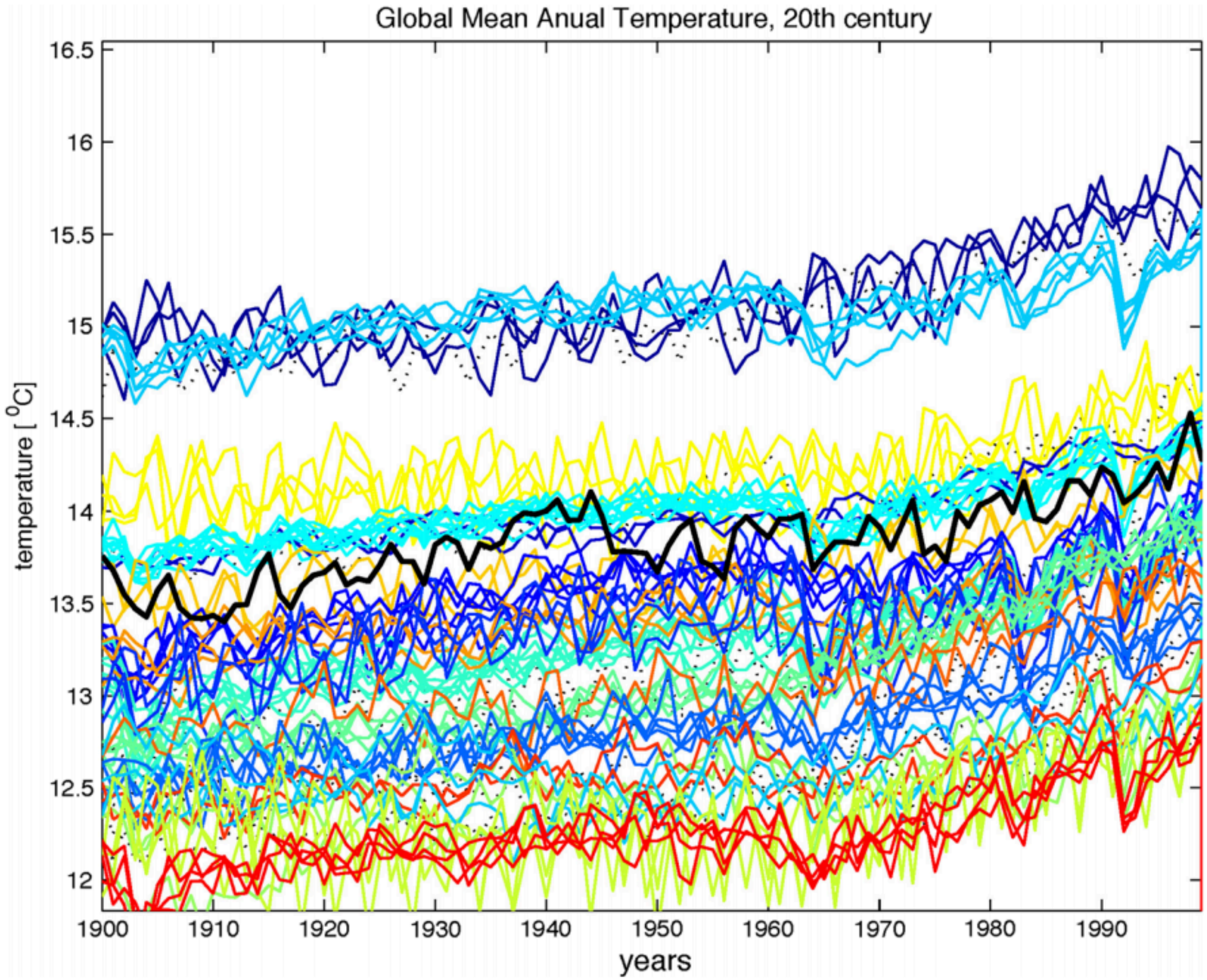
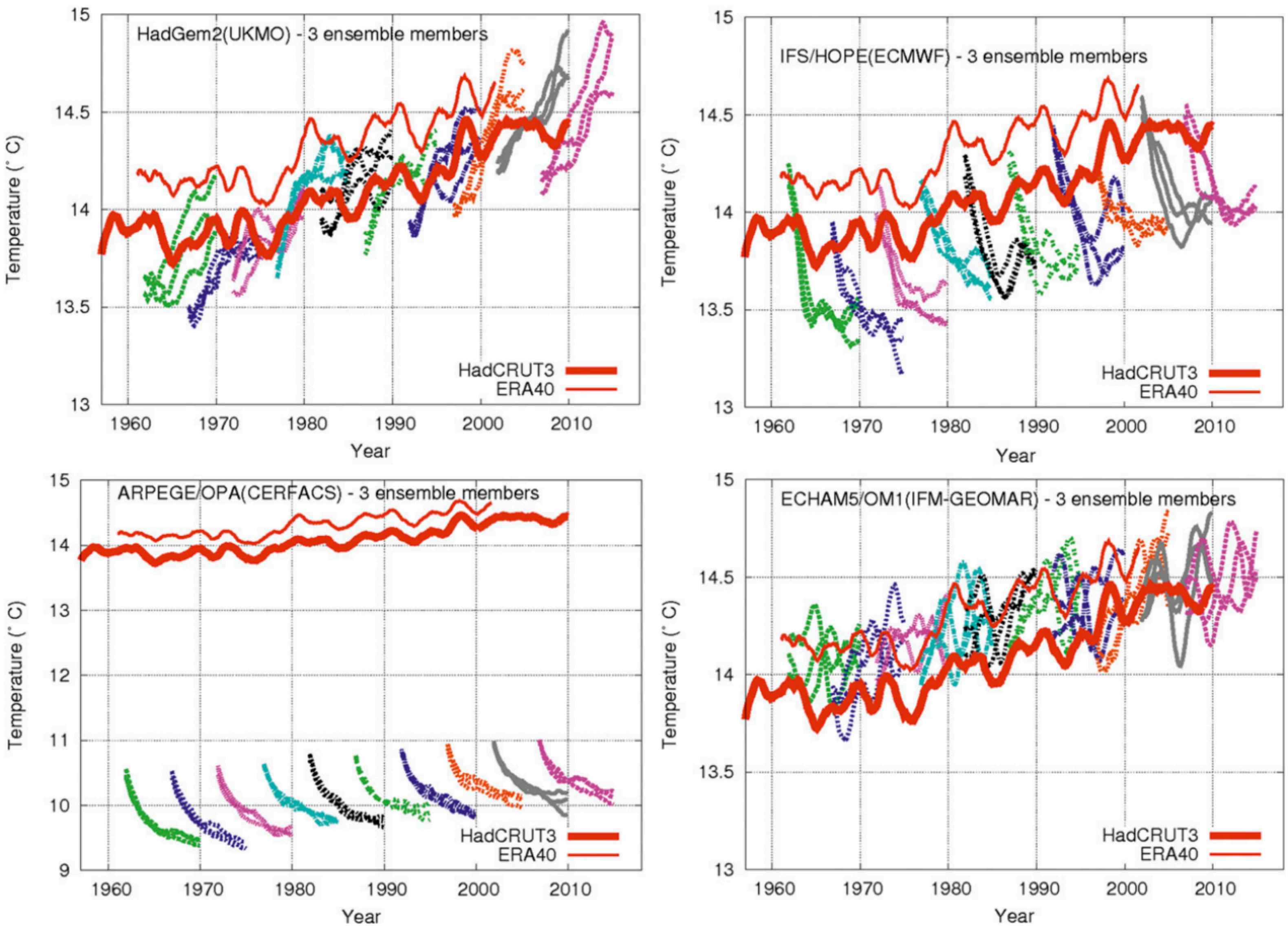


Fig. 1 Model global mean temperatures over the period 1900–2000 for the CMIP5 ensemble

Frigg, R., Smith, L.A. and Stainforth, D.A. (2015) 'An assessment of the foundational assumptions in high-resolution climate projections: the case of UKCP09', Synthese.

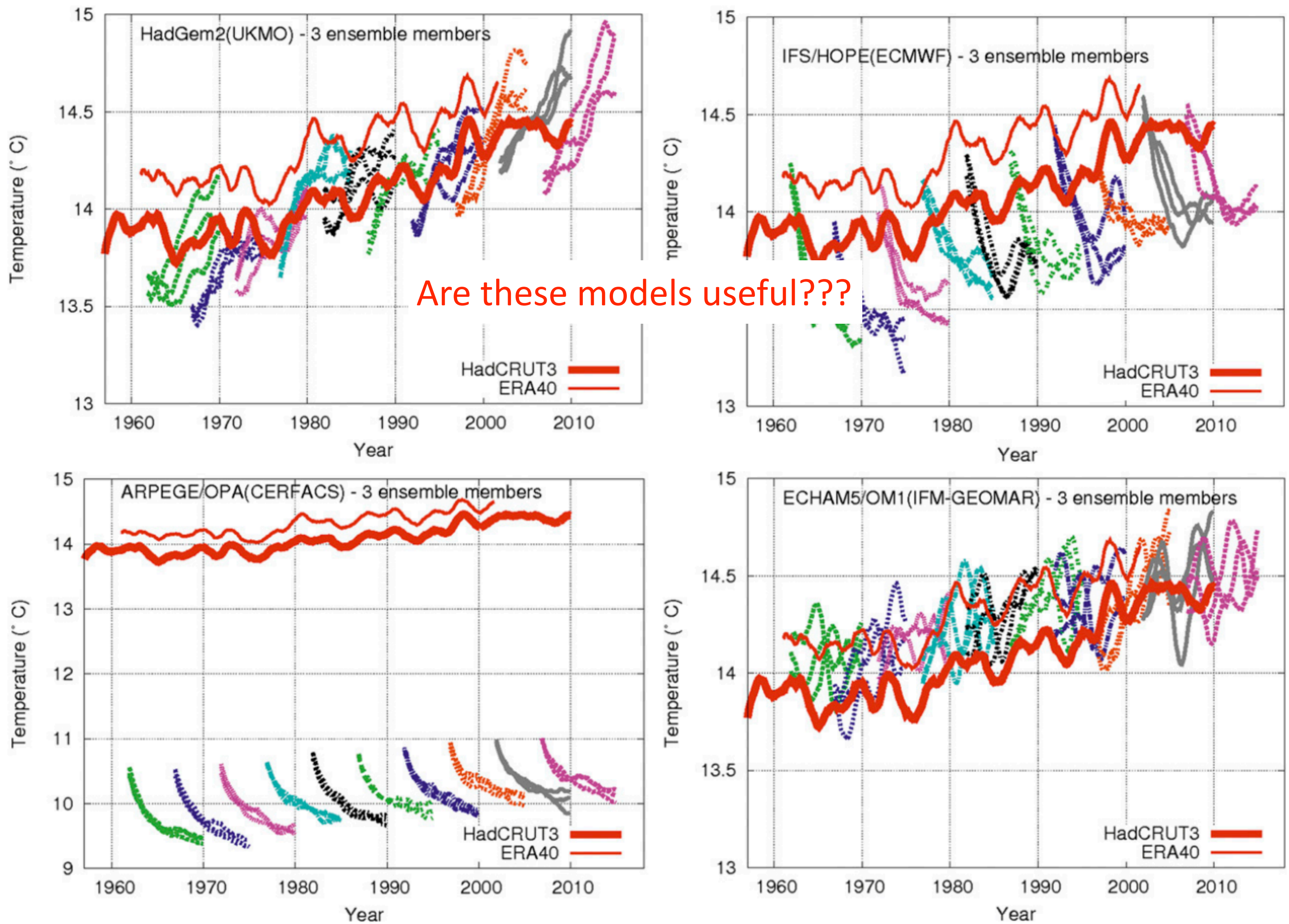
Climate in Practice: Decadal predictions



Are these models useful???

Suckling, E.B. and Smith, L.A. (2013) 'An evaluation of decadal probability forecasts from state-of-the-art climate models', Journal of Climate, 26 (23): 9334-9347.

Climate in Practice: Decadal predictions



Are these models useful???

We need a reference!

Climate in Practice: Decadal predictions

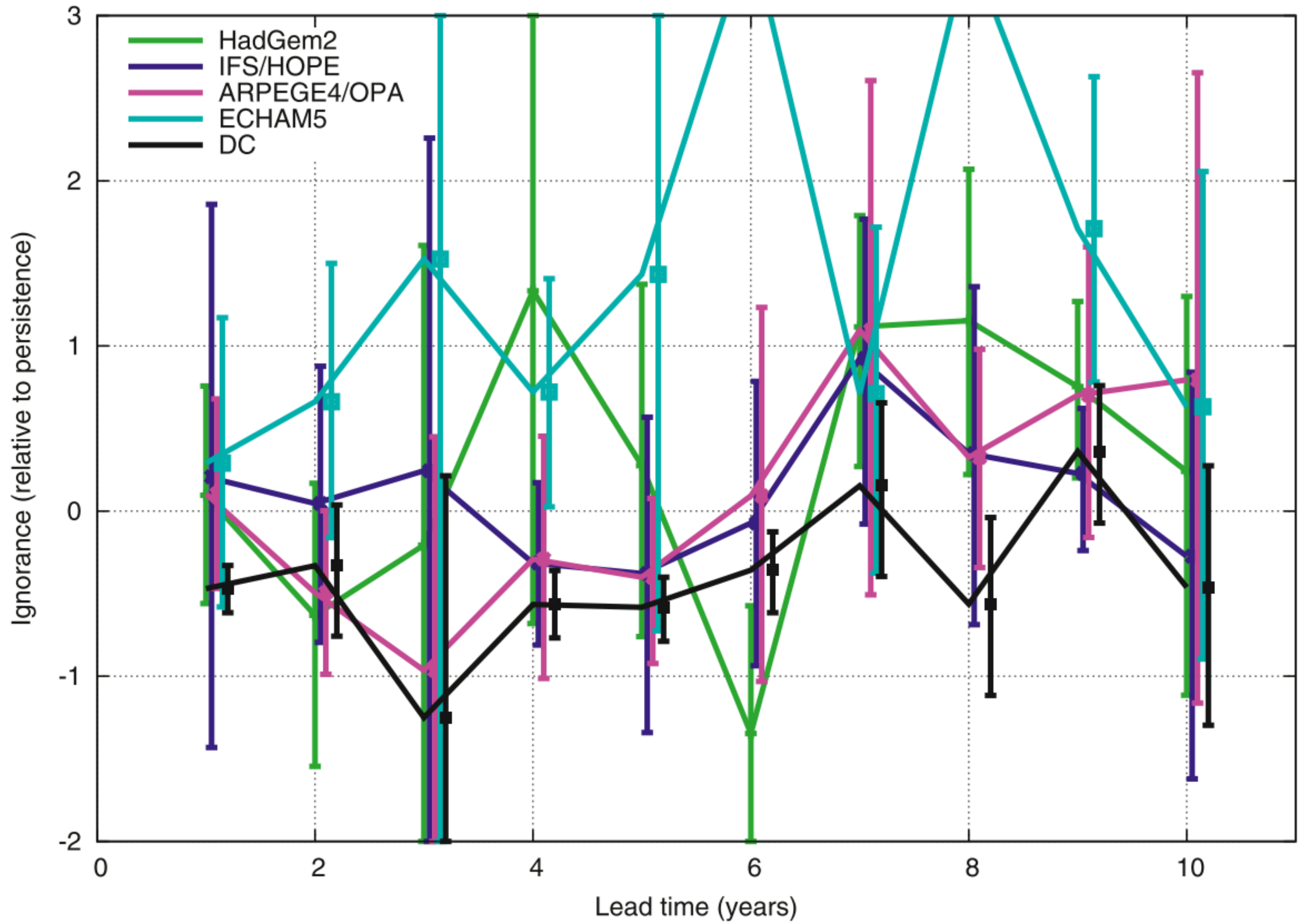
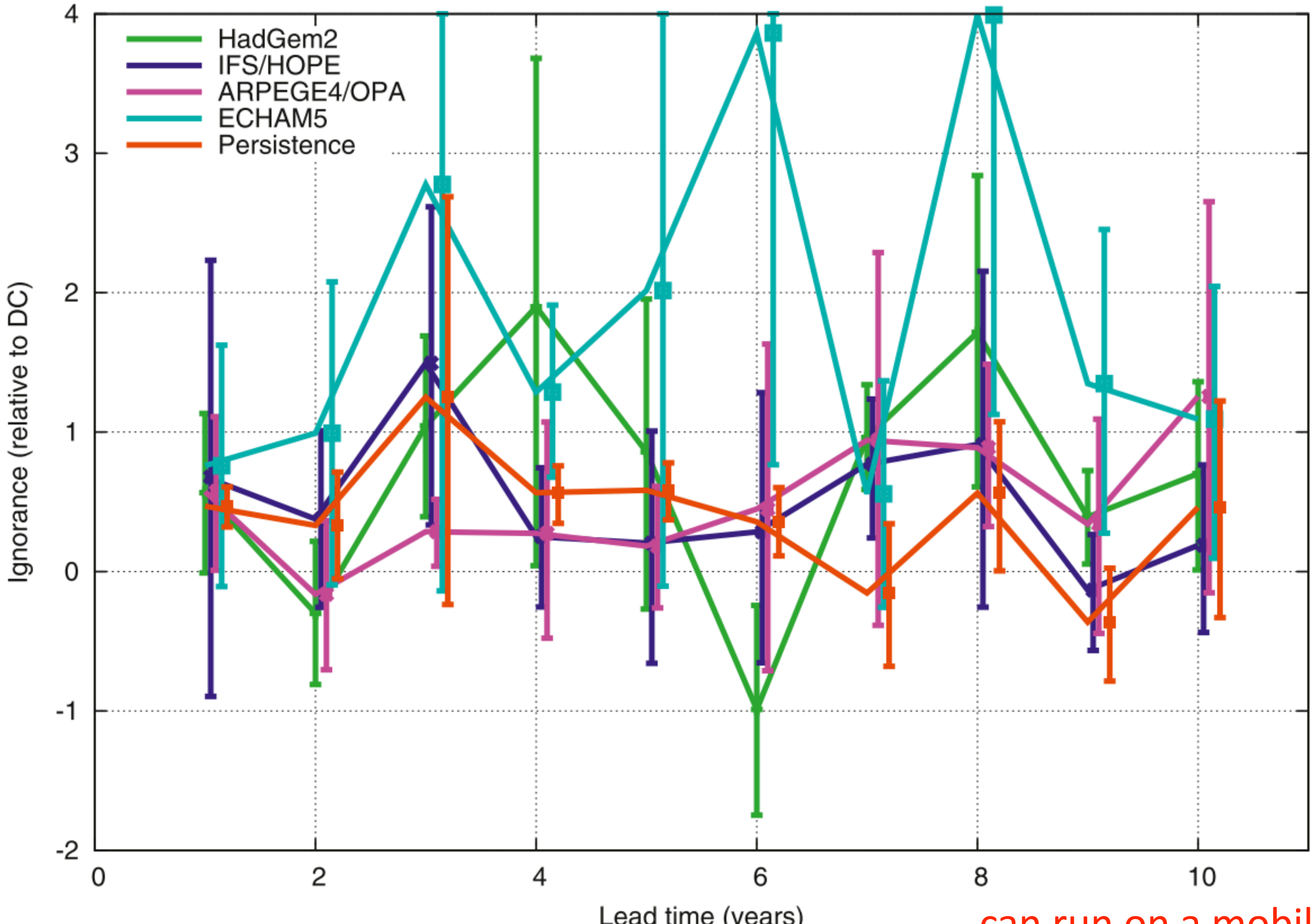


FIG. 8. Ignorance of the ENSEMBLES models and DC relative to persistence forecasts as a function of lead time. The DC model has negative relative ignorance scores up to 6 years ahead, indicating it is significantly more skillful than persistence forecasts at early lead times. The ENSEMBLES models tend to have positive scores, particularly at longer lead times, with bootstrap resampling intervals that overlap with the zero skill line. The bootstrap resampling intervals are illustrated at the 10th–90th percentile level.

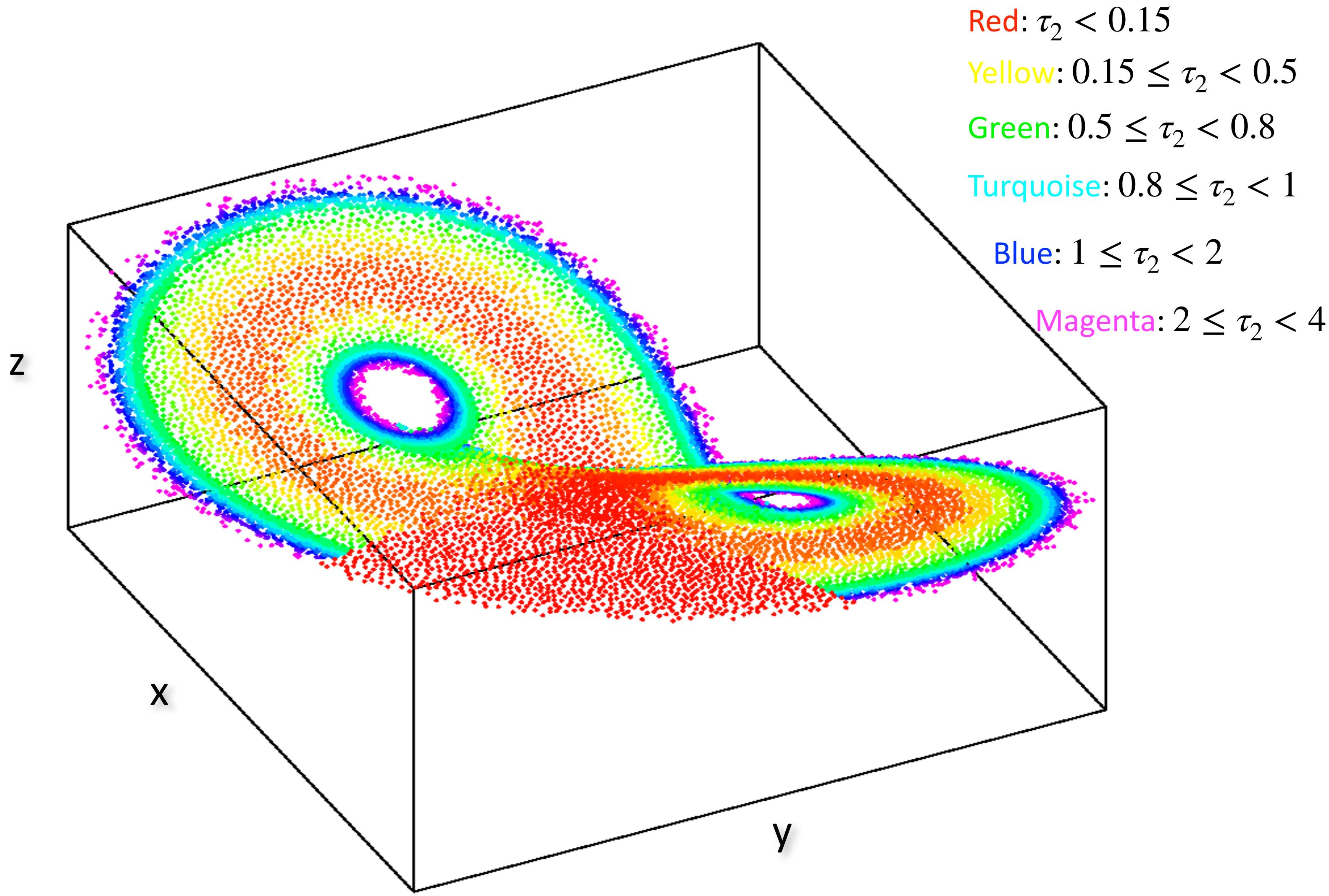
Climate in Practice: Decadal predictions



can run on a mobile phone!

FIG. 9. Ignorance of the ENSEMBLES models relative to DC as a function of lead time. The bootstrap resampling intervals are illustrated at the 10th–90th percent level. Note that the simulation models tend to have positive scores (less skill) than the DC model at every lead time.

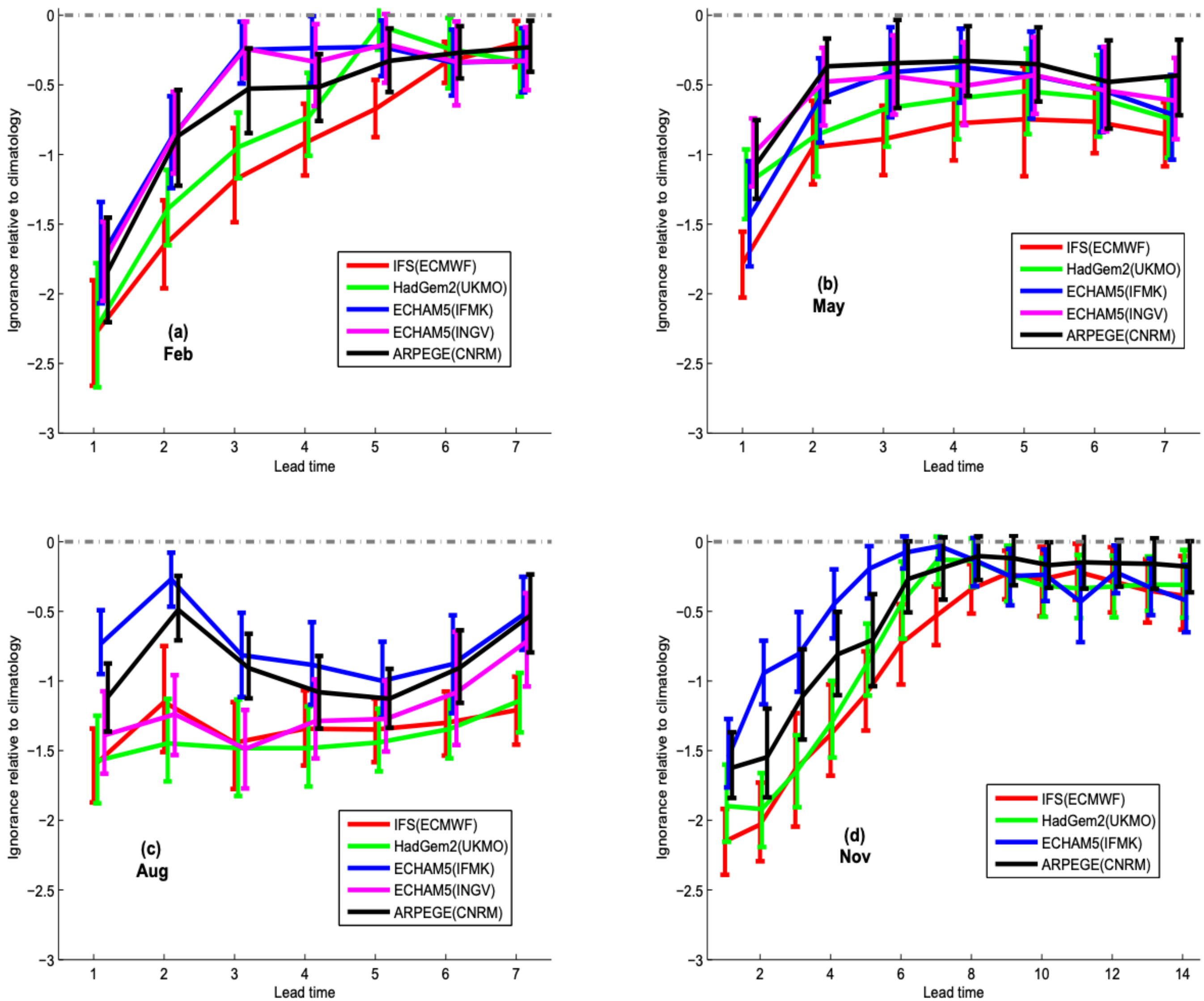
Most people think chaos is unpredictable, but exponential-on-average does not mean exponential!



Local Doubling Times on Lorenz '63

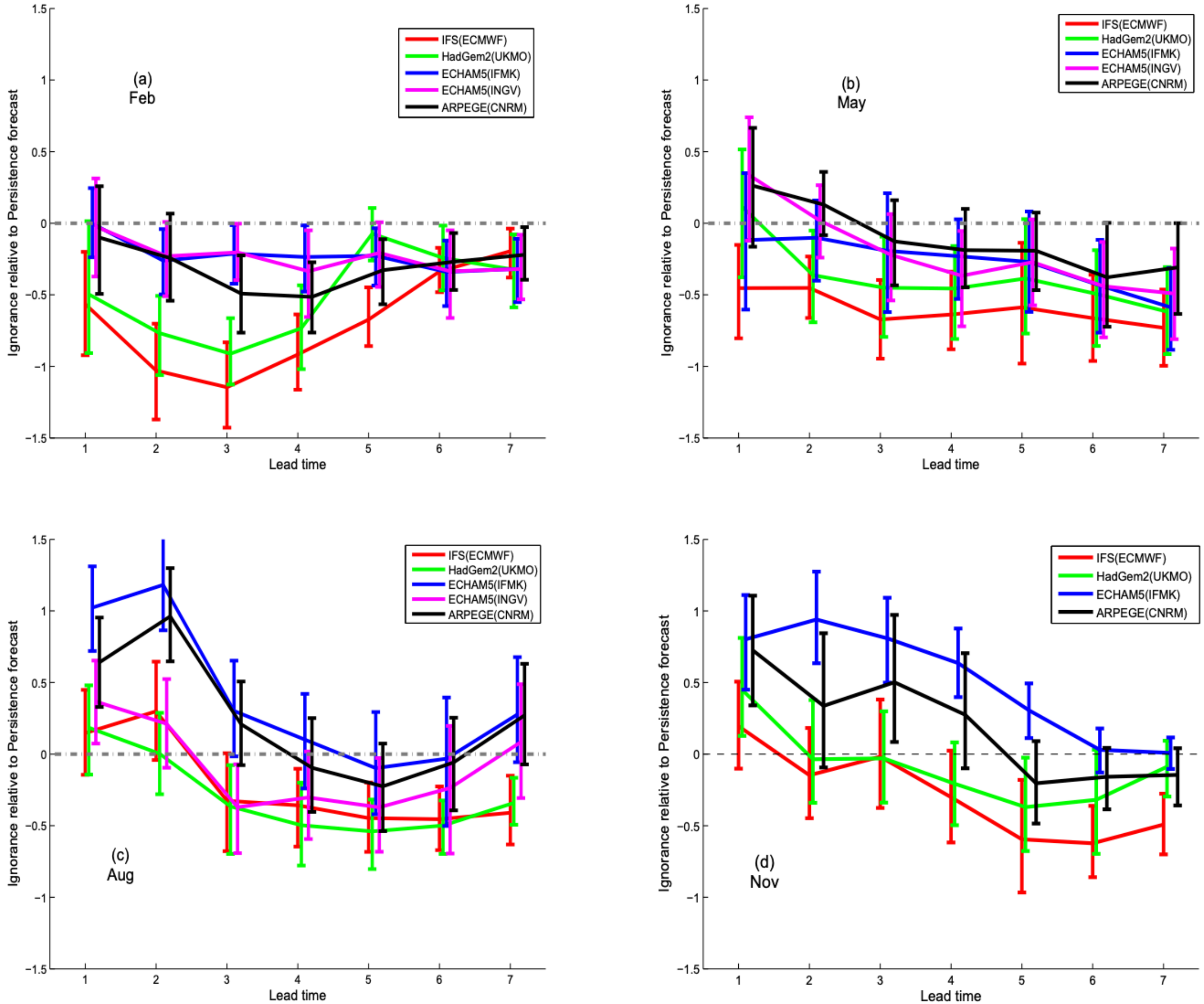
Of course it is hard to get people to bet with you on the Lorenz Differential Equations!
And how would we know if "today" was a good day to bet?

Predictability of seasonal models



Ignorance score of each model from ENSEMBLES for the Nino3.4 index relative to climatology as a function of lead time in months. Zero Ignorance indicates a model has no skill relative to climatology.

Predictability of seasonal models



Ignorance score of each model from ENSEMBLES for the Nino3.4 index relative to persistence forecasts.

Smith, L.A., Du, H., Suckling, E.B. and Niehoerster, F. (2014) 'Probabilistic skill in ensemble seasonal forecasts', Quarterly Journal of the Royal Meteorological Society.

Multi-model ensemble vs Single model ensemble

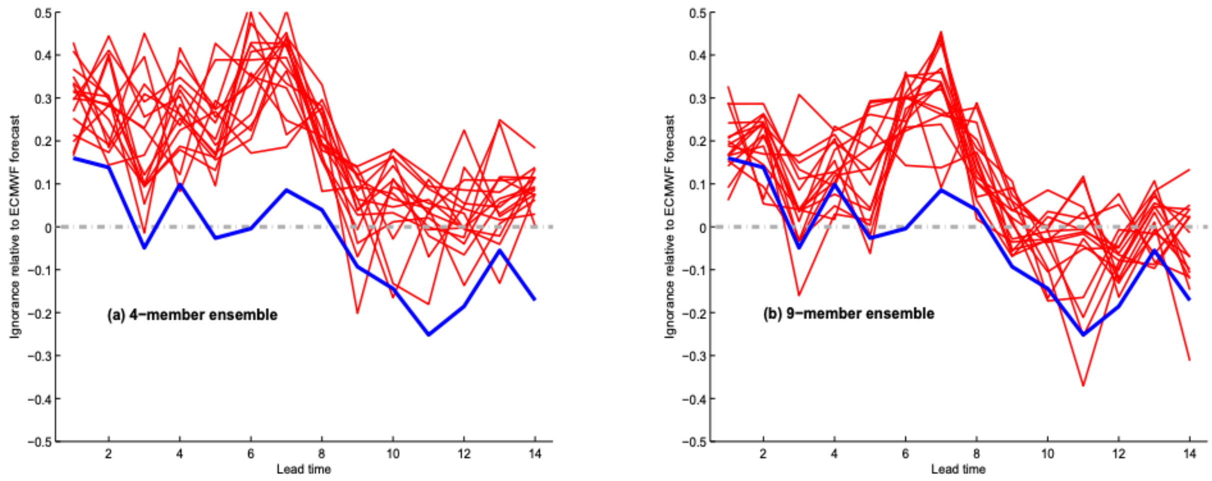
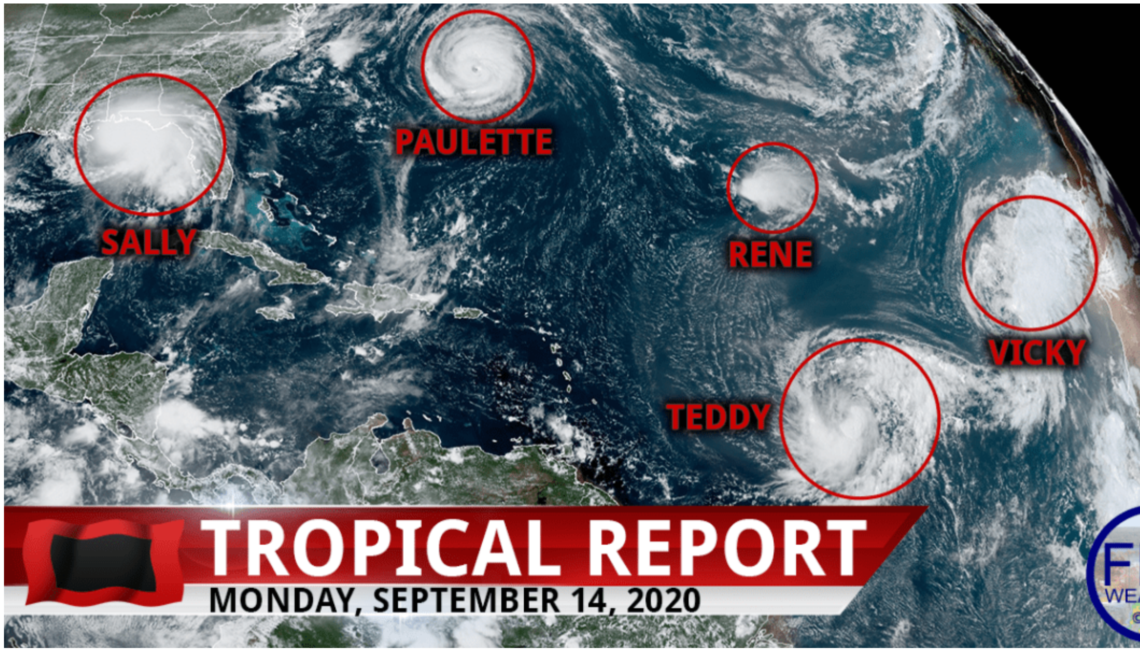
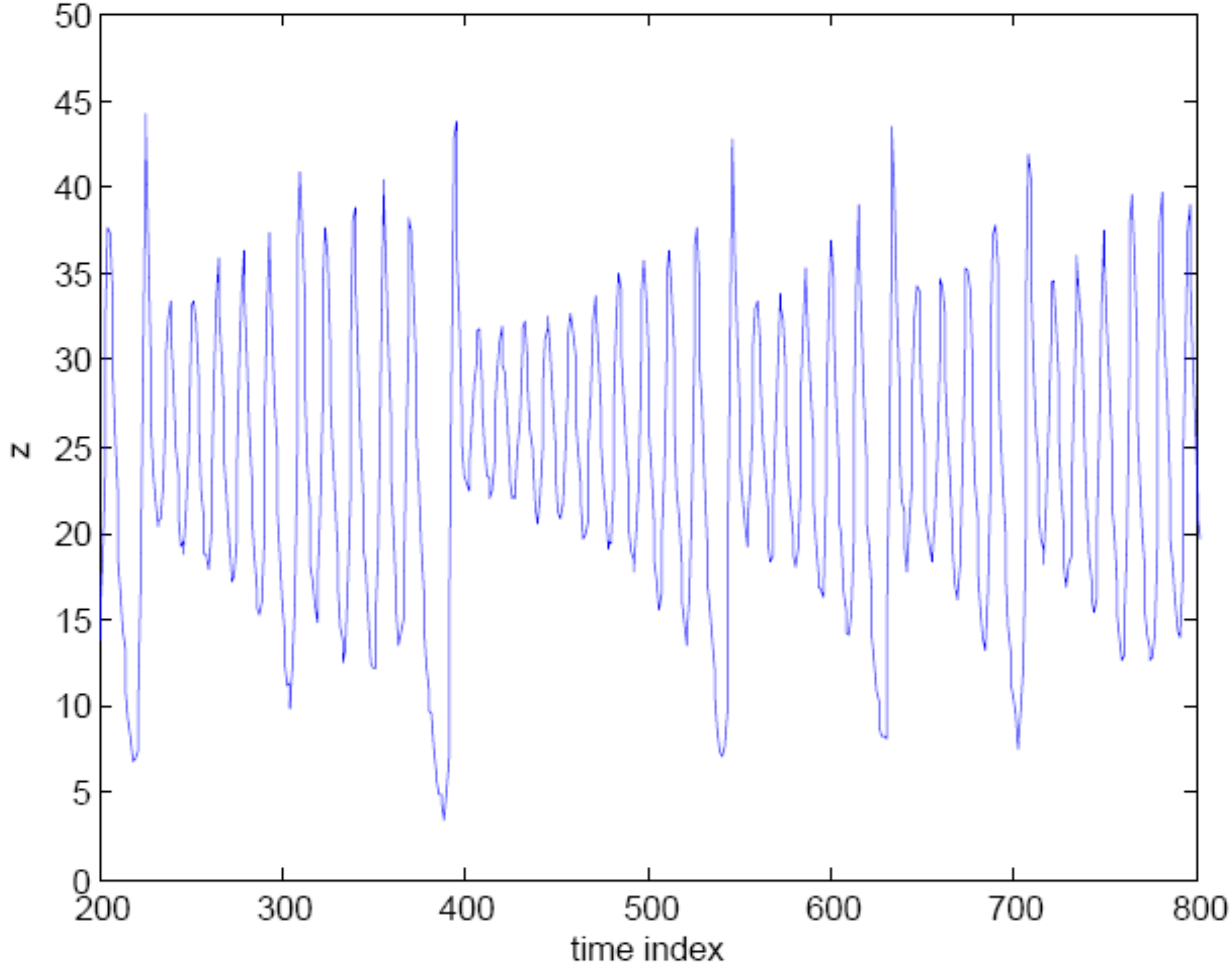


Figure 10: Ignorance of multi-model forecasts as a function of lead time in months for the Nino3.4 index, launched in November, relative to the nine-member IFS(ECMWF) forecast. The blue line represents the multi-model forecast using all 36 ensemble members from the four ENSEMBLES models, equally weighted. The red lines are multi-model forecasts using randomly drawn combinations of four-members (a) and nine-members (b) from the full ensemble. The four-member multi-model forecasts are shown to perform substantially worse than the nine-member IFS(ECMWF) ensemble (that is Ignorance scores are often above zero) and worse than the full 36-member multi-model ensemble. The nine-member multi-model forecasts perform better in general than the four-member forecasts, and to a similar level of skill as the nine-member IFS(ECMWF) ensemble at lead times beyond eight months.

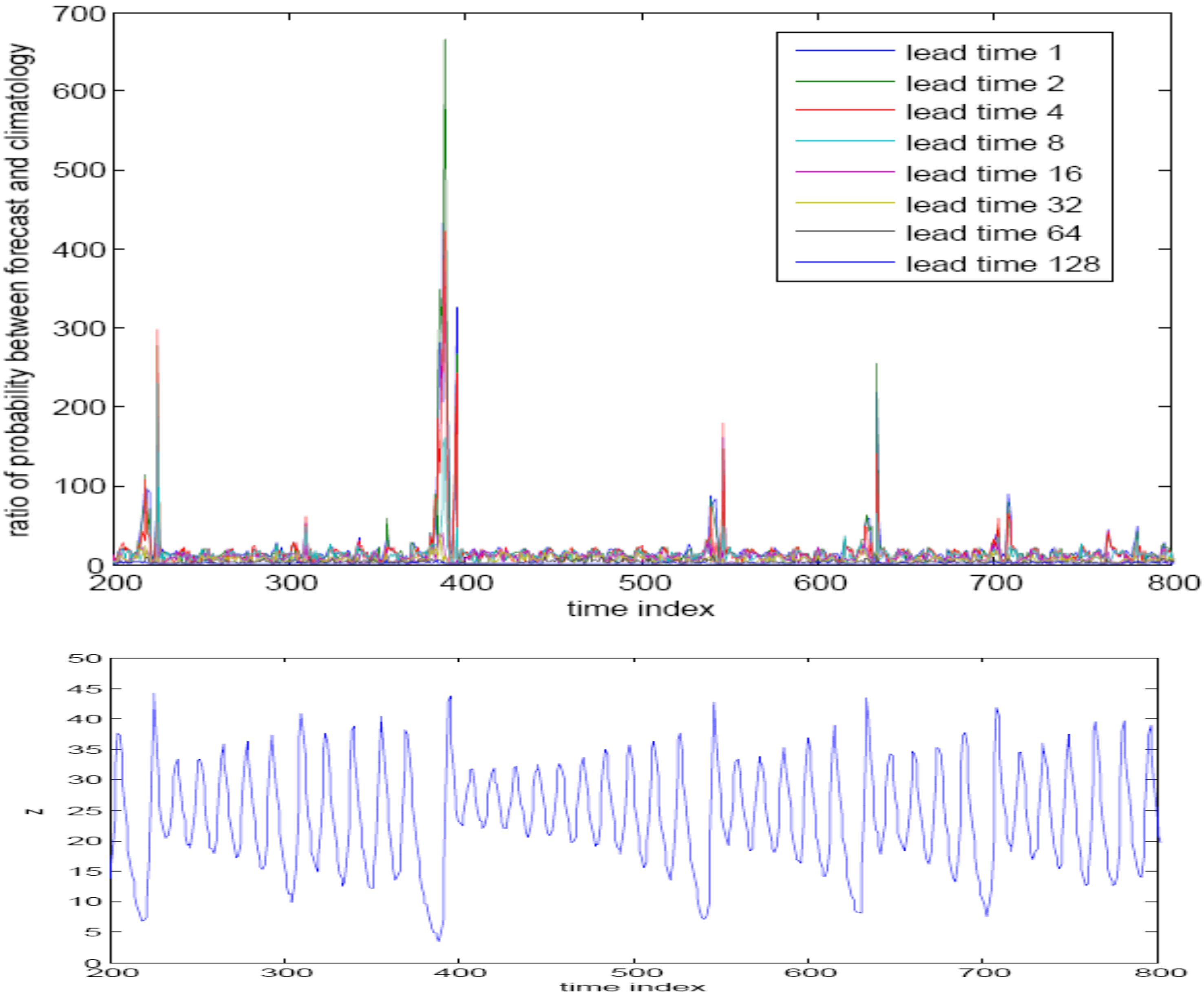
Predicting Extremes



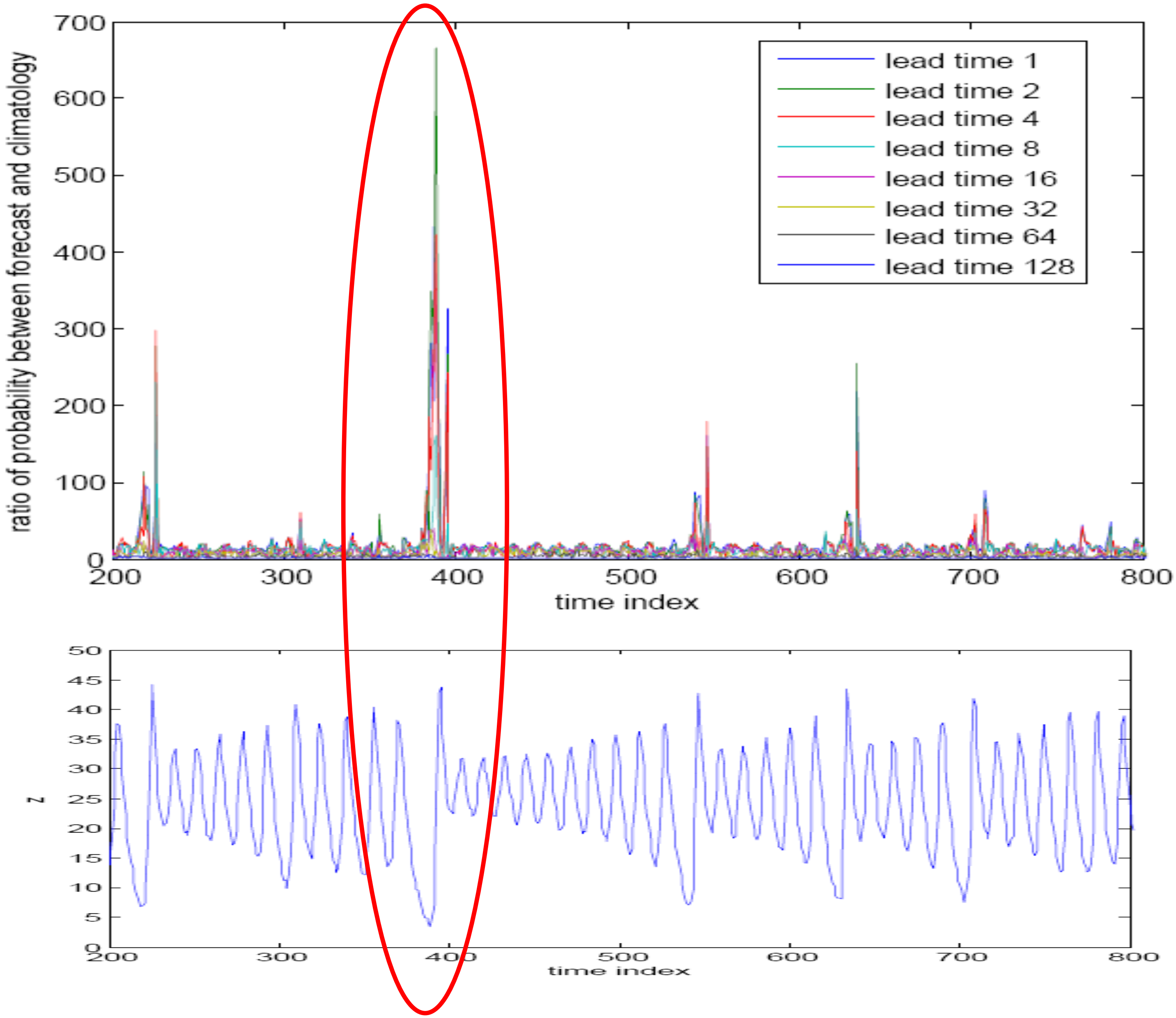
Consider noisy observations of Lorenz63 Z

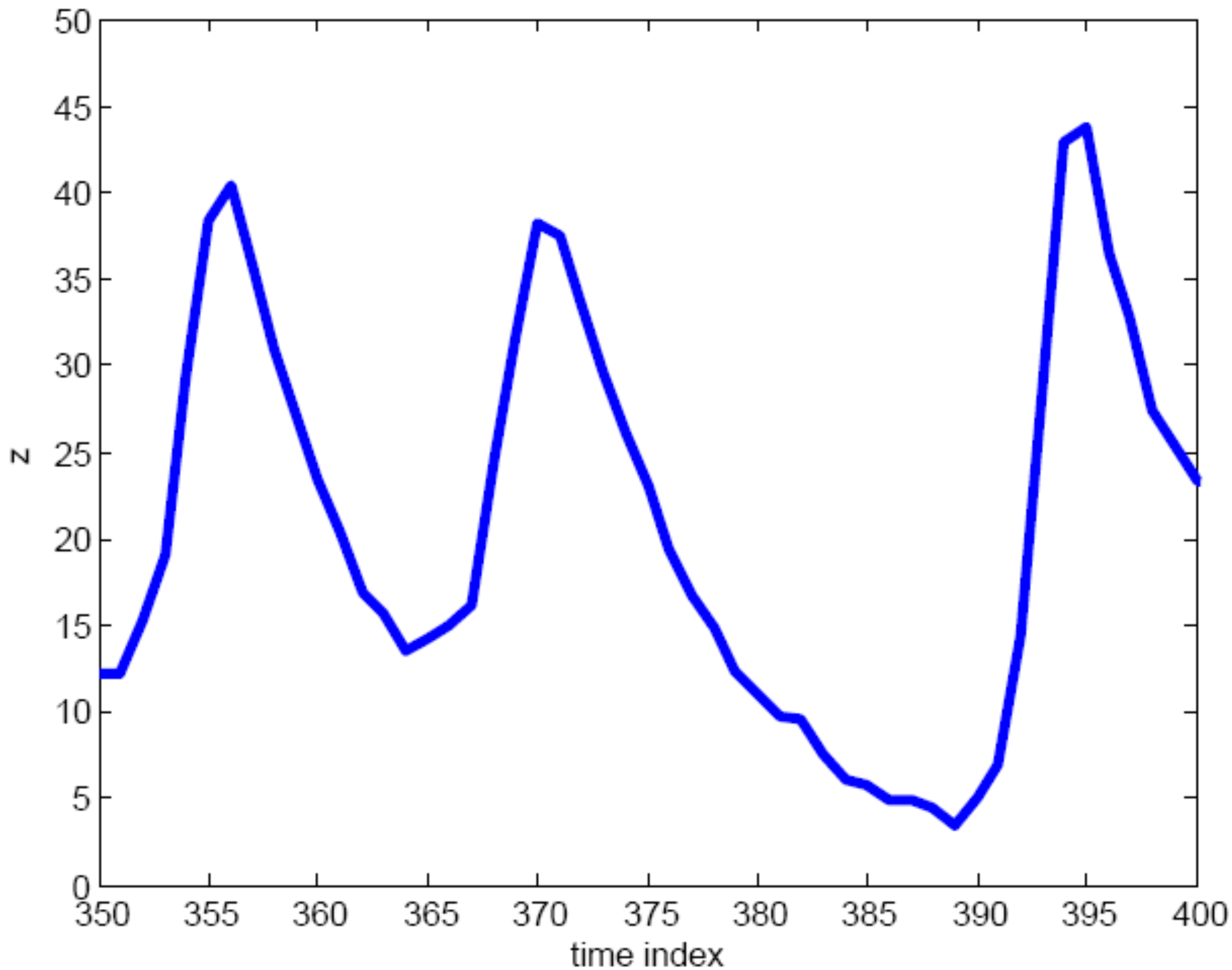


Forecast with different lead time



Forecast with different lead time





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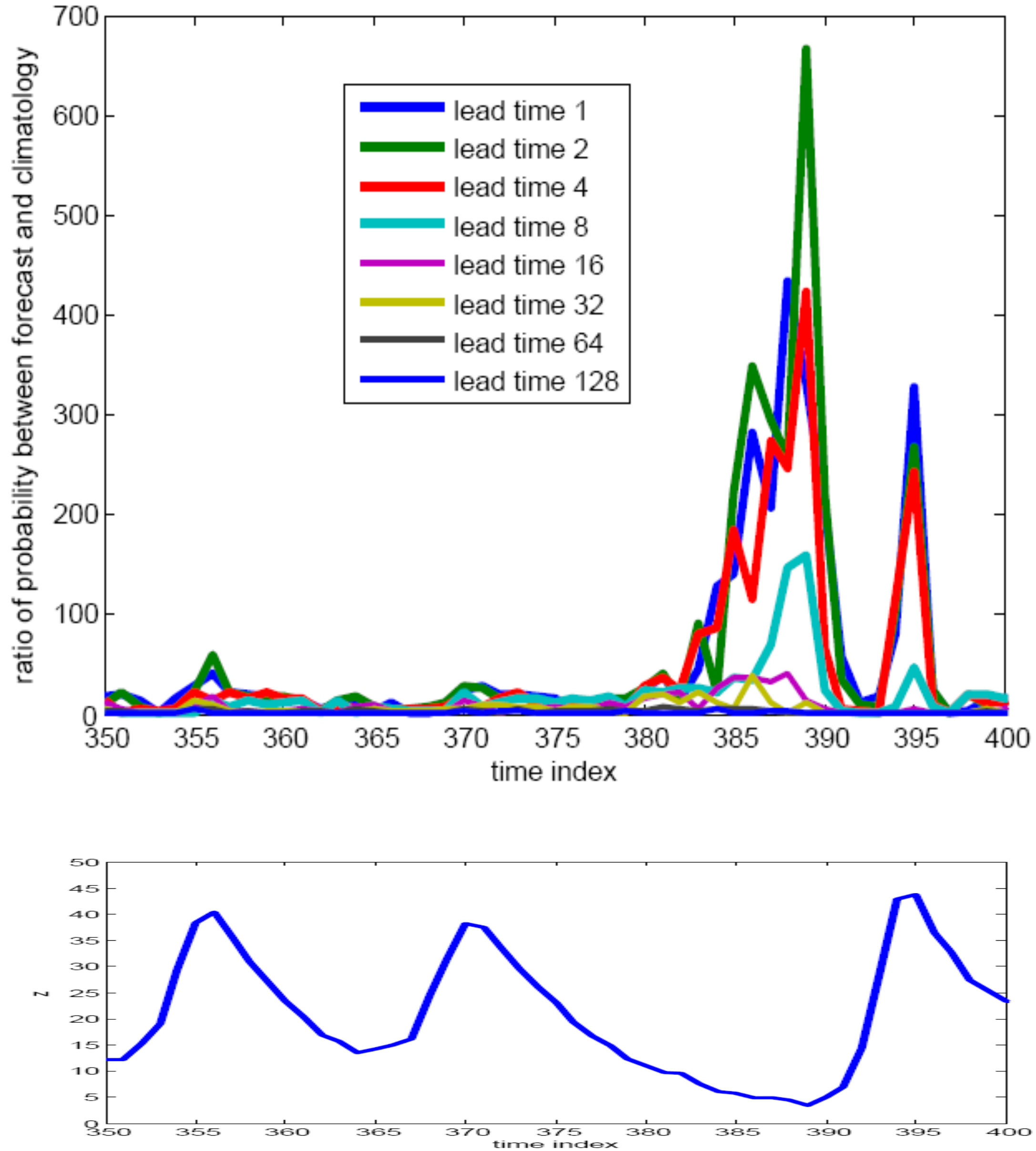
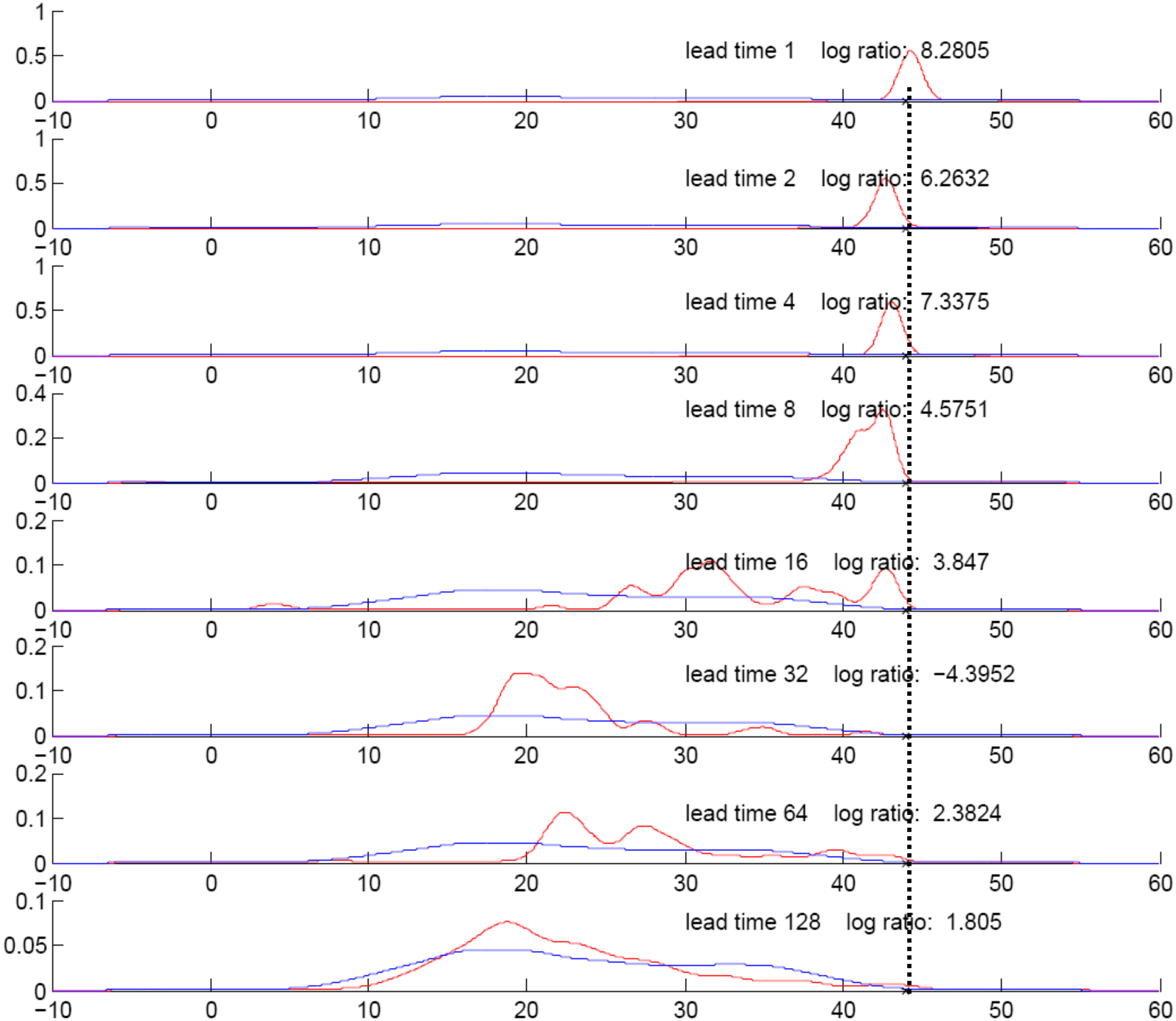
time index

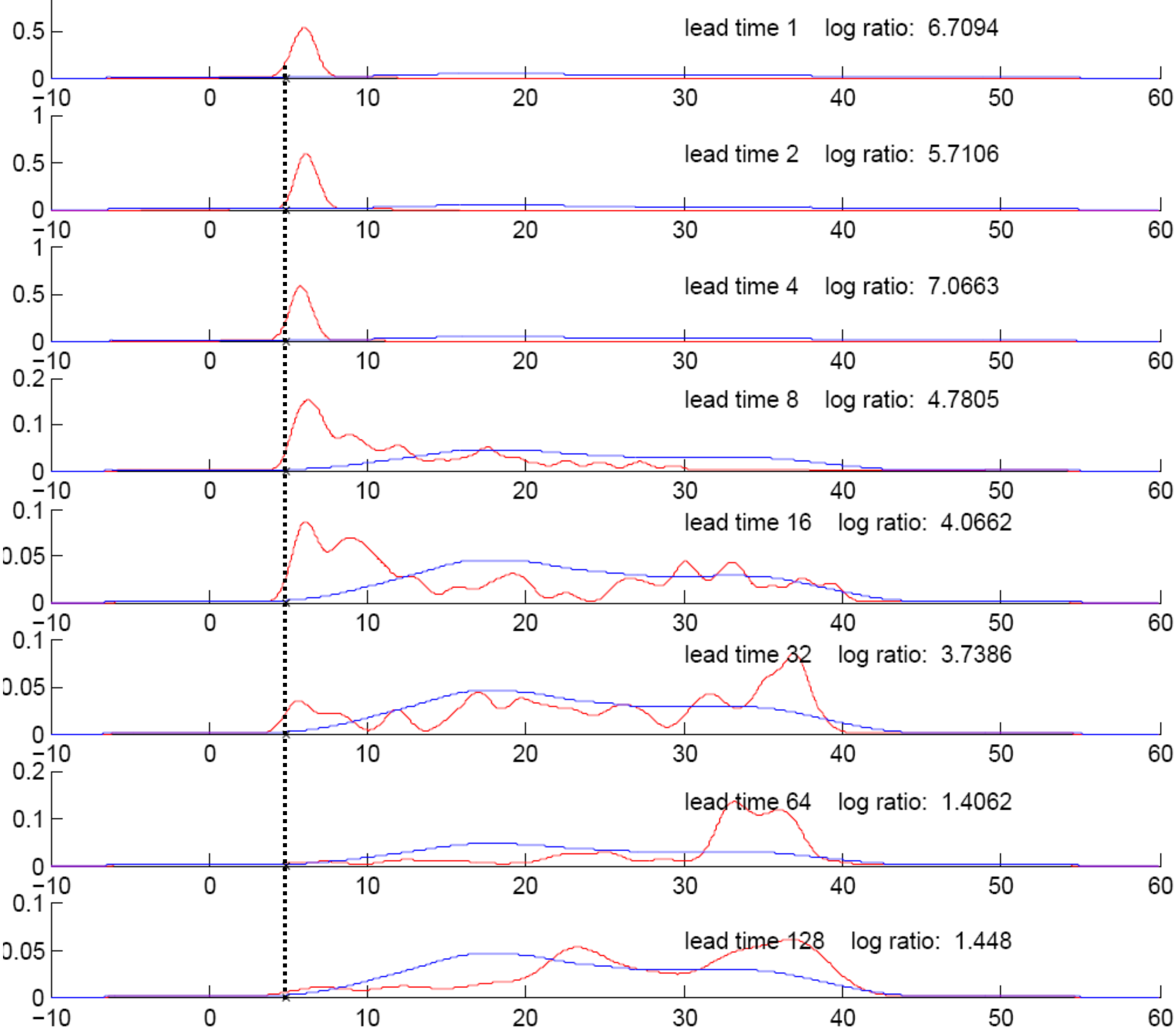


Why lead time2 is better than lead time1

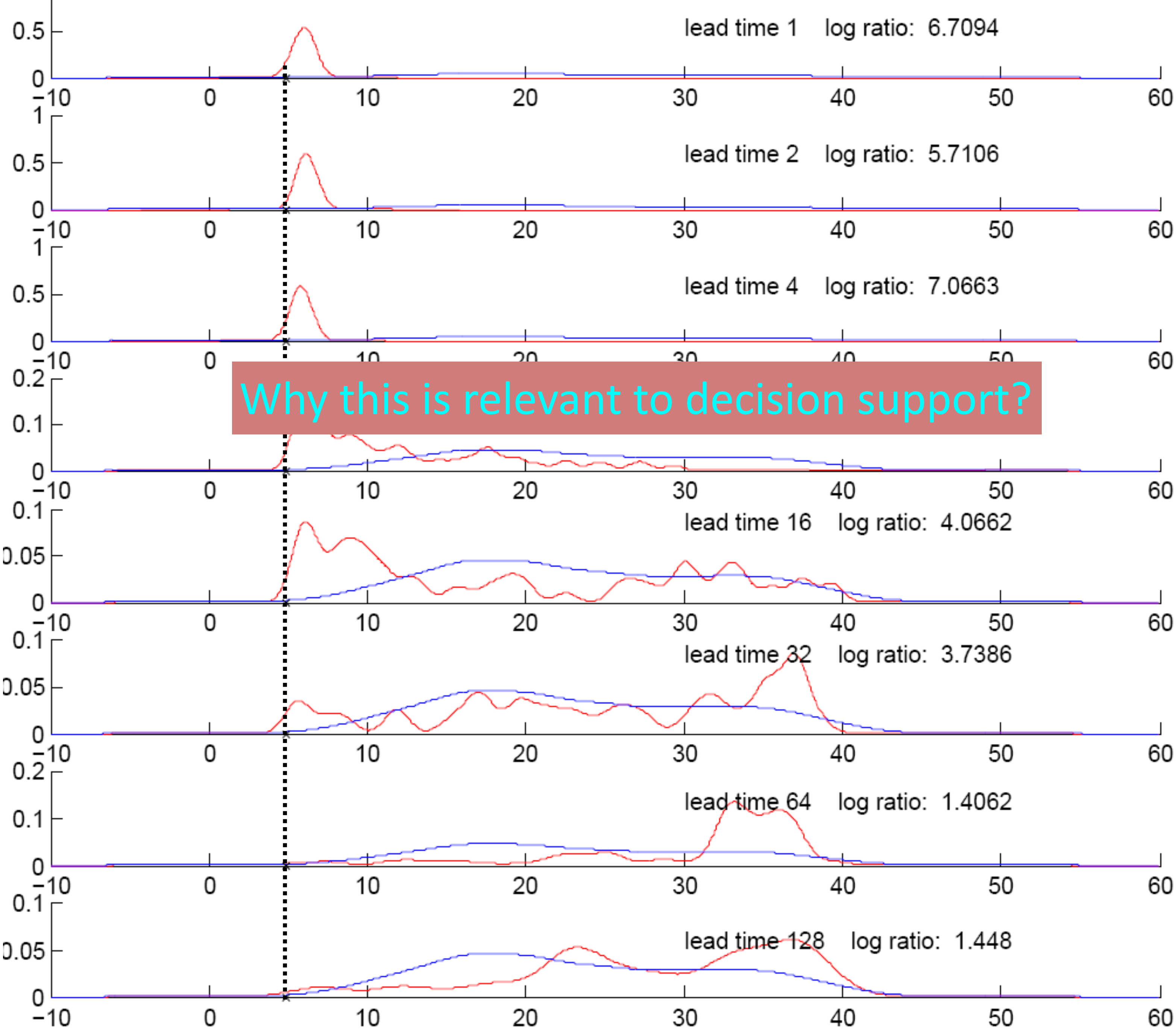
Consider the base 2 log of the forecast pdf divided by the climatological pdf.

Extreme/Rare threshold is 1/200





DA can improve the predictability even further!


Why this is relevant to decision support?
lead time 1 log ratio: 6.7094
lead time 2 log ratio: 5.7106
lead time 4 log ratio: 7.0663
lead time 16 log ratio: 4.0662
lead time 32 log ratio: 3.7386
lead time 64 log ratio: 1.4062
lead time 128 log ratio: 1.448

Extensive reading

Gareth James, Daniela Witten, Trevor Hastie, Robert Tibshirani. An Introduction to Statistical Learning : with Applications in R. New York :Springer, 2013.

Hastie, T., Tibshirani, R., & Friedman, J. H. (2009). The elements of statistical learning: data mining, inference, and prediction. 2nd ed. New York: Springer.

A Very Short Introduction to Chaos (2007) Oxford University Press.

Ott, E. (2002). Chaos in Dynamical Systems (2nd ed.). Cambridge: Cambridge University Press.

Wilks, D.S., Statistical Methods in the Atmospheric Sciences, Academic Press, 2 edition (2005).

Du, H., Smith, L.A. (2017) 'Rising Above Chaotic Likelihoods', SIAM/ASA Journal on Uncertainty Quantification, (5) 246-258.

Du, H. and Smith, L.A. (2014) 'Pseudo-orbit data assimilation part I: the perfect model scenario', Journal of the Atmospheric Sciences, 71 (2), 469-482.

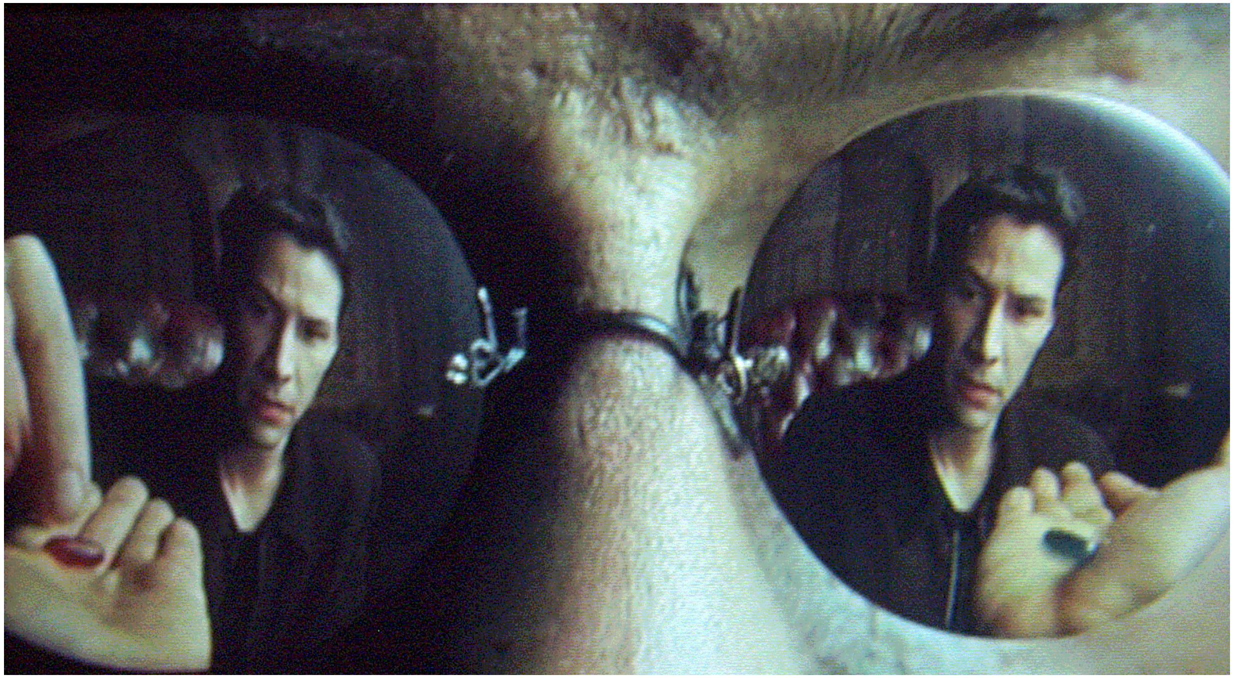
Du, H. and Smith, L.A. (2014) 'Pseudo-orbit data assimilation part II: assimilation with imperfect models', Journal of the Atmospheric Sciences, 71 (2), 483-495.

Du, H., Smith, L.A. (2017) 'Multi-model Cross Pollination in Time', Physica D, (353–354), 31-38.

Bröcker, J. and Smith, L.A. (2007) 'Scoring probabilistic forecasts: the importance of being proper', Weather and Forecasting, 22 (2): 382-388.

Du, H., 'Beyond Strictly Proper Score: The Important of Being Local', Weather and Forecasting, 36(2), 457-468, 2021.

Now you have to make yet another choice. You can pretend you took the blue pill and wake–up in your bed and still attempt to happily do mathematics…



But you took the red pill. I hope these lectures not only planted valid seeds of doubt in your mind, but also improved your personal toolkit in a useful way, and will be of help when you give coherent real world statistical advice knowing all models are wrong.

"Remember that all I am offering is the truth. Nothing more"

Morpheus

Durham University

World Ranking:

QS World University Rankings: 82th

UK Ranking:

6th in the Times and Sunday Times Good University Guide 2021;
4th in the Guardian University Guide 2021;
7th in The Complete University Guide 2021。

College system，‘Doxbridge’



Harry Potter course to be offered at Durham University

Module will focus on 'social, cultural and educational context', but no word on whether Expelliarmus will be applied to students with poor grades

谢谢大家！！！