

IPHT

# Gauge Groups K3

---

**Lilian Chabrol**

*Institut de Physique Théorique, Université Paris Saclay, CEA, CNRS  
Orme des Merisiers  
91191 Gif-sur-Yvette Cedex, France*

*E-mail:* [lilian.chabrol@ipht.fr](mailto:lilian.chabrol@ipht.fr)

ABSTRACT: Gauge groups of elliptically fibered K3s constructed from reflexive polyhedras from one complex parameter (Picard 19) to five complex parameters (Picard 15) or up to 10 moduli for K3s with only two fibrations.  $M\#$  means the polytope *ReflexivePolyhedras*(3, #) in SageMath.

M0	$\frac{SO(16) \times SO(16)}{Z_2}$	$\frac{SU(12) \times E_6}{Z_3}$	$E_8 \times E_8$	$\frac{E_7 \times E_7 \times SU(4)}{Z_2}$
M2	$\frac{E_7 \times SO(20)}{Z_2}$	$\frac{U1 \times SU(18)}{Z_3}$	$E_8 \times E_8 \times SU(2)$	

**Figure 1:** Gauge groups Picard 19 (i.e. one moduli)

M3	$SO(14) \times E_7$	$SO(14) \times SU(9)$	$\frac{SU(12) \times SO(8)}{Z_2}$	$\frac{E_6 \times E_6 \times SU(3)SU(3)}{Z_3}$	$E_8 \times E_8 \times Z_3$		
M4	$E_8 \times E_8 \times Z_3$	$E_6 \times SO(14) \times SU(3)$	$E_7 \times E_7$	$\frac{SU(10) \times SO(12)}{Z_2}$	$\frac{SU(9) \times SU(9)}{Z_3}$		
M5	$E_7 \times E_7 \times SU(2)$	$SU(10) \times E_6$	$\frac{SO(16) \times SO(12) \times SU(2)}{Z_2}$	$\frac{E_7 \times SO(12) \times SU(4)}{Z_2}$	$E_8 \times E_7$	$\frac{SU(6) \times SU(12)}{Z_3}$	
M6	$E_6 \times E_7 \times SU(3)$	$E_7 \times E_8$	$E_8 \times E_8 \times Z_3$	$SO(14) \times SO(14)$	$SO(10) \times SU(11)$	$\frac{E_6 \times SU(9) \times SU(3)}{Z_3}$	$\frac{SU(8) \times SO(16)}{Z_2}$
M7	$E_7 \times E_8$	$\frac{SU(10) \times E_7}{Z_2}$	$\frac{SU(3) \times SU(15)}{Z_3}$	$E_6 \times SO(18)$			
M10	$\frac{SO(16) \times SO(16)}{Z_2}$	$\frac{E_7 \times E_7 \times SU(2)SU(2)}{Z_2}$	$E_8 \times E_8 \times Z_4$	$\frac{SU(16) \times U1}{Z_2}$			
M11	$\frac{SO(16) \times E_7 \times SU(2)}{Z_2}$	$E_8 \times E_8 \times Z_4$	$E_8 \times E_7 \times SU(2)$	$\frac{SO(12) \times SO(20)}{Z_2}$	$SU(16) \times U1$		
M16	$SO(18) \times E_6$	$\frac{SU(15) \times U1 \times SU(3)}{Z_3}$	$E_7 \times E_8$	$\frac{SU(10) \times E_7}{Z_2}$			
M88	$E_8 \times E_8$	$\frac{SO(32) \times U1}{Z_2}$					

**Figure 2:** Gauge groups Picard 18 (i.e. two moduli)

---

## Contents

### 1 2 Fibers

**7**

---

M14	$\frac{E_7 \times SU(8) \times SU(2)}{Z_2}$	$SO(16) \times E_6$	$E_8 \times E_7 \times Z_4$	$SU(14) \times SU(2)$	$E_8 \times E_6$	$SO(10) \times SO(18)$		
M15	$E_7 \times E_6 \times SU(2)$	$SO(14) \times SO(14)$	$\frac{SO(16) \times SU(8)}{Z_2}$	$E_7 \times E_8 \times Z_4$	$\frac{SU(14) \times SU(2)}{Z_2}$			
M20	$E_7 \times E_8 \times Z_3$	$E_6 \times SO(14)$	$E_6 \times SO(10) \times SU(3) \times SU(2)$	$SU(6) \times SO(14) \times SU(3)$	$E_7 \times SO(12)$	$SO(12) \times SU(8)$	$SU(10) \times SO(8) \times SU(2)$	$SU(9) \times SU(7)$
M21	$E_7 \times SO(12) \times SU(2)$	$SU(6) \times SU(10)$	$E_7 \times E_7$	$\frac{SO(12) \times SO(12) \times SU(2) \times SU(2)}{Z_2}$	$E_7 \times E_7 \times Z_2$	$SU(8) \times E_6$	$\frac{SO(12) \times SO(12) \times SU(4)}{Z_2}$	$\frac{SO(10) \times SO(8) \times SU(2) \times SU(2)}{Z_2}$
M22	$SO(10) \times SU(9) \times SU(2)$	$SO(14) \times SU(7)$	$SO(14) \times SU(2)$	$SO(14) \times SO(12)$	$E_6 \times E_6 \times SU(3)$	$SU(10) \times SO(8)$	$E_7 \times SO(10) \times SU(2)$	$\frac{E_6 \times SU(6) \times SU(3) \times SU(2)}{Z_2}$
M23	$\frac{SU(10) \times SO(12)}{Z_2}$	$E_6 \times SO(14) \times SU(2)$	$E_7 \times SU(8)$	$SO(12) \times E_8$	$E_7 \times E_7$			
M24	$E_7 \times E_7$	$SO(10) \times SU(8) \times SU(3)$	$E_6 \times E_6 \times SU(2)$	$SO(12) \times SO(12)$	$\frac{SU(8) \times SU(8)}{Z_2}$			
M25	$SO(10) \times SO(14) \times SU(2)$	$SO(10) \times SU(9)$		$SO(12) \times E_6 \times SU(3)$	$E_7 \times E_7$	$E_8 \times E_7 \times Z_3$	$SU(7) \times E_6 \times SU(3)$	$\frac{SU(6) \times SU(6) \times SU(3)}{Z_2}$
M26	$SU(9) \times E_6$	$E_6 \times SU(9)$	$E_7 \times E_7$	$SO(10) \times SO(16)$	$E_6 \times E_8$	$\frac{SU(6) \times E_7 \times SU(4)}{Z_2}$	$\frac{SU(3) \times SU(12) \times SU(3)}{Z_3}$	
M27	$SU(8) \times SO(14)$	$\frac{SU(4) \times SU(12)}{Z_4}$	$E_6 \times E_6 \times SU(2) \times SU(2)$	$E_7 \times SO(12)$				
M28	$E_6 \times E_7 \times SU(2)$	$SU(10) \times SO(10)$	$SO(10) \times E_7 \times SU(3)$	$SU(5) \times SU(11)$	$SO(14) \times SO(12)$	$E_6 \times E_8$	$E_6 \times SU(8) \times SU(2)$	$\frac{SU(6) \times SO(10) \times SU(2)}{Z_2}$
M29	$E_7 \times E_6$	$E_7 \times E_7 \times Z_2$	$\frac{SO(12) \times SU(8) \times SU(2)}{Z_2}$	$\frac{SU(12) \times SU(10)}{Z_2}$	$SO(12) \times E_6 \times SU(2)$	$SO(10) \times SO(14) \times SU(3)$		
M30	$E_6 \times E_6$	$E_7 \times E_7 \times Z_2$	$\frac{SO(12) \times SO(12) \times SU(2) \times SU(2)}{Z_2}$	$\frac{SU(8) \times SU(8)}{Z_2}$				
M38	$E_7 \times E_7$	$SO(14) \times E_8$	$\frac{SU(1) \times SO(24)}{Z_2}$	$\frac{U(1) \times SU(10)}{Z_2}$				
M41	$\frac{SO(12) \times SO(16) \times SU(2)}{Z_3}$	$E_7 \times E_8 \times Z_4$	$\frac{SO(12) \times E_7 \times SU(2)}{Z_2}$	$E_7 \times E_7 \times SU(2)$	$U(1) \times SU(14)$			
M47	$SO(18) \times SO(10)$	$E_8 \times E_7 \times Z_4$	$SU(14) \times U(1) \times SU(2)$	$E_8 \times E_6$	$SO(16) \times E_6$	$\frac{SU(8) \times E_7 \times SU(2)}{Z_2}$		
M48	$E_7 \times E_7$	$SO(14) \times E_6 \times SU(2)$	$U(1) \times SU(13) \times SU(3)$	$\frac{SU(10) \times SO(12)}{Z_2}$				
M49	$E_7 \times E_6 \times SU(2)$	$\frac{SU(8) \times SO(16)}{Z_2}$	$E_7 \times E_8 \times Z_4$	$SO(14) \times SO(14)$	$\frac{SU(14) \times U(1) \times SU(2)}{Z_2}$			
M50	$SU(15) \times U(1)$	$\frac{SU(6) \times SO(20)}{Z_2}$	$SO(14) \times E_7$	$E_8 \times E_6 \times SU(2)$				
M53	$E_6 \times SU(9)$	$E_7 \times E_7$	$SO(16) \times SO(10)$	$\frac{U(1) \times SU(12) \times SU(3) \times SU(3)}{Z_3}$				
M104	$E_7 \times E_8$	$\frac{SO(28) \times SU(2)}{Z_2}$						
M117	$\frac{SO(23) \times U(1) \times SU(4)}{Z_4}$	$\frac{SU(16) \times U(1)}{Z_2}$						
M121	$E_7 \times E_8$	$\frac{SO(28) \times U(1) \times SU(2)}{Z_2}$	$E_7 \times E_7$					
M230	$\frac{SU(16) \times U(1)}{Z_2}$	$E_7 \times E_7$						

**Figure 3:** Gauge groups Picard 17 (i.e. three moduli)



M109	$SO(22) \times SU(2)$ $\mathbb{Z}_2$	$E_7 \times E_6$	$SU(6) \times E_6$	$SO(20) \times SU(2)$	$SU(6) \times E_7$	$E_6 \times E_6$		
M124	$SU(12) \times SU(2) \times SU(2)$	$E_6 \times SU(4) \times SU(3)$	$SU(18) \times SU(2) \times SU(3)$	$SU(12) \times SU(2)$	$SU(12) \times SU(2)$			
M143	$SU(6) \times E_7$	$E_6 \times SO(12)$	$SU(18) \times SU(2) \times SU(3)$	$SU(12) \times SU(2)$	$SU(12) \times SU(2)$	$SO(10) \times SO(8)$		
M161	$SO(10) \times E_7$	$SU(7) \times SO(10)$	$SU(4) \times E_6 \times SU(2)SU(3)$	$SU(2) \times SU(9) \times SU(2)SU(3)$	$SU(2) \times SU(9) \times SU(2)SU(3)$	$SO(14) \times SO(8)$		
M165	$E_6 \times SO(12)$	$E_7 \times SU(7)$	$SO(18) \times SU(3) \times SU(2)$	$SU(6) \times E_7$	$SU(6) \times E_7$	$SU(2) \times SU(12)$		
M168	$E_7 \times E_6 \times Z_4$	$E_6 \times SU(6) \times SU(2)$	$E_7 \times SU(6) \times SU(2)SU(3)$	$SU(11) \times SU(2)$	$SU(11) \times SU(2)$	$SO(16) \times SU(5)$		
M170	$SO(12) \times SU(6) \times SU(2)$	$SO(10) \times SO(10) \times SU(2)SU(2)$	$SO(12) \times E_7 \times Z_4$	$SU(2) \times SU(10)$	$SU(2) \times SU(10)$	$SO(8) \times SO(14)$		
M171	$E_7 \times SO(8) \times SU(2)$	$SU(6) \times E_6$	$SO(12) \times SO(12)$	$SU(4) \times SU(10)$	$SU(4) \times SU(10)$	$SO(8) \times SO(12) \times SU(2)$		
M172	$E_7 \times SO(10)$	$SO(12) \times SO(10) \times SU(2)$	$E_7 \times SO(12) \times Z_4$	$SU(6) \times E_6$	$SU(6) \times E_6$	$SO(14) \times SO(10)$		
M178	$E_6 \times SU(7)$	$E_6 \times SU(7)$	$SO(16) \times SU(4)$	$E_7 \times SU(4) \times SU(3)$	$SU(11) \times SU(2) \times SU(2)$	$SO(9) \times E_6$		
M179	$SO(14) \times SU(5)$	$E_6 \times E_6$	$SO(10) \times SO(12)$	$SO(10) \times SU(7) \times SU(2)$	$SU(0) \times SU(7) \times SU(2)$	$SO(10) \times SU(7) \times SU(2)$		
M180	$SO(12) \times SU(7)$	$SO(14) \times SU(6)$	$SU(11) \times SU(2)$	$E_6 \times SO(10) \times SU(2)$	$E_6 \times SO(10)$	$SU(5) \times E_7$		
M182	$SO(10) \times E_6 \times SU(2)$	$SU(4) \times SO(14) \times SU(2)$	$E_6 \times SO(10)$	$SU(7) \times SU(7)$	$SU(7) \times SU(7)$	$SO(10) \times SO(8)$		
M188	$SU(6) \times E_7$	$SU(6) \times SU(10)$	$E_6 \times SO(10)$	$SU(4) \times SU(11)$	$SU(4) \times SU(11)$			
M189	$SU(3) \times SU(11)$	$E_7 \times SO(10)$	$SU(10) \times SU(10) \times SU(2)$	$SO(12) \times E_6$	$SO(12) \times E_6$	$SO(18) \times SU(2) \times SU(3)$		
M190	$E_6 \times SO(10)$	$SU(10) \times SU(10) \times SU(2)$	$SO(12) \times SO(10)$	$E_7 \times SU(5) \times SU(2)$	$SO(16) \times SU(3) \times SU(3)$	$E_6 \times E_7 \times Z_4$		
M191	$SU(6) \times E_6$	$SU(10) \times SU(2) \times SU(3)$	$SU(9) \times SU(5)$	$SO(12) \times SO(10)$	$SO(10) \times E_7$			
M192	$SU(10) \times SU(12) \times SU(2)$	$E_6 \times SO(10)$	$SU(7) \times SU(7)$	$SU(6) \times SO(12)$	$SU(6) \times SO(12)$			
M196	$SO(10) \times E_6$	$SO(10) \times SO(10)$	$SU(10) \times SU(10) \times SU(2)$	$SU(10) \times SU(10) \times SU(2)$	$SU(10) \times SU(10) \times SU(2)$			
M197	$SO(13) \times SO(8) \times SU(12) \times SU(2)$	$SU(6) \times SU(6)$	$SO(12) \times SO(12) \times Z_4$	$SO(12) \times SO(12) \times Z_2$	$SO(12) \times SO(12) \times Z_2$			
M198	$SO(10) \times SO(10) \times SU(2)$	$SU(6) \times SU(6)$	$SU(6) \times SO(8) \times SU(2)SU(2)$	$SU(3) \times SU(9)$	$SU(3) \times SU(9)$			
M199	$SO(12) \times SU(6)$	$SO(10) \times SU(6) \times SU(2)SU(2)$	$SU(6) \times SU(6) \times SU(2)SU(2)$	$SO(14) \times SU(5)$	$SO(14) \times SU(5)$			
M200	$SU(9) \times SU(4)$	$SU(3) \times E_6 \times SU(3)SU(2)SU(2)$	$SU(8) \times SO(8)$	$SO(10) \times SO(8) \times SU(2)SU(2)$	$SO(10) \times SO(12)$	$SU(7) \times SU(5)$		
M201	$SU(6) \times SU(6) \times SU(2)$	$SU(10) \times SU(10) \times SU(2)$	$E_6 \times SO(12) \times Z_2$	$SO(10) \times SO(8) \times SU(2)SU(2)$	$SO(10) \times SO(12)$	$SU(6) \times SO(10)$		
M202	$SO(10) \times SU(6) \times SU(2)$	$SU(13) \times SU(6) \times SU(13)SU(13)$	$SU(7) \times SO(8)$	$SU(4) \times E_7 \times SU(2)$	$SU(4) \times E_7 \times SU(2)$	$SU(6) \times SO(12)$		
M203	$SU(8) \times SU(6)$	$SO(8) \times E_6 \times SU(2)$	$SU(7) \times SO(12)$	$SO(14) \times SU(7)$	$SO(14) \times SU(7)$	$SO(12) \times E_6$		
M204	$E_6 \times SU(5) \times SU(2)SU(2)$	$SO(14) \times SU(5)$	$SU(3) \times SU(10)$	$SU(3) \times SU(10)$	$SU(3) \times SU(10)$	$SO(12) \times E_6$		
M205	$SU(5) \times E_6 \times SU(3)$	$SO(8) \times SU(8)$	$SO(10) \times SU(7)$	$SU(10) \times SO(12)$	$SU(10) \times SO(12)$	$SO(12) \times E_6$		
M206	$SU(9) \times E_6$	$SU(5) \times SU(9)$	$SO(10) \times SO(12)$	$SO(10) \times SO(10)$	$SO(10) \times SO(10)$	$SO(12) \times E_6$		
M207	$SU(9) \times SU(3) \times SU(2)$	$SO(10) \times SO(10) \times SU(2)$	$E_6 \times SU(6)$	$SU(7) \times SO(10)$	$SU(7) \times SO(10)$	$SO(12) \times E_6$		
M208	$SU(6) \times SO(10) \times SU(2)$	$SO(12) \times SO(8) \times SU(2)$	$SU(6) \times SO(12)$	$SU(6) \times SO(12)$	$SU(6) \times SO(12)$	$SO(12) \times E_6$		
M209	$SU(6) \times SO(10) \times SU(2)$	$SO(10) \times E_7$	$SU(7) \times SO(10)$	$SU(7) \times SO(10)$	$SU(7) \times SO(10)$	$SO(12) \times E_6$		
M210	$SO(10) \times SO(10) \times SU(3)$	$SO(12) \times SU(5) \times SU(2)$	$SO(10) \times SO(10) \times SU(2)$	$SU(6) \times E_6 \times Z_4$	$SU(6) \times E_6 \times Z_4$	$SO(12) \times E_6$		
M211	$SO(40) \times SO(12)$	$SO(10) \times E_6$	$SU(6) \times SO(12)$	$SU(6) \times SO(12)$	$SU(6) \times SO(12)$	$SO(12) \times E_6$		
M212	$SU(7) \times SO(8)$	$SO(10) \times SO(12)$	$SU(6) \times SU(7) \times SU(2)$	$SU(6) \times SU(7) \times SU(2)$	$SU(6) \times SU(7) \times SU(2)$	$SO(12) \times E_6$		
M213	$SO(12) \times SO(10) \times SU(2)$	$SU(8) \times SU(5)$	$E_6 \times SU(5)$	$SU(6) \times SO(12)$	$SU(6) \times SO(12)$	$SO(12) \times E_6$		
M214	$SU(4) \times SU(9)$	$SU(6) \times SO(12)$	$E_7 \times SO(8)$	$SO(10) \times SO(12)$	$SO(10) \times SO(12)$	$SO(12) \times E_6$		
M215	$SO(10) \times E_6$	$E_6 \times E_6$	$SU(10) \times SU(10) \times SU(12) \times SU(2)$	$SO(10) \times SO(12)$	$SO(10) \times SO(12)$	$SO(12) \times E_6$		

Figure 5: Gauge groups Picard 15 (i.e. five moduli), Part 1

M216	$SO(8) \times E_6$	$SO(12) \times SU(5) \times SU(2)$	$\frac{SU(4) \times SU(8) \times SU(2)SU(2)}{Z_2}$	$SU(7) \times SU(6)$	$SO(10) \times SO(10) \times SU(2)$	$SU(3) \times SU(9) \times SU(3)$	$SO(10) \times E_7 \times Z_2$	$E_6 \times SO(10)$	$SO(12) \times SO(10)$
M217	$SO(10) \times SO(10)$	$E_6 \times SU(6)$	$E_6 \times E_6$	$\frac{SU(6) \times SU(6) \times SU(4)}{Z_2}$		$SU(3)$			
M218	$E_6 \times E_6 \times Z_2$	$SU(6) \times SU(6)$	$SO(10) \times SO(10)$	$\frac{SU(6) \times SU(6) \times SU(2)SU(2)}{Z_2}$					
M219	$SU(6) \times SO(10)$	$E_6 \times SO(10)$	$\frac{SU(4) \times SU(8) \times SU(2)SU(2)}{Z_2}$	$SU(5) \times SU(7) \times SU(3)$	$SO(10) \times SO(10)$				
M238	$SO(22) \times U1 \times SU(2)$	$SU(6) \times E_8$	$E_7 \times E_6$						
M272	$E_6 \times E_6$	$\frac{SU(12) \times U1 \times SU(2)SU(2)}{Z_4}$	$SU(2) \times SO(20)$						
M277	$E_7 \times SO(10)$	$U1 \times SU(13)$							
M291	$U1 \times SU(12)$	$\frac{E_7 \times SU(2) \times SU(4)SU(2)SU(2)}{Z_2}$	$SO(16) \times SO(8)$						
M300	$SO(12) \times E_6$	$SU(2) \times SU(12)$	$SO(18) \times U1 \times SU(2)SU(3)$	$E_7 \times SU(6)$					
M301	$E_7 \times SO(8) \times SU(2)$	$SU(12) \times U1$	$\frac{SO(16) \times SU(2) \times SU(2)SU(4)}{Z_2}$	$SO(12) \times SO(12)$					
M305	$E_7 \times SU(7)$	$SO(18) \times SU(3) \times SU(2)$	$E_6 \times SO(12)$	$U1 \times SU(12) \times SU(2)$					
M309	$E_7 \times SO(10)$	$\frac{SU(12) \times SU(2) \times SU(2)}{Z_2}$	$E_6 \times E_6$	$SO(20) \times U1 \times SU(2)$					
M310	$E_7 \times SO(10) \times SU(2)$	$SO(12) \times SO(12)$	$SU(12) \times U1$	$\frac{SU(4) \times SO(16) \times SU(2)SU(2)}{Z_2}$					
M311	$SU(4) \times SO(18)$	$U1 \times SU(13)$	$SO(8) \times E_8$	$SO(14) \times E_6$	$E_7 \times SU(6)$				
M313	$SU(13) \times U1$	$SU(3) \times SO(20)$	$SO(10) \times E_7$	$SU(5) \times E_8 \times SU(2)$					
M317	$SO(12) \times E_6$	$SU(2) \times SO(18) \times SU(3)$	$SU(12) \times U1 \times SU(2)$	$SU(6) \times E_7$					
M320	$\frac{SU(10) \times U1 \times SU(2)SU(4)}{Z_2}$	$SO(10) \times E_6$	$SU(3) \times SU(11)$	$SU(6) \times E_7$					
M344	$U1 \times SU(11) \times SU(3)$	$SO(10) \times E_6$	$\frac{SU(10) \times SU(4) \times SU(2)}{Z_2}$	$SO(12) \times E_6$	$E_7 \times SO(10)$				
M350	$SO(10) \times E_6$	$\frac{SU(10) \times SU(2) \times SU(4)}{Z_2}$	$U1 \times SU(11) \times SU(3)$						
M359	$SU(8) \times SO(8)$	$SU(7) \times SO(10)$	$U1 \times SU(9) \times SU(2)SU(3)SU(2)$						
M362	$E_6 \times SU(6) \times SU(2)$	$E_6 \times E_7 \times Z_4$	$SO(10) \times SU(8)$	$SO(8) \times SO(14)$	$U1 \times SU(11) \times SU(2)$				
M363	$SO(10) \times SU(7) \times SU(2)$	$U1 \times SU(10) \times SU(2)SU(2)$	$SO(10) \times SO(12)$	$E_6 \times E_6$					
M364	$E_6 \times SO(10) \times SU(2)$	$SO(14) \times SU(5) \times SU(2)$	$SO(12) \times SU(7)$	$SU(11) \times U1 \times SU(2)$	$SO(10) \times E_7$				
M365	$SO(8) \times E_7$	$SO(14) \times SU(6)$	$SU(12) \times U1$	$E_6 \times E_6 \times SU(2)$					

**Figure 6:** Gauge groups Picard 15 (i.e. five moduli) Part 2

M375	$SO(10) \times SO(14)$	$SU(12) \times U1$	$E_7 \times SU(5) \times SU(2)$	$SU(5) \times SO(16)$	$E_8 \times SO(10) \times Z_4$	
M376	$\frac{SU(6) \times SO(12) \times SU(2) \times SU(2)}{Z_2}$	$U1 \times SU(11)$	$SO(10) \times SO(12) \times SU(2)$	$E_7 \times E_6 \times Z_4$		
M377	$E_6 \times SU(7)$	$U1 \times SU(11) \times SU(2) \times SU(2)$	$SO(16) \times SU(4)$			
M379	$SU(10) \times U1 \times SU(2) \times SU(3)$	$SO(10) \times E_7$	$SU(6) \times E_6$	$SU(9) \times SU(5)$	$SO(12) \times SO(10)$	
M384	$\frac{SU(10) \times U1 \times SU(2) \times SU(2) \times SU(2)}{Z_2}$	$SO(10) \times E_6$	$SO(12) \times SU(6)$	$SU(7) \times SU(7)$		
M402	$SO(10) \times SO(12)$	$E_6 \times E_6$	$E_7 \times E_6 \times Z_4$	$\frac{SU(6) \times SU(8) \times SU(2)}{Z_2}$	$E_6 \times SO(10)$	$SU(11) \times U1 \times SU(2)$
M513	$SO(12) \times SO(12)$	$U1 \times SU(12)$	$\frac{SO(16) \times U1 \times SU(4) \times SU(2) \times SU(2)}{Z_2}$			
M529	$SU(2) \times SO(22)$	$SU(6) \times E_8$	$E_7 \times E_6$			
M537	$SO(12) \times E_6$	$U1 \times SO(18) \times SU(2) \times SU(3)$	$U1 \times SU(12) \times SU(2)$			
M544	$\frac{SO(20) \times SU(2) \times SU(2) \times SU(2)}{Z_2}$	$E_7 \times SO(12)$				
M549	$E_7 \times SO(10)$	$E_6 \times E_6$	$\frac{SU(12) \times SU(2) \times SU(2)}{Z_2}$			
M572	$E_6 \times SO(10)$	$U1 \times SU(11) \times SU(3)$	$\frac{SU(10) \times U1 \times SU(2) \times SU(4)}{Z_2}$			
M587	$E_7 \times SO(10)$	$SO(20) \times U1 \times SU(3)$	$SU(13) \times U1$			
M588	$\frac{SU(12) \times U1 \times SU(2) \times SU(2)}{Z_2}$	$E_6 \times E_6$	$SO(20) \times U1 \times SU(2)$			
M792	$SO(24) \times U1$	$SO(10) \times E_8$				
M859	$U1 \times SU(13)$	$SO(10) \times E_7$				
M866	$U1 \times SO(22) \times SU(2)$	$E_7 \times E_6$				
M895	$E_6 \times E_6$	$\frac{SU(12) \times U1 \times SU(2) \times SU(2)}{Z_2}$				

**Figure 7:** Gauge groups Picard 15 (i.e. five moduli) Part 3

## 1 2 Fibers

M531	$SO(18) \times U1 \times SU(2)SU(2)$	$SU(6) \times E_7$	
M936	$U1 \times SU(9) \times SU(3)SU(2)$	$SO(10) \times SO(10)$	
M959	$E_6 \times SO(10)$	$\frac{SU(10) \times SU(2) \times SU(2)SU(2)}{Z_2}$	
M987	$E_6 \times SO(8)$	$U1 \times SU(10) \times SU(3)$	
M1191	$SU(12) \times U1$	$SO(8) \times E_7$	
M1201	$SU(5) \times E_8$	$U1 \times SO(22)$	
M1220	$SO(20) \times U1 \times SU(2)$	$SO(10) \times E_7$	
M1328	$SO(10) \times E_6$	$U1 \times SU(11) \times SU(2)$	
M1346	$E_6 \times E_6$	$U1 \times SO(20)$	

**Figure 8:** Picard 14 (i.e. 6 moduli) with two fibers



M962	$U1 \times SU(9) \times SU(2)SU(2)$	$SU(5) \times E_6$	
M1084	$SU(6) \times SO(8)$	$U1 \times SU(9)$	
M1395	$U1 \times SU(9) \times SU(3)$	$E_6 \times SU(4)$	
M1430	$SU(2) \times SO(16)$	$SU(6) \times E_6$	
M1556	$SO(20) \times U1$	$E_8 \times SU(2) \times SU(3)$	
M1621	$SU(3) \times E_8 \times SU(2)$	$SO(20) \times U1$	
M1703	$SU(4) \times E_7$	$U1 \times SU(11)$	
M1728	$SO(8) \times E_6$	$U1 \times SU(10) \times SU(2)$	
M1729	$SU(5) \times E_7$	$U1 \times SO(18) \times SU(2)$	
M1740	$SO(18) \times U1$	$SO(10) \times E_6$	
M1792	$U1 \times SU(10)$	$SO(10) \times SO(10)$	
M1980	$SO(20) \times U1$	$E_8 \times U1 \times SU(3)SU(2)$	
M1985	$U1 \times SO(22)$	$SU(4) \times E_8$	

**Figure 9:** Picard 13 (i.e. 7 moduli) with two fibers

M1389	$SU(3) \times SO(14)$	$U1 \times SU(9)$	
M1432	$SU(3) \times E_6 \times SU(3)$	$U1 \times SO(14)$	
M1675	$E_6 \times SU(3)$	$SU(9) \times U1 \times SU(3)$	
M1701	$SO(14) \times SU(4)$	$SU(2) \times E_7$	
M1872	$SU(5) \times SO(10)$	$SU(2) \times SU(8)$	
M1902	$SO(8) \times SO(8)$	$U1 \times SU(8) \times SU(2)$	
M2014	$SO(16) \times SU(2)$	$E_7 \times U1 \times SU(3)SU(2)$	
M2021	$E_7 \times SU(2) \times SU(2)$	$SU(10) \times U1$	
M2037	$SO(16) \times U1 \times SU(3)$	$SU(2) \times E_7 \times SU(2)$	
M2054	$U1 \times SO(18)$	$SU(2) \times E_8$	
M2055	$SO(16) \times U1 \times SU(2)$	$E_7 \times SU(2) \times SU(3)$	
M2126	$U1 \times SU(11)$	$SU(3) \times E_7$	
M2133	$SU(10) \times U1$	$E_7 \times SU(3)$	
M2134	$SU(3) \times E_7 \times SU(2)$	$SO(16) \times U1 \times SU(2)$	
M2236	$SU(5) \times E_6$	$U1 \times SO(16)$	
M2204	$SU(4) \times E_6$	$U1 \times SU(9) \times SU(2)$	
M2197	$U1 \times SU(9)$	$SO(10) \times SO(8)$	
M2153	$SO(16) \times U1$	$SO(10) \times SO(10)$	
M2407	$SO(16) \times U1 \times SU(2)$	$E_7 \times U1 \times SU(3)SU(2)$	
M2408	$SU(3) \times E_7$	$U1 \times SU(11)$	
M2421	$U1 \times SO(18) \times SU(2)$	$SU(4) \times E_7$	
M2428	$U1 \times E_8 \times SU(2)$	$SO(18) \times U1$	
M2732	$U1 \times SO(20)$	$SU(2) \times E_8 \times SU(2)$	

**Figure 10:** Picard 12 (i.e. 8 moduli) with two fibers

M1529	$SU(4) \times SU(4)$	$SO(8) \times SO(8) \times Z_2$	
M1663	$E_6 \times SU(3)$	$U1 \times SU(9) \times SU(2)$	
M1734	$SO(14) \times U1$	$SU(3) \times E_6 \times SU(2)$	
M1878	$SO(10) \times SU(3) \times SU(2)$	$U1 \times SU(7)$	
M2015	$U1 \times E_6 \times SU(3)SU(2)$	$SO(14) \times U1$	
M2058	$SU(9) \times U1 \times SU(2)$	$E_6 \times U1 \times SU(3)$	
M2064	$E_7 \times SU(2)$	$SU(2) \times E_8 \times Z_4$	
M2154	$SO(14) \times SU(2)$	$SU(2) \times E_7$	
M2155	$SU(2) \times SO(14)$	$E_6 \times SU(2) \times SU(2)$	
M2169	$E_6 \times SU(2)$	$U1 \times SU(8) \times SU(2)$	
M2240	$SU(2) \times SO(12)$	$SU(4) \times SO(10) \times SU(2)$	
M2328	$U1 \times SU(7) \times SU(2)$	$SO(8) \times SU(4)$	
M2390	$U1 \times E_8 \times SU(2) \times Z_4$	$E_7 \times SU(2)$	
M2398	$E_7 \times SU(2)$	$SU(10) \times U1$	
M2427	$SO(16) \times U1$	$SU(2) \times E_8 \times Z_4$	
M2441	$E_7 \times U1 \times SU(2)$	$SU(2) \times E_8 \times Z_4$	
M2472	$SO(14) \times SU(2)$	$U1 \times SU(9)$	
M2473	$SO(12) \times U1 \times SU(3)SU(2)$	$SU(2) \times SO(12) \times SU(2)$	
M2501	$SU(2) \times E_7$	$U1 \times SU(9)$	
M2503	$SU(2) \times E_6 \times SU(2)$	$SU(8) \times U1 \times SU(2)$	
M2517	$SU(8) \times U1 \times SU(3)$	$E_6 \times SU(2)$	
M2525	$U1 \times SO(14) \times SU(2)$	$SU(2) \times E_7$	
M2527	$U1 \times E_7 \times SU(2)$	$SO(14) \times SU(2)$	
M2529	$SU(2) \times E_6 \times SU(3)$	$SO(14) \times U1$	
M2530	$SU(2) \times E_6 \times SU(2)$	$U1 \times SO(14) \times SU(2)$	
M2537	$E_7 \times U1$	$SU(3) \times SO(14)$	
M2551	$SU(8) \times U1$	$U1 \times SO(12) \times SU(2)SU(2)$	

**Figure 11:** Picard 11 (i.e. 9 moduli) with two fibers (Part 1)

M2567	$SU(8) \times U1$	$SO(8) \times SO(8)$	
M2571	$SU(3) \times SO(12)$	$U1 \times SU(9)$	
M2600	$U1 \times SU(9) \times SU(2)$	$SU(3) \times E_6$	
M2606	$SU(3) \times E_6$	$U1 \times SU(8) \times SU(2)$	
M2607	$SU(5) \times SO(10)$	$U1 \times SO(14)$	
M2609	$SO(14) \times U1$	$SU(3) \times E_6 \times SU(2)$	
M2636	$U1 \times SU(8)$	$SU(4) \times SO(10)$	
M2727	$U1 \times SO(16)$	$U1 \times E_8 \times SU(2) \times Z_4$	
M2744	$U1 \times E_8 \times SU(2) \times Z_4$	$E_7 \times U1 \times SU(2)$	
M2787	$U1 \times E_8$	$SU(3) \times E_7$	
M2794	$U1 \times SU(10)$	$E_7 \times U1 \times SU(2)$	
M2800	$U1 \times E_8$	$SO(16) \times U1$	
M2803	$E_7 \times SU(2)$	$SU(10) \times U1$	
M2804	$U1 \times E_7$	$SO(14) \times U1 \times SU(3)$	
M2835	$U1 \times E_6 \times SU(3)SU(2)$	$SO(14) \times U1$	
M2836	$E_7 \times U1 \times SU(2)$	$U1 \times SU(9)$	
M2860	$SU(3) \times E_6$	$SU(9) \times U1 \times SU(2)$	
M2864	$U1 \times E_7 \times SU(2)$	$SO(14) \times U1 \times SU(2)$	
M2874	$U1 \times SO(16)$	$SU(4) \times E_6$	
M3071	$U1 \times E_8$	$E_7 \times U1 \times SU(3)$	
M3112	$U1 \times SO(16) \times SU(2)$	$SU(2) \times E_7 \times SU(2)$	
M3120	$SU(10) \times U1$	$SU(2) \times E_7$	
M3340	$SO(18) \times U1$	$U1 \times E_8$	

**Figure 12:** Picard 11 (i.e. 9 moduli) with two fibers (Part 2)

M1598	$SU(2) \times SU(6)$	$SO(12) \times SU(2) \times SU(2) \times Z_4$	
M1795	$SU(2) \times SU(7)$	$SO(8) \times SU(3)$	
M1886	$SU(2) \times SU(5) \times SU(2)$	$E_6 \times SU(3) \times Z_3$	
M2027	$SU(6) \times U1 \times SU(3)SU(2)$	$SO(10) \times SU(2)$	
M2061	$E_7 \times U1$	$E_6 \times SU(2)$	
M2156	$U1 \times E_6$	$E_7 \times SU(2) \times Z_4$	
M2159	$U1 \times SO(12)$	$SU(2) \times E_6$	
M2171	$SO(10) \times SU(3)$	$U1 \times SU(8)$	
M2246	$SO(10) \times U1$	$SU(2) \times SO(10) \times SU(2)$	
M2248	$U1 \times SU(7)$	$SU(3) \times SO(10)$	
M2249	$U1 \times SO(10) \times SU(2)$	$SU(3) \times SU(6) \times SU(2)$	
M2266	$SO(10) \times SU(2)$	$SU(7) \times U1$	
M2267	$SU(2) \times SO(12)$	$SU(7) \times U1$	
M2354	$U1 \times SU(6)$	$SU(4) \times SU(4)$	
M2440	$U1 \times E_7$	$E_6 \times U1 \times SU(2)$	
M2464	$SU(6) \times U1 \times SU(2)SU(3)$	$U1 \times SO(10) \times SU(2)$	
M2526	$E_7 \times SU(2) \times Z_4$	$SO(12) \times SU(2)$	
M2531	$U1 \times E_6 \times SU(2)$	$SO(12) \times U1$	
M2533	$SO(14) \times U1$	$U1 \times SU(8)$	
M2534	$SO(10) \times U1 \times SU(3)$	$U1 \times SU(8)$	

**Figure 13:** Picard 10 (i.e. 10 moduli) with two fibers (Part 1/4)

M2573	$SO(10) \times SU(2)$	$SU(2) \times SU(7)$	
M2608	$SU(2) \times E_6$	$SU(2) \times SU(7)$	
M2619	$SU(3) \times SO(8)$	$U1 \times SU(7) \times SU(2)$	
M2622	$SU(2) \times SO(10) \times SU(2)$	$SU(2) \times SO(12)$	
M2638	$SU(4) \times SO(8)$	$SU(2) \times SU(6)$	
M2640	$SU(3) \times SU(6)$	$SU(2) \times SO(10)$	
M2696	$U1 \times SU(6) \times SU(2)$	$SU(4) \times SU(4)$	
M2728	$E_6 \times U1$	$E_7 \times U1 \times SU(2) \times Z_4$	
M2784	$SO(12) \times SU(2)$	$U1 \times SU(8)$	
M2788	$SU(3) \times E_6$	$U1 \times E_8 \times Z_3$	
M2793	$SU(2) \times E_6 \times SU(2)$	$SO(14) \times U1$	
M2797	$U1 \times E_7 \times SU(2) \times Z_4$	$SO(12) \times SU(2)$	
M2811	$SU(8) \times U1 \times SU(2)$	$SU(2) \times E_6$	
M2859	$SU(8) \times SU(2)$	$U1 \times E_6 \times SU(2)$	
M2861	$E_7 \times SU(2) \times Z_4$	$SU(8) \times U1$	
M2862	$U1 \times E_6$	$E_7 \times SU(2) \times Z_4$	
M2863	$SU(2) \times E_7 \times Z_4$	$U1 \times SO(12) \times SU(2)$	
M2865	$U1 \times E_6$	$SU(2) \times SO(12)$	
M2868	$SO(10) \times SU(2) \times SU(3)$	$SO(12) \times U1$	
M2869	$SO(12) \times SU(2)$	$SU(8) \times U1$	

**Figure 14:** Picard 10 (i.e. 10 moduli) with two fibers (Part 2/4)

M2875	$SU(2) \times E_6$	$E_7 \times U1$	
M2877	$SU(2) \times SO(12)$	$U1 \times E_7$	
M2912	$U1 \times E_6 \times SU(2)$	$SU(7) \times SU(2)$	
M2914	$SO(12) \times U1 \times SU(2)$	$SU(7) \times U1$	
M2917	$SO(10) \times SU(2)$	$U1 \times SU(7) \times SU(2)$	
M2918	$SU(6) \times U1 \times SU(3)SU(2)$	$SO(10) \times SU(2)$	
M2919	$U1 \times SU(7)$	$SO(10) \times SU(2) \times SU(2)$	
M2942	$SU(2) \times E_6$	$U1 \times SU(7) \times SU(2)$	
M2947	$SU(7) \times SU(3)$	$U1 \times E_6$	
M2948	$SU(3) \times SO(10) \times SU(2)$	$U1 \times SO(12)$	
M2955	$SU(2) \times E_6$	$SO(12) \times U1$	
M2956	$SU(2) \times SO(10) \times SU(2)$	$SO(12) \times U1 \times SU(2)$	
M2970	$U1 \times SU(7)$	$SU(4) \times SO(8)$	
M2982	$E_6 \times U1$	$U1 \times SU(7) \times SU(2)$	
M2990	$U1 \times SU(7)$	$SO(10) \times SU(3)$	
M3002	$SU(5) \times SU(5)$	$SO(12) \times U1$	
M3005	$SO(10) \times SU(3)$	$U1 \times SU(8)$	
M3066	$U1 \times E_8 \times Z_3$	$SU(9) \times U1$	
M3067	$E_6 \times U1 \times SU(3)$	$U1 \times E_8 \times Z_3$	
M3068	$SO(14) \times U1$	$U1 \times E_8 \times Z_3$	
M3102	$E_6 \times U1$	$E_7 \times U1 \times SU(2) \times Z_4$	
M3103	$SU(2) \times E_6$	$SU(8) \times U1 \times SU(2)$	

**Figure 15:** Picard 10 (i.e. 10 moduli) with two fibers (Part 3/4)

M3109	$U1 \times SU(8)$	$E_7 \times U1 \times SU(2) \times Z_4$	
M3114	$SU(2) \times E_6$	$U1 \times E_7$	
M3116	$SO(12) \times U1 \times SU(2)$	$U1 \times E_7 \times SU(2) \times Z_4$	
M3144	$U1 \times SU(8)$	$SO(12) \times SU(2)$	
M3145	$SO(10) \times U1 \times SU(3)SU(2)$	$SO(12) \times U1$	
M3149	$SO(12) \times U1 \times SU(2)$	$U1 \times SU(8)$	
M3167	$E_7 \times U1$	$U1 \times SU(8)$	
M3174	$SU(7) \times U1 \times SU(3)$	$E_6 \times U1$	
M3176	$SO(14) \times U1$	$SU(4) \times SO(10)$	
M3183	$SU(8) \times U1 \times SU(2)$	$U1 \times E_6 \times SU(2)$	
M3184	$U1 \times SO(12) \times SU(2)$	$U1 \times E_6$	
M3186	$U1 \times E_7$	$SU(3) \times SO(12)$	
M3188	$SU(2) \times E_7 \times Z_4$	$SO(12) \times SU(2) \times SU(2)$	
M3189	$U1 \times E_6 \times SU(2)$	$U1 \times E_7$	
M3192	$U1 \times SO(12) \times SU(2)$	$U1 \times E_7$	
M3219	$U1 \times E_6 \times SU(2)$	$SU(7) \times U1 \times SU(2)$	
M3230	$SO(10) \times SU(3)$	$U1 \times SU(8)$	
M3231	$SO(14) \times U1$	$U1 \times SU(8)$	
M3258	$U1 \times E_6 \times SU(2)$	$U1 \times SO(12)$	
M3378	$U1 \times E_7$	$U1 \times E_6 \times SU(2)$	
M3381	$SO(14) \times SU(2)$	$U1 \times E_7$	
M3385	$U1 \times E_7$	$SO(12) \times U1 \times SU(3)$	
M3426	$SU(2) \times E_6$	$SU(8) \times U1 \times SU(2)$	
M3438	$U1 \times SU(9)$	$E_7 \times U1$	
M3440	$U1 \times SO(14)$	$SU(2) \times E_6 \times SU(2)$	
M3569	$E_7 \times SU(2)$	$U1 \times E_8 \times Z_4$	
M3598	$U1 \times E_7$	$U1 \times SU(9)$	
M3608	$SO(14) \times U1 \times SU(2)$	$U1 \times E_7$	
M3745	$U1 \times SO(16)$	$U1 \times E_8 \times Z_4$	
M3751	$E_7 \times U1 \times SU(2)$	$U1 \times E_8 \times Z_4$	

**Figure 16:** Picard 10 (i.e. 10 moduli) with two fibers (Part 4/4)