Gauge Groups K3

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ABSTRACT: Gauge groups of elliptically fibered K3s constructed from reflexive polyhedras from one complex parameter (Picard 19) to five complex parameters (Picard 15) or up to 10 moduli for K3s with only two fibrations. M# means the polytope ReflexivePolyhedras(3, #) in SageMath. Additional contributions of U(1) factors should be considered so that the gauge group is of rank p-2 where p is the Picard number.

| M0 | $\frac{SO(16)\times SO(16)}{Z_2}$ | $\frac{SU(12)\times E_6}{Z_3}$ | $E_8 \times E_8$ | $\frac{E_7 \times E_7 \times SU(4)}{Z_2}$ |
|----|-----------------------------------|--------------------------------|-------------------------------|---|
| M2 | $\frac{E_7 \times SO(20)}{Z_2}$ | $\frac{\times SU(18)}{Z_3}$ | $E_8 \times E_8 \times SU(2)$ | |

Figure 1: Gauge groups Picard 19 (i.e. one moduli)

| М3 | $SO(14) \times E_7$ | $SO(14) \times SU(9)$ | $\frac{SU(12)\times SO(8)}{Z_2}$ | $\frac{E_6 \times E_6 \times SU(3)SU(3)}{Z_3}$ | $E_8 \times E_8 \times Z_3$ | | |
|-----|--|--|---|--|---------------------------------|---|----------------------------------|
| M4 | $E_8 \times E_8 \times Z_3$ | $E_6 \times SO(14) \times SU(3)$ | $E_7 \times E_7$ | $\frac{SU(10)\times SO(12)}{Z_2}$ | $\frac{SU(9)\times SU(9)}{Z_3}$ | | |
| M5 | $E_7 \times E_7 \times SU(2)$ | $SU(10) \times E_6$ | $\frac{SO(16)\times SO(12)\times SU(2)}{Z_2}$ | $\frac{E_7 \times SO(12) \times SU(4)}{Z_2}$ | $E_8 \times E_7$ | $\frac{SU(6)\times SU(12)}{Z_3}$ | |
| M6 | $E_6 \times E_7 \times SU(3)$ | $E_7 \times E_8$ | $E_8 \times E_8 \times Z_3$ | $SO(14) \times SO(14)$ | $SO(10) \times SU(11)$ | $\frac{E_6 \times SU(9) \times SU(3)}{Z_3}$ | $\frac{SU(8)\times SO(16)}{Z_2}$ |
| Μ7 | $E_7 \times E_8$ | $\frac{SU(10)\times E_7}{Z_2}$ | $\frac{SU(3)\times SU(15)}{Z_3}$ | $E_6 \times SO(18)$ | | | |
| M10 | $\frac{SO(16)\times SO(16)}{Z_2}$ | $\frac{E_7 \times E_7 \times SU(2)SU(2)}{Z_2}$ | $E_8 \times E_8 \times Z_4$ | $\frac{SU(16)}{Z_2}$ | | | |
| M11 | $\frac{SO(16)\times E_7\times SU(2)}{Z_2}$ | $E_8 \times E_8 \times Z_4$ | $E_8 \times E_7 \times SU(2)$ | $\frac{SO(12)\times SO(20)}{Z_2}$ | SU(16) | | |
| M16 | $SO(18) \times E_6$ | $\frac{SU(15)\times SU(3)}{Z_3}$ | $E_7 \times E_8$ | $\frac{SU(10)\times E_7}{Z_2}$ | | | |
| M88 | $E_8 \times E_8$ | $\frac{SO(32)}{Z_2}$ | | | | | |

Figure 2: Gauge groups Picard 18 (i.e. two moduli)

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| M14 | $E_7 \times SU(8) \times SU(2)$ Z_2 | $SO(16) \times E_6$ | $E_8 	imes E_7 	imes Z_4$ | $SU(14) \times SU(2)$ | $E_8 	imes E_6$ | $SO(10) \times SO(18)$ | | | |
|------|---|-----------------------------------|--|--|-----------------------------------|---|---|---|--|
| M15 | $E_7 \times E_6 \times SU(2)$ | $SO(14) \times SO(14)$ | $\frac{SO(16)\times SU(8)}{Z_2}$ | $E_7 	imes E_8 	imes Z_4$ | $\frac{SU(14) \times SU(2)}{Z_2}$ | | | | |
| M20 | $E_7 \times E_8 \times Z_3$ | $E_6 \times SO(14)$ | $E_6 \times SO(10) \times SU(3)SU(2)$ | $SU(6)\times SO(14)\times SU(3)$ | $E_7 \times SO(12)$ | $SO(12) \times SU(8)$ | $\frac{SU(10)\times SO(8)\times SU(2)}{Z_2}$ | $SU(9) \times SU(7)$ | |
| M21 | $E_7 \times SO(12) \times SU(2)$ | $SU(6) \times SU(10)$ | $E_7 	imes E_7$ | $\frac{SO(12)\times SO(12)\times SU(2)SU(2)}{Z_2}$ | $E_7 \times E_7 \times Z_2$ | $SU(8) \times E_6$ | $\frac{SO(12)\times SO(12)\times SU(4)}{Z_2}$ | $\frac{SO(16)\times SO(8)\times SU(2)SU(2)}{Z_2}$ | |
| M22 | $SO(10) \times SU(9) \times SU(2)$ | $SO(14) \times SU(7)$ | $E_8 \times E_7 \times Z_3$ | $SO(14) \times SO(12)$ | $E_6 \times E_6 \times SU(3)$ | $SU(10) \times SO(8)$ | $E_7 \times SO(10) \times SU(2)$ | $\frac{E_6 \times SU(6) \times SU(3)SU(3)}{Z_3}$ | |
| M23 | $\frac{SU(10)\times SO(12)}{Z_2}$ | $E_6 \times SO(14) \times SU(2)$ | $E_7 \times SU(8)$ | $SO(12) \times E_8$ | $E_7 	imes E_7$ | $SU(3) \times SU(13)$ | $SO(18) \times SU(6)$ | | |
| M24 | $E_7 	imes E_7$ | $SO(10)\times SU(8)\times SU(3)$ | $E_6 \times E_6 \times SU(2)$ | $SO(12) \times SO(12)$ | $\frac{SU(8)\times SU(8)}{Z_2}$ | | | | |
| M25 | $SO(10) \times SO(14) \times SU(2)$ | $SO(10) \times SU(9)$ | $E_6 	imes E_7$ | $SO(12) \times E_6 \times SU(3)$ | $E_7 	imes E_7$ | $E_8 	imes E_7 	imes Z_3$ | $SU(7) \times E_6 \times SU(3)$ | $\frac{SU(6)\times SU(9)\times SU(3)}{Z_3}$ | $\frac{SU(8)\times SO(12)\times SU(2)}{Z_2}$ |
| M26 | $SU(9) \times E_6$ | $E_6 \times SU(9)$ | $E_7 	imes E_7$ | $SO(10) \times SO(16)$ | $E_6 	imes E_8$ | $\frac{SU(6) \times E_7 \times SU(4)}{Z_2}$ | $SU(3)\times SU(12)\times SU(3)$ Z_3 | | |
| M27 | $SU(8) \times SO(14)$ | $\frac{SU(4)\times SU(12)}{Z_2}$ | $E_6 \times E_6 \times SU(2)SU(2)$ | $E_7 \times SO(12)$ | | | | | |
| M28 | $E_6 \times E_7 \times SU(2)$ | $SU(10) \times SO(10)$ | $SO(10) \times E_7 \times SU(3)$ | $SU(5) \times SU(11)$ | $SO(14) \times SO(12)$ | $E_6	imes E_8$ | $E_6 \times SU(8) \times SU(2)$ | $SU(6) \times SO(16) \times SU(2)$ Z_2 | |
| M29 | $E_7 	imes E_6$ | $E_7 \times E_7 \times Z_2$ | $SO(12) \times SU(8) \times SU(2)$ Z_2 | $\frac{SU(6) \times SU(10)}{Z_2}$ | $SO(12) \times E_6 \times SU(2)$ | $SO(10)\times SO(14)\times SU(3)$ | $SU(8) \times SU(8)$ | | |
| M30 | $E_6	imes E_6$ | $E_7 	imes E_7 	imes Z_2$ | $\frac{SO(12)\times SO(12)\times SU(2)SU(2)}{Z_2}$ | $\frac{SU(8) \times SU(8)}{Z_2}$ | | | | | |
| M38 | $E_7 	imes E_7$ | $SO(14) \times E_8$ | $\frac{SU(4) \times SO(24)}{Z_2}$ | $\frac{\times SU(16)}{Z_2}$ | | | | | |
| M41 | $\overline{SO(12)\times SO(16)\times SU(2)}$ Z_2 | $E_7 	imes E_8 	imes Z_4$ | $\frac{SO(12) \times E_7 \times SU(2)SU(2)}{Z_2}$ | $E_7 \times E_7 \times SU(2)$ | $\times SU(14)$ | | | | |
| M47 | $SO(18) \times SO(10)$ | $E_8 	imes E_7 	imes Z_4$ | $SU(14) \times SU(2)$ | $E_8 	imes E_6$ | $SO(16) \times E_6$ | $\frac{SU(8) \times E_7 \times SU(2)}{Z_2}$ | | | |
| M48 | $E_7 \times E_7$ | $SO(14) \times E_6 \times SU(2)$ | $\times SU(13) \times SU(3)$ | $\frac{SU(10)\times SO(12)}{Z_2}$ | | | | | |
| M49 | $E_7 \times E_6 \times SU(2)$ | $\frac{SU(8)\times SO(16)}{Z_2}$ | $E_7 	imes E_8 	imes Z_4$ | $SO(14) \times SO(14)$ | $\frac{SU(14) \times SU(2)}{Z_2}$ | | | | |
| M50 | SU(15) | $\frac{SU(6) \times SO(20)}{Z_2}$ | $SO(14) \times E_7$ | $E_8 \times E_6 \times SU(2)$ | | | | | |
| M53 | $E_6 \times SU(9)$ | $E_7 	imes E_7$ | $SO(16) \times SO(10)$ | $\frac{\times SU(12) \times SU(3)SU(3)}{Z_3}$ | | | | | |
| M104 | $E_7 	imes E_8$ | $\frac{SO(28) \times SU(2)}{Z_2}$ | | | | | | | |
| M117 | $\frac{SO(24)\times SU(4)}{Z_2}$ | $\frac{SU(16)}{Z_2}$ | $E_7 	imes E_7$ | | | | | | |
| M221 | $E_7 	imes E_8$ | $\frac{SO(28) \times SU(2)}{Z_2}$ | | | | | | | |
| M230 | $\frac{SU(16)}{Z_2}$ | $E_7 \times E_7$ | | | | | | | |

Figure 3: Gauge groups Picard 17 (i.e. three moduli)

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|-------|---|---|--|---|---|--|--|----------------------------------|--|
| MEI | $SO(12) \times E_8 \times Z_4$ | $SU(1) \times E_8$ $E_7 \times SU(6) \times SU(2)$ | SO(14) × SO(10) × SU(2) | Eex | $SU(8) \times SO(12) \times SU(2)$ | SU(12) × SU(2) | E-× E-× Z. | ž × | $SO(16) \times SU(6)$ |
| M52 | $SU(4) \times E_7 \times SU(2)SU(4)$ Z_3 | $SO(10) \times E_8$ | SO(16) × SO(8) | | $SU(2) \times SU(12) \times SU(2)$ | $E_6 \times SU(8)$ | $E_6 \times SU(8)$ | $SO(10) \times SO(16)$ | |
| M57 | $SO(10) \times E_8$ | $SO(18) \times SU(5)$ | $E_7 \times SU(7)$ | $SU(13) \times SU(2)$ | $E_6 \times SO(14)$ | | | | |
| M58 | $SO(10) \times SU(9)$ | $E_6 \times E_7$ | $SO(14) \times SO(10)$ | $SU(7) \times E_6 \times SU(2)$ | $SU(2) \times SU(11) \times SU(3)$ | | | | |
| M60 | $SO(14) \times SU(7)$ | $E_6 \times E_6 \times SU(2)$ | $SO(14) \times SU(7)$ | $SO(10) \times E_7$ | $SU(12)\times SU(2)\times SU(2)$ Z_2 | | | | |
| M64 | $SO(12) \times E_7$ | $E_6 \times E_6$ | $\frac{SU(4)\times SU(12)}{Z_2}$ | $SO(20) \times SU(2) \times SU(2) SU(2)$ Z_2 | | | | | |
| M65 | $SO(12) \times SO(12)$ | $E_6 \times E_6$ | $SO(16) \times SU(4) \times SU(4)$ Z_2 | $E_7 \times E_7 \times Z_4$ | $E_7 \times SO(10) \times SU(2)$ | $\frac{SU(8) \times SU(8)}{Z_2}$ | $SU(12) \times SU(2) \times SU(2)$ Z_2 | | |
| M67 | $SO(12) \times SO(12) \times SU(2)$ | $SO(8) \times SO(12) \times SU(2)SU(2)SU(2)$ Z_2 | | $SO(12) \times E_7 \times Z_2$ | $SU(6) \times E_6$ | | | | |
| M68 | $SO(14) \times SU(6)$ | $SO(8) \times SU(9)$ | $E_6 \times SU(3) \times SU(3)SU(3)SU(3)$ Z_3 | | | | | | |
| M69 | $SO(12) \times SU(8)$ | $E_7 \times SU(6)$ | $SO(12) \times E_7$ | $SO(14) \times SU(6) \times SU(2)$ | $E_6 \times SO(10) \times SU(2)SU(2)$ | $SU(11) \times SU(3)$ | | | |
| - | $SO(10) \times SU(6) \times SU(3)SU(2)$ | $E_6 \times SO(10) \times SU(2)$ | $SU(7) \times SU(7)$ | $SU(8) \times SO(8) \times SU(2)$ | $E_7 \times E_7 \times Z_3$ | $SO(12) \times SO(12)$ | $SO(12) \times SU(6)$ | | |
| M71 | $SO(12) \times SU(6) \times SU(2)$ | $SO(12) \times E_6$ | $SU(8) \times SU(6)$ | $E_7 \times SO(12) \times Z_2$ | $SU(8)\times SO(8)\times SU(2)SU(2)$ Z_2 | $SO(8) \times E_6 \times SU(2)SU(2)$ | $SO(10) \times SO(10) \times SU(2)SU(3)$ | $SO(10) \times SO(14)$ | |
| MT2 | $SO(16) \times SU(5)$ | $SO(10) \times SU(9)$ | $E_6 \times SU(7) \times SU(2)$ | $SO(10) \times E_8$ | $SO(14) \times SO(10)$ | $SU(3) \times SU(11) \times SU(2)$ | $SU(5) \times E_7 \times SU(3)$ | $E_6 \times SU(8)$ | $E_7 	imes E_6$ |
| MT3 | $SO(10) \times SU(8) \times SU(2)$ | $SO(14) \times SU(6)$ | $SU(6) \times E_6 \times SU(2)SU(2)$ | $SO(12) \times SO(12)$ | $SU(10) \times SU(4)$ | $E_6 \times E_6 \times SU(2)$ | $SO(8) \times E_7 \times SU(2)$ | | |
| MT4 | $SU(7) \times SO(10) \times SU(2)$ | $E_7 \times E_7 \times Z_3$ | $SU(6) \times SU(6) \times SU(3)SU(3)$ Z_3 | $SO(10) \times SO(12) \times SU(2)$ | $E_6 \times E_6$ | $SU(8) \times SO(8)$ | | | |
| MT5 | $E_6 \times E_7$ | $SO(10) \times SU(8)$ | $SO(8) \times SO(14)$ | $SO(10) \times SU(8)$ | $SU(9) \times SU(3) \times SU(3)SU(3)$ Z_3 | $E_6 \times SU(6) \times SU(3)$ | | | |
| MT6 | $E_6 \times SO(10) \times SU(3)$ | $SO(10) \times SU(8)$ | $SO(10) \times SO(14)$ | $SU(5) \times E_6 \times SU(3)SU(2)$ | $SO(12) \times SU(7)$ | $E_7 \times SO(10)$ | $SO(14) \times SU(5) \times SU(2)$ | $SO(8) \times SU(9)$ | $SU(9) \times SU(5) \times SU(2)$ $E_6 \times E_8 \times Z_3$ |
| M77 | $SU(3) \times SU(10) \times SU(3)$ | $SO(12) \times E_7$ | $SU(7) \times E_6$ | $SO(12) \times SU(6) \times SU(4)$ Z_2 | $SU(6) \times SU(9)$ | $SO(10) \times SO(12) \times SU(2)$ | $E_6 \times E_7$ | | |
| MT8 | $E_7 \times SO(10)$ | $SO(10) \times SO(12) \times SU(2)$ | $SO(10) \times SO(12) \times SU(3)$ | $SO(14) \times SO(8) \times SU(2)$ | $SU(6) \times SU(8) \times SU(2)$ | $E_7 \times E_6$ | $SU(5) \times SU(9)$ | $SU(6) \times E_6 \times SU(2)$ | $\frac{SU(6)\times SO(12)\times SU(2)SU(2)}{Z_2} SU(8)\times SO(10) SO(12)\times E_6\times SU(2) E_6\times E_7\times Z_2$ |
| 97IV | $SU(7) \times SO(10)$ | $E_6 \times SO(12)$ | $SU(8) \times SO(8) \times SU(2)SU(2)$ Z_2 | $SO(10) \times SO(10) \times SU(2)SU(2)$ | $E_7 \times E_7 \times Z_3$ | $SU(6) \times SU(7) \times SU(3)$ | | | |
| MB0 | $SU(7) \times E_7$ | $E_6 \times SO(12)$ | $E_6 \times E_7$ | $SU(12) \times SU(3)$ | $SU(6) \times SU(10)$ Z_2 | $SO(18) \times SU(3) \times SU(3)$ | | | |
| M81 | $E_6 \times E_6$ | $SO(12) \times SO(10)$ | $SU(7) \times SO(10) \times SU(2)$ | $SU(8) \times SU(6) \times SU(2)$ Z_2 | $E_6 \times SO(10) \times SU(2)$ | $SU(8) \times SU(5) \times SU(3)$ | $E_7 \times E_6$ | $SU(7)\times SO(10)\times SU(3)$ | |
| M82 | $SU(7) \times SO(12)$ | $SO(10) \times E_6 \times SU(2)$ | $E_6 \times SO(12)$ | $SU(5) \times SO(14) \times SU(3)$ | $SU(8) \times SU(7)$ | $SO(10) \times E_7$ | $\frac{SU(4)\times SU(10)\times SU(2)}{Z_2}$ | | |
| M83 | $SU(5) \times SU(10)$ | $SO(10) \times E_7 \times SU(2)$ | $SO(12) \times SO(12)$ | $\frac{SU(4)\times SO(16)\times SU(2)SU(2)}{Z_2}$ | $E_6 \times SU(7)$ | | | | |
| M84 | $E_7 \times E_6 \times Z_2$ | $E_6 \times SO(10)$ | $SU(7) \times SU(7)$ | $SU(6) \times SO(12) \times SU(2)SU(2)$ Z_2 | $E_6 \times E_6$ | $SU(6) \times SU(8) \times SU(2)$ Z_2 | $SO(12) \times SO(10) \times SU(2)$ | | |
| MIOI | $E_6 \times E_8$ | SO(26) | | | | | | | |
| M125 | $E_8 \times SO(10) \times SU(2)$ | SU(14) | $SO(14) \times E_7$ | $SU(4) \times SO(20) \times SU(2)$ Z_2 | $SO(12) \times E_7$ | | | | |
| MI39 | $E_6 \times E_7$ | $\frac{SU(14)\times SU(2)}{Z_2}$ | $SU(3) \times SO(22)$ | $SU(7) \times E_8$ | | | | | |
| M140 | $\frac{SU(14) \times SU(2)}{\mathbb{Z}_2}$ | $E_7 \times E_6$ | $SO(22) \times SU(3)$ | | | | | | |
| M141 | $E_7 \times SO(12)$ | SU(14) | $SO(20) \times SU(2) \times SU(4)$ Z_2 | | | | | | |
| M144 | $SU(12) \times SU(4)$ Z_2 | $\frac{SU(12) \times SU(4)}{Z_2}$ | $SO(12) \times E_7$ | $E_6 \times E_6$ | | | | | |
| MI54 | $SU(8) \times SO(10)$ | $SU(9) \times SU(3)SU(3)SU(3)$ Z_3 | | | | | | | |
| M155 | $E_7 \times E_7 \times Z_4$ | $\times SU(12)$ | $SO(12)\times SO(12)\times SU(2)SU(2)$ Z_2 | | | | | | |
| MIS7 | $SO(10) \times E_8$ | $E_7 \times SU(7)$ | $SU(13) \times SU(2)$ | $SO(18) \times SU(5)$ | $E_6 \times SO(14)$ | | | | |
| M158 | $E_6 \times SU(7) \times SU(2)$ | $SO(10) \times SU(9)$ | $SO(10) \times SO(14)$ | $E_7 \times E_6$ | $\times SU(11) \times SU(3)SU(2)$ | | | | |
| M159 | $SU(12) \times SU(2) SU(2)$ Z_2 | $SO(14) \times SU(7)$ | $SO(10) \times E_7$ | $E_6 \times E_6 \times SU(2)$ | | | | | |
| M166 | $SU(8) \times SO(12) \times SU(2)$ Z_2 | $SO(10) \times SO(14) \times SU(2)$ | $SO(12) \times E_6 \times SU(2)$ | $SU(12) \times SU(2)$ | $E_7 \times E_6$ | $E_7 \times E_7 \times Z_4$ | | | |
| M167 | $E_8 \times E_6 \times Z_4$ | $E_6 \times E_7 \times SU(2)$ | $\times SU(13)$ | $SU(6) \times SO(16) \times SU(2)$ Z_2 | $SO(12) \times SO(14)$ | $E_7 \times SO(10) \times SU(2)$ | | | |
| M169 | $E_7 \times E_7 \times Z_4$ | $E_6 \times SU(8)$ | $\times SU(12) \times SU(2)SU(2)$ | $SO(16) \times SO(8)$ | | | | | |
| M176 | $SO(16) \times SO(10)$ | $SU(6) \times E_7 \times SU(2)SU(2)$ Z_2 | $E_6 \times E_8 \times Z_4$ | $\times SU(13)$ | | | | | |
| MIS6 | $SU(8) \times SU(8)$ Z_2 | $SU(12) \times SU(2)SU(2)$ Z_2 | $E_7 \times E_7 \times Z_4$ | $E_6 \times E_6$ | $SO(12) \times SO(12)$ | | | | |
| MIS7 | $E_6 \times E_7$ | $SO(12) \times E_6$ | $\frac{SU(6)\times SU(10)}{Z_2}$ | $SU(12) \times SU(3)$ | | | | | |
| M247 | $SO(24)\times SU(2)\times SU(2)$ Z_2 | $E_7 \times E_7$ | $SO(12) \times E_8$ | | | | | | |
| M248 | $E_7 \times SO(12)$ | ×SU(14) | $SO(20) \times SU(2)SU(4)$ Z_2 | | | | | | |
| MZ70 | $\frac{SU(14) \times SU(2)}{Z_2}$ | $E_7 \times E_6$ | | | | | | | |
| M275 | $\frac{SU(14) \times SU(2)}{Z_2}$ | $E_7 \times E_6$ | $SO(22) \times SU(3)$ | | | | | | |
| M282 | $E_6 \times E_6$ | $\frac{SU(12) \times SU(4)}{Z_2}$ | | | | | | | |
| M473 | SO(26) | $E_8 \times E_6$ | | | | | | | |
| M476 | $E_7 \times E_7$ | $SO(24)\times SU(2)SU(2)$ Z_2 | | | | | | | |
| M497 | $E_7 \times E_6$ | $SU(14) \times SU(2)$ Z_2 | | | | | | | |
| | | | | | | | | | |

Figure 4: Gauge groups Picard 16 (i.e. four moduli)

| M 109 | $SO(22) \times SU(2)$ | $E_7 \times E_6$ | $SU(6) \times E_8$ | | | | | | | |
|-------|---|--|--|---|--|--|---------------------------------------|--|--|-----------------------|
| M124 | $SU(12)\times SU(2)\times SU(2)$ Z_2 | $E_8 \times SU(4) \times SU(3)$ | $SO(20) \times SU(2)$ | $SO(10) \times E_7$ | $E_6 \times E_6$ | | | | | |
| M143 | $SU(6) \times E_7$ | $E_6 \times SO(12)$ | $SO(18) \times SU(2) \times SU(3)$ | $SU(12) \times SU(2)$ | | | | | | |
| M161 | $SO(10) \times E_7$ | $SU(7) \times SO(10)$ | $SU(4) \times E_6 \times SU(2)SU(3)$ | $SU(2) \times SU(9) \times SU(2)SU(3)$ | $SU(8) \times SO(8)$ | $SO(10) \times SU(7)$ | $SO(14) \times SO(8)$ | | | |
| M165 | $E_6 \times SO(12)$ | $E_7 \times SU(7)$ | $SO(18) \times SU(3) \times SU(2)$ | $SU(6) \times E_7$ | $SU(5) \times E_8$ | $SU(2) \times SU(12)$ | | | | |
| 891W | $E_7 \times E_6 \times Z_4$ | $E_6 \times SU(6) \times SU(2)$ | $E_7 \times SU(3) \times SU(2)SU(3)$ | $SU(11) \times SU(2)$ | $SU(8) \times SO(10)$ | $SO(16) \times SU(5)$ | $SO(8) \times SO(14)$ | | | |
| M170 | $SO(12) \times SU(6) \times SU(2)$ | $SO(10) \times SO(10) \times SU(2)SU(2)$ | $SO(12) \times E_7 \times Z_4$ | $SU(2) \times SU(10)$ | | | | | | |
| M171 | $E_7 \times SO(8) \times SU(2)$ | $SU(6) \times E_6$ | $SO(12) \times SO(12)$ | $SU(4) \times SU(10)$ | $SO(16) \times SU(2) \times SU(2)SU(2)SU(2)$ Z_2 | | | | | |
| M172 | $E_7 \times SO(10)$ | $SO(12) \times SO(10) \times SU(2)$ | $E_7 \times SO(12) \times Z_4$ | $SU(6) \times E_6$ | $SO(12) \times SU(4) \times SU(2)SU(4)$ Z_2 | $SU(6) \times SU(8)$ | $SO(8) \times SO(12) \times SU(2)$ | $SU(10) \times SU(2) \times SU(2)$ | | |
| M178 | $E_6 \times SU(7)$ | $E_6 \times SU(7)$ | $SO(16) \times SU(4)$ | $E_7 \times SU(4) \times SU(3)$ | $SO(14) \times SO(10)$ | $SU(11) \times SU(2) \times SU(2)$ | $SO(8) \times E_8$ | | | |
| M179 | $SO(14) \times SU(5)$ | $E_6 \times E_6$ | $SO(10) \times SO(12)$ | $SO(10) \times SU(7) \times SU(2)$ | $SU(6) \times E_6 \times SU(2)$ | $SU(10) \times SU(2) \times SU(2)$ | $SO(10) \times SU(7) \times SU(2)$ | | | |
| M180 | $SO(12) \times SU(7)$ | $SO(14) \times SU(6)$ | $SU(11) \times SU(2)$ | $E_6 \times SO(10) \times SU(2)$ | $E_7 \times SO(10)$ | $SO(14) \times SU(5) \times SU(2)$ | $SU(5) \times E_7$ | | | |
| M182 | $SO(10) \times E_6 \times SU(2)$ | $SU(4) \times SO(14) \times SU(3)$ | $E_6 \times SO(10)$ | $SU(10)\times SU(2)\times SU(2)SU(2)$ Z_2 | $SU(7) \times SU(7)$ | $SU(6) \times SO(12)$ | $E_7 \times SO(8)$ | $SO(10) \times E_6$ | | |
| M188 | $SU(6) \times E_7$ | $SU(2)\times SU(10)\times SU(4)$ Z_2 | $E_6 \times SO(10)$ | $SU(3) \times SU(11)$ | | | | | | |
| 081 M | $SU(3) \times SU(11)$ | $E_7 \times SO(10)$ | $SU(10)\times SU(4)\times SU(2)$ Z_2 | $SO(12) \times E_6$ | $E_6 \times SO(10)$ | $SU(6) \times E_7$ | $SO(18) \times SU(2) \times SU(2)$ | | | |
| M190 | $E_6 \times SO(10)$ | $SU(6)\times SU(8)\times SU(2)$ Z_2 | $SO(12) \times SO(10)$ | $E_7 \times SU(5) \times SU(2)$ | $SO(16) \times SU(3) \times SU(3)$ | $SU(11) \times SU(2)$ | $E_6 \times E_7 \times Z_4$ | $E_6 \times E_6$ | | |
| 161M | $SU(6) \times E_6$ | $SU(10) \times SU(2) \times SU(3)$ | $SU(9) \times SU(5)$ | $SO(12) \times SO(10)$ | $SO(10) \times E_7$ | | | | | |
| M192 | $SU(10) \times SU(2) \times SU(2) SU(2)$ Z_2 | $E_6 \times SO(10)$ | $SU(7) \times SU(7)$ | $SU(6) \times SO(12)$ | | | | | | |
| 961 M | $SO(10) \times E_6$ | $SO(10) \times SO(10)$ | $SU(8) \times SU(4) \times SU(4)$ Z_2 | $SU(10)\times SU(2)\times SU(2)SU(2)$ Z_2 | | | | | | |
| 761M | $SO(8) \times SO(8) \times SU(2)SU(2)SU(2)$ Z_2 | $SU(6) \times SU(6)$ | $SO(12) \times SO(12) \times Z_2$ | | | | | | | |
| 861M | $SO(10) \times SO(10) \times SU(2)$ | $SU(6) \times SU(6)$ | $SU(6) \times SO(8) \times SU(2)SU(2)$ | $SO(12) \times SO(12) \times Z_2$ | | | | | | |
| M 199 | $SO(12) \times SU(6)$ | $SO(10) \times SU(6) \times SU(2)SU(2)$ | $SU(3) \times SU(9)$ | | | | | | | |
| M200 | $SU(9) \times SU(4)$ | $SU(3) \times E_6 \times SU(3)SU(2)SU(2)$ | $SU(8) \times SO(8)$ | $SO(14) \times SU(5)$ | $SO(12) \times SU(6)$ | | | | | |
| M201 | $SU(6) \times SU(6) \times SU(2)$ | $SU(6)\times SO(8)\times SU(2)SU(2)SU(2)$ Z_2 | $E_6 \times SO(12) \times Z_2$ | $SO(10) \times SO(8) \times SU(2)SU(2)$ | $SO(10) \times SO(12)$ | $SU(7) \times SU(5)$ | $SU(6) \times SO(10)$ | | | |
| M202 | $SO(10) \times SU(6) \times SU(2)$ | $SU(3)\times SU(6)\times SU(3)SU(3)SU(3)$ Z_3 | $SU(7) \times SO(8)$ | | | | | | | |
| M203 | $SU(8) \times SU(6)$ | $SO(8) \times E_6 \times SU(2)$ | $SU(7) \times SO(12)$ | $\boxed{SO(14) \times SU(3) \times SU(2)SU(3)}$ | $SU(10) \times SU(3)$ | $E_7 \times SU(5)$ | $SU(6) \times SO(12)$ | $SO(12) \times E_6$ | | |
| M204 | $E_6 \times SU(5) \times SU(2)SU(2)$ | $SO(14) \times SU(5)$ | $SU(3) \times SU(10)$ | $SU(4) \times E_7 \times SU(2)$ | $SU(8) \times SO(10)$ | | | | | |
| M205 | $SU(5) \times E_6 \times SU(3)$ | $SO(8) \times SU(8)$ | $SO(10) \times SU(7)$ | $SU(4) \times SO(14)$ | $SU(9) \times SU(3) \times SU(2)SU(2)$ | | | | | |
| M206 | $SU(6) \times E_6$ | $SU(5) \times SU(9)$ | $SO(10) \times SO(12)$ | $SO(16) \times SU(3) \times SU(2)$ | $SU(3) \times SU(10) \times SU(2)$ | $E_7 \times SO(10)$ | $SU(7) \times E_6$ | $E_7 \times SU(5) \times SU(2)$ | | |
| M207 | $SU(9) \times SU(3) \times SU(2)$ | $SO(10) \times SO(10) \times SU(2)$ | $E_6 \times SU(6)$ | $SU(5) \times SO(12) \times SU(3)$ | $SU(6) \times SU(7) \times SU(2)$ | $SU(5) \times SO(12) \times SU(2)$ | $SO(12) \times E_6$ | $E_7 \times SO(10)$ | $SO(10) \times SU(7)$ | |
| M208 | $SU(6) \times SO(10) \times SU(2)$ | $SO(12) \times SO(8) \times SU(2)$ | $SU(6) \times SU(6) \times SU(2)SU(2)$ | $E_6 \times E_6 \times Z_2$ | $SU(4) \times SU(8)$ | | | | | |
| M209 | $SU(6) \times SO(10) \times SU(2)$ | $SO(10) \times E_7$ | $SU(7) \times SO(10)$ | $SU(5) \times E_6 \times SU(2)$ | $SO(12) \times SO(8)$ | $SO(10) \times SU(6) \times SU(3)$ | $E_6 \times E_6$ | $SU(8) \times SU(3) \times SU(3)SU(2)$ | $SU(5) \times SU(8)$ | |
| M210 | $SO(10) \times SO(10) \times SU(3)$ | $SO(12) \times SU(5) \times SU(2)$ | $E_7 \times SO(10) \times Z_2$ | $SU(8) \times SU(5)$ | $E_6 \times SU(4) \times SU(2)SU(2)$ | $SU(8) \times SO(8)$ | $SO(8) \times SO(14)$ | | | |
| M211 | $SO(10) \times SO(12)$ | $SO(10) \times E_6$ | $SU(5) \times SO(12)$ | $SU(7) \times SO(8) \times SU(2)$ | $SU(7) \times SU(6)$ | $E_6 \times SO(8) \times SU(2)$ | $SU(5)\times SO(10)\times SU(3)SU(2)$ | $SU(4) \times SU(8) \times SU(2)$ | $SO(10) \times SU(6) \times SU(2)$ | |
| M212 | $SU(7) \times SO(8)$ | $SO(10) \times SO(12)$ | $SU(5) \times SU(7) \times SU(2)$ | $SO(10) \times SO(10) \times SU(2)$ | $SU(7) \times SO(8) \times SU(2)$ | $E_6 \times SO(10)$ | $E_6 \times E_7 \times Z_3$ | $SU(5)\times SO(10)\times SU(2)SU(2)$ | $SU(5) \times SU(6) \times SU(2)SU(3)$ S | $SU(6) \times SO(10)$ |
| M213 | $SO(12) \times SO(10) \times SU(2)$ | $SU(8) \times SU(5)$ | $E_6 \times SU(5)$ | $SO(10) \times E_7 \times Z_2$ | $SU(7) \times SU(6)$ | $SO(12) \times SU(4) \times SU(2)SU(2)SU(2)$ Z_2 | $SO(12) \times SO(8) \times SU(2)$ | | | |
| M214 | $SU(4) \times SU(9)$ | $SU(6) \times SO(12)$ | $E_7 \times SO(8)$ | $SO(10) \times SO(12)$ | $SU(5) \times E_6 \times SU(2)$ | $SU(7) \times SO(10)$ | $E_6 \times SO(10) \times SU(2)$ | $SU(8) \times SU(5) \times SU(2)$ | $SO(14) \times SU(4) \times SU(2)$ | |
| M215 | $SO(10) \times E_6$ | $E_6 \times E_6$ | $SU(6) \times SU(6) \times SU(2)SU(2)$ Z_2 | $SO(10) \times SO(10)$ | $SO(8) \times SO(12)$ | $SU(7) \times SU(5) \times SU(2)$ | $SO(10) \times SU(6) \times SU(2)$ | $SO(10) \times SO(10) \times SU(2)$ | $E_6 \times E_6 \times Z_2$ | |

Figure 5: Gauge groups Picard 15 (i.e. five moduli), Part 1

| M216 | $SO(8) \times E_6$ | $SO(12) \times SU(5) \times SU(2)$ | $SU(4) \times SU(8) \times SU(2)SU(2)$ | $SU(7) \times SU(6)$ | $SO(10) \times SO(10) \times SU(2)$ | $SO(10) \times E_7 \times Z_2$ | $E_6 \times SO(10)$ | $E_6 \times SO(10) \mid SO(12) \times SO(10) \mid$ |
|------|--|---|---|--|-------------------------------------|--------------------------------|---------------------|--|
| M217 | $SO(10) \times SO(10)$ | $E_6 \times SU(6)$ | $E_6 	imes E_6$ | $\frac{SU(6)\times SU(6)\times SU(4)}{Z_2}$ | $SU(3) \times SU(9) \times SU(3)$ | | | |
| M218 | $E_6 \times E_6 \times Z_2$ | $SU(6) \times SU(6)$ | $SO(10) \times SO(10)$ | $\frac{SU(6) \times SU(6) \times SU(2) \times SU(2)}{Z_2}$ | | | | |
| M219 | $SU(6) \times SO(10)$ | $E_6 \times SO(10)$ | $\frac{SU(4) \times SU(8) \times SU(2)SU(2)}{Z_2}$ | $SU(5) \times SU(7) \times SU(3)$ | $SO(10) \times SO(10)$ | | | |
| M238 | $SO(22) \times SU(2)$ | $SU(6) \times E_8$ | $E_7 	imes E_6$ | | | | | |
| M272 | $E_6 	imes E_6$ | $\frac{SU(12) \times SU(2)SU(2)}{Z_2}$ | $SU(2) \times SO(20)$ | | | | | |
| M277 | $E_7 \times SO(10)$ | SU(13) | | | | | | |
| M291 | SU(12) | $\frac{E_7 \times SU(2) \times SU(4) SU(2) SU(2)}{Z_2}$ | $SO(16) \times SO(8)$ | | | | | |
| M300 | $SO(12) \times E_6$ | $SU(2) \times SU(12)$ | $SO(18) \times SU(2)SU(3)$ | $E_7 \times SU(6)$ | | | | |
| M301 | $E_7 \times SO(8) \times SU(2)$ | SU(12) | $\frac{SO(16) \times SU(2) \times SU(2) \times SU(4)}{Z_2}$ | $SO(12) \times SO(12)$ | | | | |
| M305 | $E_7 \times SU(7)$ | $SO(18) \times SU(3) \times SU(2)$ | $E_6 \times SO(12)$ | $SU(12) \times SU(2)$ | | | | |
| M309 | $E_7 \times SO(10)$ | $\frac{SU(12)\times SU(2)\times SU(2)}{Z_2}$ | $E_6 	imes E_6$ | $SO(20) \times SU(2)$ | | | | |
| M310 | $E_7 \times SO(10) \times SU(2)$ | $SO(12) \times SO(12)$ | SU(12) | $\frac{SU(4)\times SO(16)\times SU(2)SU(2)}{Z_2}$ | | | | |
| M311 | $SU(4) \times SO(18)$ | SU(13) | $SO(8) \times E_8$ | $SO(14) \times E_6$ | $E_7 \times SU(6)$ | | | |
| M313 | SU(13) | $SU(3) \times SO(20)$ | $SO(10) 	imes E_7$ | $SU(5) \times E_8 \times SU(2)$ | | | | |
| M317 | $SO(12) \times E_6$ | $SU(2) \times SO(18) \times SU(3)$ | $SU(12) \times SU(2)$ | $SU(6)	imes E_7$ | | | | |
| M320 | $\frac{SU(10) \times SU(2)SU(4)}{Z_2}$ | $SO(10) \times E_6$ | $SU(3) \times SU(11)$ | $SU(6)	imes E_7$ | | | | |
| M344 | $SU(11) \times SU(3)$ | $SO(10) \times E_6$ | $\frac{SU(10) \times SU(4) \times SU(2)}{Z_2}$ | $SO(12) \times E_6$ | $E_7 	imes SO(10)$ | | | |
| M350 | $SO(10) \times E_6$ | $\frac{SU(10) \times SU(2) \times SU(4)}{Z_2}$ | $SU(11) \times SU(3)$ | | | | | |
| M359 | $SU(8) \times SO(8)$ | $SU(7) \times SO(10)$ | $SU(9) \times SU(2)SU(3)SU(2)$ | | | | | |
| M362 | $E_6 \times SU(6) \times SU(2)$ | $E_6	imes E_7	imes Z_4$ | $SO(10) \times SU(8)$ | $SO(8) \times SO(14)$ | $SU(11) \times SU(2)$ | | | |
| M363 | $SO(10) \times SU(7) \times SU(2)$ | $SU(10) \times SU(2)SU(2)$ | $SO(10) \times SO(12)$ | $E_6 	imes E_6$ | | | | |
| M364 | $E_6 \times SO(10) \times SU(2)$ | $SO(14) \times SU(5) \times SU(2)$ | $SO(12) \times SU(7)$ | $SU(11) \times SU(2)$ | $SO(10) 	imes E_7$ | | | |
| M365 | $SO(8) \times E_7$ | $SO(14) \times SU(6)$ | SU(12) | $E_6 \times E_6 \times SU(2)$ | | | | |

Figure 6: Gauge groups Picard 15 (i.e. five moduli) Part 2

| - | | | | | | U(2) | | | | | | | | | | | | |
|---|---------------------------------|---|----------------------------|----------------------------|---|---|---|-----------------------|----------------------------|---|--|--|-----------------------|--|---------------------|---------------------|-----------------------|----------------------------|
| | | | | | | $SU(11) \times SU(2)$ | | | | | | | | | | | | |
| | $E_8 \times SO(10) \times Z_4$ | | | $SO(12) \times SO(10)$ | | $E_6 \times SO(10)$ | | | | | | | | | | | | |
| | $SU(5) \times SO(16)$ | $E_7 	imes E_6 	imes Z_4$ | | $SU(9) \times SU(5)$ | $SU(7) \times SU(7)$ | $\frac{SU(6) \times SU(8) \times SU(2)}{Z_2}$ | | | | | | | | | | | | |
| | $E_7 \times SU(5) \times SU(2)$ | $SO(10) \times SO(12) \times SU(2)$ | $SO(16) \times SU(4)$ | $SU(6) \times E_6$ | $SO(12) \times SU(6)$ | $E_7 	imes E_6 	imes Z_4$ | $\frac{SO(16) \times SU(4)SU(2)SU(2)}{Z_2}$ | $E_7	imes E_6$ | $SU(12) \times SU(2)$ | | $\frac{SU(12) \times SU(2) \times SU(2)}{Z_2}$ | $\frac{SU(10) \times SU(2)SU(4)}{Z_2}$ | SU(13) | $SO(20) \times SU(2)$ | | | | |
| | SU(12) | SU(11) | $SU(11) \times SU(2)SU(2)$ | $SO(10) \times E_7$ | $SO(10) \times E_6$ | $E_6 \times E_6$ | SU(12) | $SU(6) 	imes E_8$ | $SO(18) \times SU(2)SU(3)$ | $E_7 \times SO(12)$ | $E_6 	imes E_6$ | $SU(11) \times SU(3)$ | $SO(20) \times SU(3)$ | $E_6 	imes E_6$ | $SO(10) \times E_8$ | $SO(10) \times E_7$ | $E_7	imes E_6$ | $SU(12) \times SU(2)SU(2)$ |
| | $SO(10) \times SO(14)$ | $\frac{SU(6) \times SO(12) \times SU(2)SU(2)}{Z_2}$ | $E_6 \times SU(7)$ | $SU(10) \times SU(2)SU(3)$ | $\frac{SU(10) \times SU(2)SU(2)SU(2)}{Z_2}$ | $SO(10) \times SO(12)$ | $SO(12) \times SO(12)$ | $SU(2) \times SO(22)$ | $SO(12) \times E_6$ | $\frac{SO(20) \times SU(2) \times SU(2)SU(2)}{Z_2}$ | $E_7 	imes SO(10)$ | $E_6 \times SO(10)$ | $E_7 	imes SO(10)$ | $\frac{SU(12) \times SU(2)SU(2)}{Z_2}$ | SO(24) | SU(13) | $SO(22) \times SU(2)$ | <u> </u> |
| | M375 | M376 | M377 | M379 | M384 | M402 | M513 | M529 | M537 | M544 | M549 | M572 | M587 | M588 | M792 | M859 | 998M | Moor |

Figure 7: Gauge groups Picard 15 (i.e. five moduli) Part 3

1 2 Fibers

| M531 | $SO(18) \times SU(2)SU(2)$ | $SU(6) \times E_7$ | |
|-------|----------------------------|---|--|
| M936 | $SU(9) \times SU(3)SU(2)$ | $SO(10) \times SO(10)$ | |
| M959 | $E_6 \times SO(10)$ | $\frac{SU(10)\times SU(2)\times SU(2)SU(2)}{Z_2}$ | |
| M987 | $E_6 \times SO(8)$ | $SU(10) \times SU(3)$ | |
| M1191 | SU(12) | $SO(8) \times E_7$ | |
| M1201 | $SU(5) \times E_8$ | SO(22) | |
| M1220 | $SO(20) \times SU(2)$ | $SO(10) \times E_7$ | |
| M1328 | $SO(10) \times E_6$ | $SU(11) \times SU(2)$ | |
| M1346 | $E_6 \times E_6$ | SO(20) | |

Figure 8: Picard 14 (i.e. 6 moduli) with two fibers

| M962 | $SU(9) \times SU(2)SU(2)$ | $SU(5) \times E_6$ |
|-------|---------------------------------|---------------------------------|
| M1084 | $SU(6) \times SO(8)$ | SU(9) |
| M1395 | $SU(9) \times SU(3)$ | $E_6 \times SU(4)$ |
| M1430 | $SU(2) \times SO(16)$ | $SU(6) \times E_6$ |
| M1556 | SO(20) | $E_8 \times SU(2) \times SU(3)$ |
| M1621 | $SU(3) \times E_8 \times SU(2)$ | SO(20) |
| M1703 | $SU(4) \times E_7$ | SU(11) |
| M1728 | $SO(8) \times E_6$ | $SU(10) \times SU(2)$ |
| M1729 | $SU(5) \times E_7$ | $SO(18) \times SU(2)$ |
| M1740 | SO(18) | $SO(10) \times E_6$ |
| M1792 | SU(10) | $SO(10) \times SO(10)$ |
| M1980 | SO(20) | $E_8 \times SU(3)SU(2)$ |
| M1985 | SO(22) | $SU(4) \times E_8$ |

Figure 9: Picard 13 (i.e. 7 moduli) with two fibers

| M1389 | $SU(3) \times SO(14)$ | SU(9) |
|-------|---------------------------------|---------------------------------|
| M1432 | $SU(3) \times E_6 \times SU(3)$ | SO(14) |
| M1675 | $E_6 \times SU(3)$ | $SU(9) \times SU(3)$ |
| M1701 | $SO(14) \times SU(4)$ | $SU(2) \times E_7$ |
| M1872 | $SU(5) \times SO(10)$ | $SU(2) \times SU(8)$ |
| M1902 | $SO(8) \times SO(8)$ | $SU(8) \times SU(2)$ |
| M2014 | $SO(16) \times SU(2)$ | $E_7 \times SU(3)SU(2)$ |
| M2021 | $E_7 \times SU(2) \times SU(2)$ | SU(10) |
| M2037 | $SO(16) \times SU(3)$ | $SU(2) \times E_7 \times SU(2)$ |
| M2054 | SO(18) | $SU(2) \times E_8$ |
| M2055 | $SO(16) \times SU(2)$ | $E_7 \times SU(2) \times SU(3)$ |
| M2126 | SU(11) | $SU(3) \times E_7$ |
| M2133 | SU(10) | $E_7 \times SU(3)$ |
| M2134 | $SU(3) \times E_7 \times SU(2)$ | $SO(16) \times SU(2)$ |
| M2236 | $SU(5) \times E_6$ | SO(16) |
| M2204 | $SU(4) \times E_6$ | $SU(9) \times SU(2)$ |
| M2197 | SU(9) | $SO(10) \times SO(8)$ |
| M2153 | SO(16) | $SO(10) \times SO(10)$ |
| M2407 | $SO(16) \times SU(2)$ | $E_7 \times SU(3)SU(2)$ |
| M2408 | $SU(3) \times E_7$ | SU(11) |
| M2421 | $SO(18) \times SU(2)$ | $SU(4) \times E_7$ |
| M2428 | $E_8 \times SU(2)$ | SO(18) |
| M2732 | SO(20) | $SU(2) \times E_8 \times SU(2)$ |

Figure 10: Picard 12 (i.e. 8 moduli) with two fibers

| M1529 | $SU(4) \times SU(4)$ | $SO(8) \times SO(8) \times Z_2$ |
|-------|------------------------------------|------------------------------------|
| M1663 | $E_6 \times SU(3)$ | $SU(9) \times SU(2)$ |
| M1734 | SO(14) | $SU(3) \times E_6 \times SU(2)$ |
| M1878 | $SO(10) \times SU(3) \times SU(2)$ | SU(7) |
| M2015 | $E_6 \times SU(3)SU(2)$ | SO(14) |
| M2058 | $SU(9) \times SU(2)$ | $E_6 \times SU(3)$ |
| M2064 | $E_7 \times SU(2)$ | $SU(2) \times E_8 \times Z_4$ |
| M2154 | $SO(14) \times SU(2)$ | $SU(2) \times E_7$ |
| M2155 | $SU(2) \times SO(14)$ | $E_6 \times SU(2) \times SU(2)$ |
| M2169 | $E_6 \times SU(2)$ | $SU(8) \times SU(2)$ |
| M2240 | $SU(2) \times SO(12)$ | $SU(4) \times SO(10) \times SU(2)$ |
| M2328 | $SU(7) \times SU(2)$ | $SO(8) \times SU(4)$ |
| M2390 | $E_8 \times SU(2) \times Z_4$ | $E_7 \times SU(2)$ |
| M2398 | $E_7 \times SU(2)$ | SU(10) |
| M2427 | SO(16) | $SU(2) \times E_8 \times Z_4$ |
| M2441 | $E_7 \times SU(2)$ | $SU(2) \times E_8 \times Z_4$ |
| M2472 | $SO(14) \times SU(2)$ | SU(9) |
| M2473 | $SO(12) \times SU(3)SU(2)$ | $SU(2) \times SO(12) \times SU(2)$ |
| M2501 | $SU(2) \times E_7$ | SU(9) |
| M2503 | $SU(2) \times E_6 \times SU(2)$ | $SU(8) \times SU(2)$ |
| M2517 | $SU(8) \times SU(3)$ | $E_6 \times SU(2)$ |
| M2525 | $SO(14) \times SU(2)$ | $SU(2) \times E_7$ |
| M2527 | $E_7 \times SU(2)$ | $SO(14) \times SU(2)$ |
| M2529 | $SU(2) \times E_6 \times SU(3)$ | SO(14) |
| M2530 | $SU(2) \times E_6 \times SU(2)$ | $SO(14) \times SU(2)$ |
| M2537 | E_7 | $SU(3) \times SO(14)$ |
| M2551 | SU(8) | $SO(12) \times SU(2)SU(2)$ |
| | | |

Figure 11: Picard 11 (i.e. 9 moduli) with two fibers (Part 1)

| M2567 | SU(8) | $SO(8) \times SO(8)$ |
|-------|-------------------------------|---------------------------------|
| M2571 | $SU(3) \times SO(12)$ | SU(9) |
| M2600 | $SU(9) \times SU(2)$ | $SU(3) \times E_6$ |
| M2606 | $SU(3) \times E_6$ | $SU(8) \times SU(2)$ |
| M2607 | $SU(5) \times SO(10)$ | SO(14) |
| M2609 | SO(14) | $SU(3) \times E_6 \times SU(2)$ |
| M2636 | SU(8) | $SU(4) \times SO(10)$ |
| M2727 | SO(16) | $E_8 \times SU(2) \times Z_4$ |
| M2744 | $E_8 \times SU(2) \times Z_4$ | $E_7 \times SU(2)$ |
| M2787 | E_8 | $SU(3) \times E_7$ |
| M2794 | SU(10) | $E_7 \times SU(2)$ |
| M2800 | E_8 | SO(16) |
| M2803 | $E_7 \times SU(2)$ | SU(10) |
| M2804 | E_7 | $SO(14) \times SU(3)$ |
| M2835 | $E_6 \times SU(3)SU(2)$ | SO(14) |
| M2836 | $E_7 \times SU(2)$ | SU(9) |
| M2860 | $SU(3) \times E_6$ | $SU(9) \times SU(2)$ |
| M2864 | $E_7 \times SU(2)$ | $SO(14) \times SU(2)$ |
| M2874 | SO(16) | $SU(4) \times E_6$ |
| M3071 | E_8 | $E_7 \times SU(3)$ |
| M3112 | $SO(16) \times SU(2)$ | $SU(2) \times E_7 \times SU(2)$ |
| M3120 | SU(10) | $SU(2) \times E_7$ |
| M3340 | SO(18) | E_8 |

Figure 12: Picard 11 (i.e. 9 moduli) with two fibers (Part 2)

| M1598 | $SU(2) \times SU(6)$ | $SO(12) \times SU(2) \times SU(2) \times Z_4$ |
|-------|-----------------------------------|---|
| M1795 | $SU(2) \times SU(7)$ | $SO(8) \times SU(3)$ |
| M1886 | $SU(2) \times SU(5) \times SU(2)$ | $E_6 \times SU(3) \times Z_3$ |
| M2027 | $SU(6) \times SU(3)SU(2)$ | $SO(10) \times SU(2)$ |
| M2061 | E_7 | $E_6 \times SU(2)$ |
| M2156 | E_6 | $E_7 \times SU(2) \times Z_4$ |
| M2159 | SO(12) | $SU(2) \times E_6$ |
| M2171 | $SO(10) \times SU(3)$ | SU(8) |
| M2246 | SO(10) | $SU(2) \times SO(10) \times SU(2)$ |
| M2248 | SU(7) | $SU(3) \times SO(10)$ |
| M2249 | $SO(10) \times SU(2)$ | $SU(3) \times SU(6) \times SU(2)$ |
| M2266 | $SO(10) \times SU(2)$ | SU(7) |
| M2267 | $SU(2) \times SO(12)$ | SU(7) |
| M2354 | SU(6) | $SU(4) \times SU(4)$ |
| M2440 | E_7 | $E_6 \times SU(2)$ |
| M2464 | $SU(6) \times SU(2)SU(3)$ | $SO(10) \times SU(2)$ |
| M2526 | $E_7 \times SU(2) \times Z_4$ | $SO(12) \times SU(2)$ |
| M2531 | $E_6 \times SU(2)$ | SO(12) |
| M2533 | SO(14) | SU(8) |
| M2534 | $SO(10) \times SU(3)$ | SU(8) |
| | | |

Figure 13: Picard 10 (i.e. 10 moduli) with two fibers (Part 1/4)

| M2573 | $SO(10) \times SU(2)$ | $SU(2) \times SU(7)$ |
|-------|------------------------------------|-------------------------------|
| M2608 | $SU(2) \times E_6$ | $SU(2) \times SU(7)$ |
| M2619 | $SU(3) \times SO(8)$ | $SU(7) \times SU(2)$ |
| M2622 | $SU(2) \times SO(10) \times SU(2)$ | $SU(2) \times SO(12)$ |
| M2638 | $SU(4) \times SO(8)$ | $SU(2) \times SU(6)$ |
| M2640 | $SU(3) \times SU(6)$ | $SU(2) \times SO(10)$ |
| M2696 | $SU(6) \times SU(2)$ | $SU(4) \times SU(4)$ |
| M2728 | E_6 | $E_7 \times SU(2) \times Z_4$ |
| M2784 | $SO(12) \times SU(2)$ | SU(8) |
| M2788 | $SU(3) \times E_6$ | $E_8 \times Z_3$ |
| M2793 | $SU(2) \times E_6 \times SU(2)$ | SO(14) |
| M2797 | $E_7 \times SU(2) \times Z_4$ | $SO(12) \times SU(2)$ |
| M2811 | $SU(8) \times SU(2)$ | $SU(2) \times E_6$ |
| M2859 | $SU(8) \times SU(2)$ | $E_6 \times SU(2)$ |
| M2861 | $E_7 \times SU(2) \times Z_4$ | SU(8) |
| M2862 | E_6 | $E_7 \times SU(2) \times Z_4$ |
| M2863 | $SU(2) \times E_7 \times Z_4$ | $SO(12) \times SU(2)$ |
| M2865 | E_6 | $SU(2) \times SO(12)$ |
| M2868 | $SO(10) \times SU(2) \times SU(3)$ | SO(12) |
| M2869 | $SO(12) \times SU(2)$ | SU(8) |

Figure 14: Picard 10 (i.e. 10 moduli) with two fibers (Part 2/4)

| M2875 | $SU(2) \times E_6$ | E_7 |
|-------|------------------------------------|------------------------------------|
| M2877 | $SU(2) \times SO(12)$ | E_7 |
| M2912 | $E_6 \times SU(2)$ | $SU(7) \times SU(2)$ |
| M2914 | $SO(12) \times SU(2)$ | SU(7) |
| M2917 | $SO(10) \times SU(2)$ | $SU(7) \times SU(2)$ |
| M2918 | $SU(6) \times SU(3)SU(2)$ | $SO(10) \times SU(2)$ |
| M2919 | SU(7) | $SO(10) \times SU(2) \times SU(2)$ |
| M2942 | $SU(2) \times E_6$ | $SU(7) \times SU(2)$ |
| M2947 | $SU(7) \times SU(3)$ | E_6 |
| M2948 | $SU(3) \times SO(10) \times SU(2)$ | SO(12) |
| M2955 | $SU(2) \times E_6$ | SO(12) |
| M2956 | $SU(2) \times SO(10) \times SU(2)$ | $SO(12) \times SU(2)$ |
| M2970 | SU(7) | $SU(4) \times SO(8)$ |
| M2982 | E_6 | $SU(7) \times SU(2)$ |
| M2990 | SU(7) | $SO(10) \times SU(3)$ |
| M3002 | $SU(5) \times SU(5)$ | SO(12) |
| M3005 | $SO(10) \times SU(3)$ | SU(8) |
| M3066 | $E_8 \times Z_3$ | SU(9) |
| M3067 | $E_6 \times SU(3)$ | $E_8 \times Z_3$ |
| M3068 | SO(14) | $E_8 \times Z_3$ |
| M3102 | E_6 | $E_7 \times SU(2) \times Z_4$ |
| M3103 | $SU(2) \times E_6$ | $SU(8) \times SU(2)$ |
| | | |

Figure 15: Picard 10 (i.e. 10 moduli) with two fibers (Part 3/4)

| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | |
|---|-------|-------------------------------|------------------------------------|
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3109 | SU(8) | $E_7 \times SU(2) \times Z_4$ |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3114 | $SU(2) \times E_6$ | E_7 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3116 | $SO(12) \times SU(2)$ | $E_7 \times SU(2) \times Z_4$ |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3144 | SU(8) | $SO(12) \times SU(2)$ |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3145 | $SO(10) \times SU(3)SU(2)$ | SO(12) |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3149 | $SO(12) \times SU(2)$ | SU(8) |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3167 | E_7 | SU(8) |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3174 | $SU(7) \times SU(3)$ | E_6 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3176 | SO(14) | $SU(4) \times SO(10)$ |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3183 | $SU(8) \times SU(2)$ | $E_6 \times SU(2)$ |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3184 | $SO(12) \times SU(2)$ | E_6 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3186 | E_7 | $SU(3) \times SO(12)$ |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3188 | $SU(2) \times E_7 \times Z_4$ | $SO(12) \times SU(2) \times SU(2)$ |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3189 | $E_6 \times SU(2)$ | E_7 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3192 | $SO(12) \times SU(2)$ | E_7 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3219 | $E_6 \times SU(2)$ | $SU(7) \times SU(2)$ |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M3230 | $SO(10) \times SU(3)$ | SU(8) |
| M3378 E_7 $E_6 \times SU(2)$ M3381 $SO(14) \times SU(2)$ E_7 M3385 E_7 $SO(12) \times SU(3)$ M3426 $SU(2) \times E_6$ $SU(8) \times SU(2)$ M3438 $SU(9)$ E_7 M3440 $SO(14)$ $SU(2) \times E_6 \times SU(2)$ M3569 $E_7 \times SU(2)$ $E_8 \times Z_4$ M3598 E_7 $SU(9)$ M3608 $SO(14) \times SU(2)$ E_7 M3745 $SO(16)$ $E_8 \times Z_4$ | M3231 | SO(14) | SU(8) |
| M3381 $SO(14) \times SU(2)$ E_7 M3385 E_7 $SO(12) \times SU(3)$ M3426 $SU(2) \times E_6$ $SU(8) \times SU(2)$ M3438 $SU(9)$ E_7 M3440 $SO(14)$ $SU(2) \times E_6 \times SU(2)$ M3569 $E_7 \times SU(2)$ $E_8 \times Z_4$ M3598 E_7 $SU(9)$ M3608 $SO(14) \times SU(2)$ E_7 M3745 $SO(16)$ $E_8 \times Z_4$ | M3258 | $E_6 \times SU(2)$ | SO(12) |
| M3385 E_7 $SO(12) \times SU(3)$ M3426 $SU(2) \times E_6$ $SU(8) \times SU(2)$ M3438 $SU(9)$ E_7 M3440 $SO(14)$ $SU(2) \times E_6 \times SU(2)$ M3569 $E_7 \times SU(2)$ $E_8 \times Z_4$ M3598 E_7 $SU(9)$ M3608 $SO(14) \times SU(2)$ E_7 M3745 $SO(16)$ $E_8 \times Z_4$ | M3378 | E_7 | $E_6 \times SU(2)$ |
| M3426 $SU(2) \times E_6$ $SU(8) \times SU(2)$ M3438 $SU(9)$ E_7 M3440 $SO(14)$ $SU(2) \times E_6 \times SU(2)$ M3569 $E_7 \times SU(2)$ $E_8 \times Z_4$ M3598 E_7 $SU(9)$ M3608 $SO(14) \times SU(2)$ E_7 M3745 $SO(16)$ $E_8 \times Z_4$ | M3381 | $SO(14) \times SU(2)$ | E_7 |
| M3438 $SU(9)$ E_7 M3440 $SO(14)$ $SU(2) \times E_6 \times SU(2)$ M3569 $E_7 \times SU(2)$ $E_8 \times Z_4$ M3598 E_7 $SU(9)$ M3608 $SO(14) \times SU(2)$ E_7 M3745 $SO(16)$ $E_8 \times Z_4$ | M3385 | E_7 | $SO(12) \times SU(3)$ |
| M3440 $SO(14)$ $SU(2) \times E_6 \times SU(2)$ M3569 $E_7 \times SU(2)$ $E_8 \times Z_4$ M3598 E_7 $SU(9)$ M3608 $SO(14) \times SU(2)$ E_7 M3745 $SO(16)$ $E_8 \times Z_4$ | M3426 | $SU(2) \times E_6$ | $SU(8) \times SU(2)$ |
| M3569 $E_7 \times SU(2)$ $E_8 \times Z_4$ M3598 E_7 $SU(9)$ M3608 $SO(14) \times SU(2)$ E_7 M3745 $SO(16)$ $E_8 \times Z_4$ | M3438 | SU(9) | E_7 |
| M3598 E_7 $SU(9)$ M3608 $SO(14) \times SU(2)$ E_7 M3745 $SO(16)$ $E_8 \times Z_4$ | M3440 | SO(14) | $SU(2) \times E_6 \times SU(2)$ |
| M3608 $SO(14) \times SU(2)$ E_7 M3745 $SO(16)$ $E_8 \times Z_4$ | M3569 | $E_7 \times SU(2)$ | $E_8 \times Z_4$ |
| M3745 $SO(16)$ $E_8 \times Z_4$ | M3598 | E_7 | SU(9) |
| · , | M3608 | $SO(14) \times SU(2)$ | E_7 |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ | M3745 | SO(16) | $E_8 	imes Z_4$ |
| | M3751 | $E_7 \times SU(2)$ | $E_8 \times Z_4$ |

Figure 16: Picard 10 (i.e. 10 moduli) with two fibers (Part 4/4)