Exercises IV: Drift diffusion model

Cognitive modeling lecture 2024/25, week 7

Questions marked with a star are bonus material.*

Exercise 1 - The math of the DDM

1a) To achieve an error rate of α in a free response paradigm with 2 choice options A and B, and with evidence samples y_i, we use this decision rule:

$$\text{decision} = \begin{cases} \text{choose } A, & \text{if } \frac{p(A|y)}{p(B|y)} > \frac{1-\alpha}{\alpha} \\ \text{choose } B, & \text{if } \frac{p(A|y)}{p(B|y)} < \frac{\alpha}{1-\alpha} \\ \text{wait,} & \text{otherwise} \end{cases} \tag{0.1}$$

Use Bayes rule to show that this equivalent to using this expression:

$$Z_T = \log \frac{p(A)}{p(B)} + \sum_{t=1}^{T} \log \frac{p(y_t|A)}{p(y_t|B)} \tag{0.2}$$

with

$$\text{decision} = \begin{cases} \text{choose } A, & \text{if } \ Z_T > \log \frac{1-\alpha}{\alpha} \\ \text{choose } B, & \text{if } \ Z_T < \log \frac{\alpha}{1-\alpha} \\ \text{wait,} & \text{otherwise} \end{cases} \tag{0.3}$$

1b)* The probability distribution of the decision variable in the DDM at time t is normally distributed as:

$$L_T = \mathcal{N}(bT, c^2T) \tag{0.4}$$

with a time-dependent mean (constant drift μ times time T) and variance (noise parameter c^2 times time T)[1].

Show that we arrive at such a distribution starting from the following assumptions:

- The decision variable L_T, is the sum of all updates Δ_t until time T: $L_T = \sum_{t=1}^{T} \Delta_t$
- Each Δ_t is the log-likelihood ratio for the current evidence sample e_t under hypothesis A versus hypothesis B: $\Delta_t = \log \frac{p(e_t|s=A)}{p(e_t|s=B)}$
- Measurements e_t are Gaussian-distributed with different means m_s depending on the true state s of the world (which can be A or B), and $m_A = -m_B$:

[1] ignoring the effect of bounds

$$p(e_t|s = \{A, B\}) = \mathcal{N}\left(m_{\{A,B\}}, \sigma^2\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(e_t - m_{\{A,B\}})^2}{2\sigma^2}\right] \tag{0.5}$$

Tip 1: Start by considering the log-likelihood ratio Δ_t for a single data point e_t, and then take the sum to derive the distribution for L_T.

Tip 2: You can rewrite $e_t = m_{\{A,B\}} + \sigma\epsilon_t$, where $\epsilon_t \sim \mathcal{N}(0, 1)$ is a standard Gaussian variable with zero mean and unit variance.

Tip 3: For independent random variables, the mean of a sum is the sum of the means. And the variance of a sum is the sum of the variances.

Exercise 2 - DDM parameters

2a) Explore the impact of the DDM parameters on reaction time distributions. Use PyDDM model GUI as set up in Part 1 of the Python notebook for this exercise sheet. Note down your observations for the effects of changing the noise parameter and the non-decision time. Can you compensate for changes in noise by also changing the drift rate and bound?

2b) People often show decision biases, even if both choice options are equally likely. Such biases can also be elicited experimentally, for example by providing cues that inform participants about which decision is more likely to be correct in the upcoming trial. In the DDM, such biases could be implemented by a change in starting point, or by a bias in drift rate (higher drift rate for one option than the other). How can we experimentally distinguish between these options?

Try to come up with a prediction about how the RT distributions for correct responses and errors would look in each case. You can also use the model GUI from above to help you. *Tip:* Consider correct responses and errors separately for the preferred (biased) option and the other option.

Exercise 3* - Fitting data using PyDDM

This task is optional, but recommended if you want to use the DDM in your project work.

In the online repository for this exercise sheet, you will find a Python notebook that walks you through using the PyDDM toolbox to simulate and fit DDM models.
Bonus question: You have analysed two datasets with choices and RTs. One dataset is from a group of middle-aged, healthy participants. The other dataset is from a group of older (but also healthy) people. You are interested in changes in decision strategies with aging. You have fit simple DDMs and have found a lower drift rate in the group of older people. How do you

interpret your results?