LESSON

1

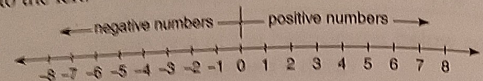
The Number Line and Signed Numbers

Key Ideas

- If numbers have like signs, add and keep the same sign.
- If numbers have unlike signs, find the difference and use the sign from the larger number.
- To subtract signed numbers, change the operation to addition and change the sign of the number you are subtracting.

Understanding Signed Numbers

Signed numbers include zero, all positive numbers, and all negative numbers. Zero is neither positive nor negative. On a number line, the positive numbers are shown to the right of zero, and the negative numbers are shown to the left.



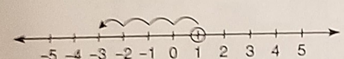
A positive number may be written with a plus (+) symbol. If a number has no symbol at all, we assume that it is positive. A negative number *must* be preceded by a minus (−) symbol.

A signed number provides two important facts. The sign tells the direction from zero, and the number tells you the distance from zero. For example, −5 lies five spaces to the left of zero, and +4 lies four spaces to the right of zero.

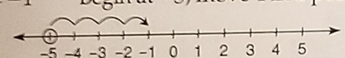
Adding and Subtracting Signed Numbers

You can use a number line to model the addition of signed numbers.

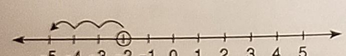
Examples: $1+(-4)=-3$ Begin at +1; move 4 in a negative direction (left).



$-5+4=-1$ Begin at −5; move 4 in a positive direction (right).



$-2+(-3)=-5$ Begin at −2; move 3 in a negative direction (left).



GED® TEST TIP

ou can use the "change ign" key (−) at the bottom f the online calculator. his changes a positive to a egative number, and vice ersa.

To add without a number line, follow these steps:

- If numbers have like signs, add the numbers and keep the same sign.
- If the numbers have unlike signs, find the difference between the two numbers and use the sign of the larger number.

Example 1: Add $15+(-25)$.

1. Since the numbers have unlike signs, subtract: $25-15=10$.
2. Use the sign from the larger number: $15+(-25)=$ **−10.**

Subtraction is the opposite of addition. To rewrite a subtraction problem as an addition problem, change the operation symbol to addition and change the sign on the number you are subtracting. Then apply the rules for adding signed numbers.

Example 2: Subtract $3-8$.

1. Change the operation symbol and the sign of the number you are subtracting:

   $3-8$ becomes $3+(-8)$.

2. Add: $3+(-8)=$ **−5**.

You can use the same rules to combine several signed numbers.

Example 3: $(-5)+6-4-(-2)=?$

1. Rewrite each subtraction as addition: $(-5)+6+(-4)+2$.
2. Add the positive terms: $6+2=8$. Add the negative terms: $-5+(-4)=-9$.
3. Combine the results: $8+-9=$ **−1**.

34
15
19

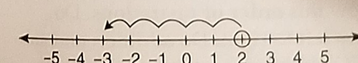
PRACTICE 1.1

A. Solve.

1. $8+(-3)$ 5	9. $(-7)-(-3)$ −4	17. $7+(-3)+(-5)-10$ −11
2. $50-5$ 45	10. $(-4)+6$ 2	18. $66+(-22)-33$ 11
3. $11-(-2)$ 13	11. $-15+(-7)$ −22	19. $-14-(-6)+18$ 10
4. $-1+2$ 1	12. $36-4$ 32	20. $80-(-15)-20$ 75
5. $-4-(-5)$ 1	13. $-60-(-10)$ −50	21. $6-(-3)+(-5)+8$ 12
6. $8-(-2)$ 10	14. $-5-6$ −11	22. $-23+(-11)-(-15)+21$ 2
7. $6-9$ −3	15. $12+13$ 25	23. $3+9-5+12-9-11$ −1
8. $2+11$ 13	16. $-55+20$ −35	24. $-7-20-(-14)$ −13

B. Choose the one best answer to each question.

Question 25 refers to the following number line.



25. The number line above shows which of the following expressions?
    - A. $2+(-3)$
    - B. $2+(-5)$
    - C. $-3+2$
    - D. $-3-(+2)$

26. At noon, the temperature in the high desert was 92°F. A scientist observed the following temperature changes over the course of the next two hours: +12°F, −5°F, +6°F, −3°F, and +13°F. What was the temperature, in degrees Fahrenheit at the end of the two-hour period?
    - A. 95°F
    - B. 103°F
    - C. 115°F
    - D. 131°F

92
12
−5
6
−3
13

Answers and explanations begin on page 671.

Multiplying and Dividing Signed Numbers

In algebra, multiplication is not shown using the times sign (×) because the symbol could be easily mistaken for the variable x. Instead, multiplication is shown using a dot or by placing two numbers next to each other. To avoid confusion, one or both of the numbers may be enclosed in parentheses. For example, the expressions $-5 \cdot 6$ and $-5(6)$ and $(-5)(6)$ all mean "−5 times 6."

In algebra, division can be written with the ÷ symbol, but it is usually shown with a line that means "divided by." The expression $\frac{20}{-5}$ means "20 divided by −5," as does 20/−5.

When multiplying or dividing signed numbers, use the following rules:

- If the signs are the same, the answer is positive.
- If the signs are different, the answer is negative.

Example 1: Multiply $4(-25)$. −100

1. Multiply the numbers only: $4 \times 25 = 100$.
2. Determine the sign. Since the signs on the numbers are different, the answer is negative: $4(-25) = \mathbf{-100}$.

Example 2: Divide $\frac{-160}{-8}$. 20

1. Divide the numbers only: $160 \div 8 = 20$.
2. Determine the sign. Since the signs on the numbers are the same, the answer is positive: $\frac{-160}{-8} = \mathbf{20}$.

A problem may contain more than two factors. Remember that each pair of negative factors will equal a positive product. Therefore, if there is an even number of negative terms, the product will be positive. If there is an odd number of negative terms, the product will be negative.

Example 3: $(-5)(6)(-1)(-2)(2) = ?$

1. Multiply: $5 \times 6 \times 1 \times 2 \times 2 = 120$.
2. There are three negative terms. Since 3 is an odd number, the product is negative: $\mathbf{-120}$.

When a problem contains more than one operation, follow this **order of operations**. Do multiplication and division operations first, working from left to right. Do addition and subtraction operations last, also working from left to right. If a problem contains division presented with a division bar, do any operations above and below the bar first and then divide.

Example 4: $(-4)(6) - \frac{3+(-9)}{-2}$

1. Multiply: $-24 - \frac{3+(-9)}{-2}$.
2. Do the operation above the fraction bar: $-24 - \frac{-6}{-2}$.
3. Divide: $-24 - (+3)$.
4. Subtract: $-24 - (+3) = -24 + (-3) = \mathbf{-27}$.

NOTE: *Notice how parentheses clarify meaning:* $4 - 5$ means "four minus five," but $4(-5)$ means "four times negative five."

PRACTICE 1.2

A. Solve. You MAY NOT use a calculator.

1. $(5)(4)$ 20
2. $(7)(-3)$ −21
3. $(-8)(6)$ −48
4. $(-2)(-9)$ 18
5. $(-10)(1)$ −10
6. $9 \div 3$ 3
7. $12 \div (-4)$ −3
8. $-25 \div 5$ −5
9. $(-18) \div (-9)$ 2
10. $\frac{40}{-8}$ −5
11. $(14)(-2)$ −28
12. $(-75) \div 25$ −3
13. $13 \div (-13)$ −1
14. $(-5)(15)$ −75
15. $\frac{18}{3}$ 6

B. Solve. You MAY use a calculator.

16. $\frac{25(4)}{-5}$ −20
17. $(-3)(-5)(2)(-10)$ −300
18. $20 \div (-5) \div (-2)$ 2
19. $\frac{6(5)}{(-3)(2)}$ −5
20. $(-11)(2)(-5)(6)$ −24 660
21. $(12)(-2) \div (-2)$ 12
22. $(-4)(-6)(-5)$
23. $50 \div (2)(-5)$
24. $(-1)(2)(-3)(2)(-1)$
25. $\frac{(3)(-4)(2)(5)}{-6}$
26. $\frac{4(-4)}{-8(-2)}$
27. $(-5)(-2)(0)(-1)$

C. Choose the <u>one best answer</u> to each question.

28. Janice is creating a computer spreadsheet. A portion of her work is shown below.

	A	B	C
1	−3	4	7
2	2	−5	−8
3	−1	3	−2

Using the information from the spreadsheet, what is the value of the expression A1*C1*A3/(B3*A3)? (*Hint:* In a spreadsheet, the symbol * means multiplication.)

A. −21
B. −7
C. $\frac{-1}{7}$
D. 21

29. The product of 2 and 8 is divided by −8. Which of the following expressions could be used to find the value of the statement?

A. $\frac{\frac{2}{8}}{-8}$
B. $2(8)(-8)$
C. $\frac{2(8)}{(-8)}$
D. $\frac{2(-8)}{8}$

30. Which of the following is a true statement about the value of the expression $(52)(-103)(-45)(-8)(3)$?

A. The result is a fraction.
B. The result is greater than 1.
C. The result is a negative number.
D. The result is a positive number.

Answers and explanations begin on page 671.

LESSON

2

Powers and Roots

Key Ideas

- In the expression 2^3, the base is 2 and the exponent is 3.
- To raise a number to a certain power, write the base the number of times shown by the exponent and then multiply.
- To find a square root, think, "What number times itself equals this number?"

Powers are a special way to show repeated multiplication. For example, suppose you needed to multiply $5 \times 5 \times 5 \times 5$. This series of operations can be expressed as "five to the fourth power." In other words, the number 5 appears in the multiplication problem four times.

We can write the operations algebraically using **exponents**. In the expression $5 \times 5 \times 5 \times 5$ above, the number 5 is the base. The exponent, a small number written above and to the right of the **base**, tells how many times the base is repeated: $5 \times 5 \times 5 \times 5 = 5^4$.

To evaluate an expression, perform the multiplication indicated by the exponent.

Example 1: Find the value of 2^5.

Write the base the number of times indicated by the exponent and then multiply:

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = \mathbf{32}$.

You may encounter some special uses of exponents on the *Mathematical Reasoning Test*. An exponent can be any positive number, or it can be 0, 1, or a negative number. Memorize the situations described below.

1. A number raised to the first power equals itself: $8^1 = 8$.
2. A number other than zero raised to the power of zero equals 1: $6^0 = 1$.
3. A number raised to a negative exponent is equal to a fraction with a numerator of 1: $4^{-2} = \frac{1}{4^2} = \frac{1}{4 \times 4} = \frac{1}{16}$.

You can use a calculator to raise numbers to any power. Use the $\boxed{x^2}$ or $\boxed{\wedge}$ key.

Example 2: Find the square of 24.

Press: 24 $\boxed{x^2}$ $\boxed{\text{enter}}$. The right side of the display reads 576.

The square of 24 is **576**.

Example 3: Find the value of 9^4.

Enter the base, press $\boxed{\wedge}$, and enter the exponent. Then press $\boxed{\text{enter}}$.

Press 9 $\boxed{\wedge}$ 4 $\boxed{\text{enter}}$. The right side of the display reads 6561.

Nine raised to the fourth power is **6561**.

NOTE: To raise a negative number to a power using the TI-30XS MultiView™ calculator, you must enter that number in parentheses: $\boxed{(}$ $\boxed{(-)}$ 4 $\boxed{)}$ $\boxed{x^2}$ $\boxed{\text{enter}}$.

To square a number, multiply the number by itself. For example, $6^2 = 6 \times 6 = 36$. In the expression $6 \times 6 = 36$, the number 36 is the square, and the number 6 is the **square root** of 36.

GED® TEST TIP

Make a list of common square roots by squaring the numbers from 1 to 15. Memorize them. You will need to know common square roots to solve geometry problems about area and right triangles.

The symbol for square root is $\sqrt{\ }$. To find a square root, think, "What number multiplied by itself is equal to the number in the bracket?"

Example 4: Find the value of $\sqrt{144}$.

You know that $12 \times 12 = 144$, so the square root of 144 is **12**.

Although $(-12) \times (-12)$ also equals 144, you will only be expected to find positive roots on the GED® Test.

You may have to approximate the value of a square root.

Example 5: What is the square root of 90?

You know that $9 \times 9 = 81$ and $10 \times 10 = 100$. Therefore, the square root of 90 is **between 9 and 10.**

You can also use your calculator to find a square root. On the TI-30SX MultiView™, you must first press the $\boxed{\text{2nd}}$ key to access the square root function. The $\sqrt{\ }$ function is directly above the $\boxed{x^2}$ key. (Other calculators may not require the use of the $\boxed{\text{2nd}}$ key.)

Example 6: Use your calculator to find $\sqrt{90}$ to the nearest tenth.

On the TI-30SX MultiView™, press $\boxed{\text{2nd}}$ $\boxed{x^2}$ 90 $\boxed{\text{enter}}$. The right side of the display reads $3\sqrt{10}$.

Press the toggle key, $\boxed{\blacktriangleleft\blacktriangleright}$, to change the format of the answer into a decimal.

The right side of the display now reads 9.486832981.

Rounding to the tenths place, $\sqrt{90} \approx \mathbf{9.5}$.

PRACTICE 2

A. Solve each expression. You MAY NOT use a calculator.

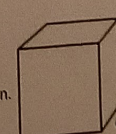
1. 3^2	4. 25^0	7. 5^3	10. $\sqrt{64}$
2. 4^1	5. $(-3)^2$	8. 4^{-2}	11. 10^{-3}
3. $\sqrt{9}$	6. $\sqrt{49}$	9. 2^4	12. $\sqrt{121}$

B. Solve each expression below. You MAY use your calculator. Round your answer to the nearest tenth.

13. 3^8	16. 20^3	19. $\sqrt{242}$	22. $\sqrt{536}$
14. $(-6)^4$	17. 1^{15}	20. $(3.3)^2$	23. 112^0
15. $\sqrt{150}$	18. $(-4)^{-2}$	21. $\sqrt{57}$	24. $(-2)^8$

C. Choose the <u>one best answer</u> to each question.

25. The cube shown below measures 6 inches on each side. You can find the volume of the cube by multiplying $\text{length} \times \text{width} \times \text{height}$. Which of the following expressions represents the volume of the cube?
   A. 6^1
   B. 6^2
   C. 6^3
   D. 6^6



6 in.
6 in.
6 in.

26. Which of the following expressions has the least value?
   A. 3^{-3}
   B. 4^0
   C. 4^1
   D. 2^{-4}

Answers and explanations begin on page 672.

LESSON

3

ALGEBRA BASICS, EXPRESSIONS, AND POLYNOMIALS

Scientific Notation

Key Ideas

- In scientific notation, a number equal to or greater than 1 and less than 10 is multiplied by a power of ten.
- Very small numbers are expressed by multiplying by a negative power of ten.
- In scientific notation, the number with the greatest power of ten has the greatest value.

Scientific notation uses the powers of ten to express very small and very large numbers. In scientific notation, a decimal number (greater than or equal to 1 and less than 10) is multiplied by a power of ten.

Look for patterns as you review these powers of ten.

$10^1 = 10$ $\quad 10^2 = 100$ $\quad 10^3 = 1{,}000$ $\quad 10^4 = 10{,}000$ $\quad$ and so on.

$10^{-1} = 0.1$ $\quad 10^{-2} = 0.01$ $\quad 10^{-3} = 0.001$ $\quad 10^{-4} = 0.0001$ $\quad$ and so on.

Did you find the patterns? In the row with positive exponents, the exponent is the same as the number of zeros in the number written in standard form. In the row with negative exponents, the exponent is the same as the number of decimal places in the number.

You can use these patterns to change scientific notation to standard form.

Example 1: Write 6.2×10^5 in standard form. $\qquad$ 6.20000

Move the decimal point five places to the *right* (the same number as the exponent). Add zeros as needed. $\qquad$ **620,000**

Example 2: Write 3.82×10^{-3} in standard form.

Move the decimal point three places to the *left* (the same number as the exponent). Add zeros as needed. $\qquad$ 0.00382

Work backward to write large and small numbers in scientific notation.

Example 3: To reach Mars, the Viking 2 spacecraft traveled 440,000,000 miles. What is the distance traveled in scientific notation?

1. Move the decimal point to the left until there is only a single digit in the ones place. $\qquad$ 4.40000000
2. Multiply by 10 raised to a power equal to the number of places you moved the decimal point. $\qquad$ 4.4×10^8

GED® TEST TIP

Always try to save time. Suppose the multiple-choice answers are written in scientific notation. Instead of changing each choice to standard notation, change your answer to scientific notation.

Example 4: Scientists find that a kind of bacteria moves at a rate of 0.00016 kilometers per hour. Write the measurement in scientific notation.

1. Move the decimal place to the right until there is a single digit in the ones place. $\qquad$ 0001.6
2. Multiply by 10 raised to a negative exponent equal to the number of places you moved the decimal point. $\qquad$ 1.6×10^{-4}

You may be asked to compare numbers in scientific notation.

Example 5: Which is greater: 4.5×10^3 or 9.8×10^4?

You don't need to change the numbers to standard notation. Simply consider the powers of ten. Multiplying by 10^4, or 10,000, must have a greater result than multiplying by 10^3, which equals 1,000. In scientific notation, the number with the greater power of 10 has the greater value. Therefore, **9.8×10^4 is greater than 4.5×10^3.**

PRACTICE 3

A. Write each number in scientific notation.

1. 2300
2. 0.00042
3. 12,400,000
4. 14,320,000,000
5. 36,000,000
6. 0.0095
7. 0.00000058
8. 150,000,000,000
9. 0.000000009

B. Convert from scientific notation to standard notation.

10. 5.173×10^{-4}
11. 3.7×10^6
12. 4.8×10^8
13. 1.7×10^{-5}
14. 7.2×10^{-3}
15. 9.16×10^5
16. 8.591×10^7
17. 9.56×10^{-6}

C. Answer the following questions.

18. Many domestic satellites maintain an orbit approximately 23,500 miles above Earth. What is that distance, in miles, in scientific notation?
19. Modern technology measures very fast transactions in nanoseconds. One nanosecond equals 1.0×10^{-9} of a second. How many seconds is a nanosecond, in standard notation?
20. The average distance of Neptune from Earth is 2.67×10^9 miles. Write the distance, in miles, in standard notation.
21. Light in the vacuum of space travels at a speed of nearly 300 million meters per second. Write the speed, in meters, in scientific notation.

D. Choose the one best answer to each question.

Questions 22 and 23 refer to the following table.

Unit	U.S. Equivalent	Metric Equivalent
1 ton	2,000 lb	0.907 metric ton
1 acre	43,560 sq ft	4,047 square m

4

22. What is the number of square feet in an acre, written in scientific notation?
    A. 0.4356×10^6
    B. 4.356×10^4
    C. 4.356×10^3
    D. 43.56×10^3

23. A shipment of goods weighs 5 tons. Which of the following expressions could be used to express the weight in metric tons?
    A. $5 \times 0.907 \times 10^{-1}$
    B. $5 \times 9.07 \times 10^1$
    C. $5 \times 9.07 \times 10^{-2}$
    D. $5 \times 9.07 \times 10^{-1}$

Answers and explanations begin on page 672.

LESSON 4

Order of Operations

When a mathematical expression contains more than one operation, its value may depend upon the order in which the operations are performed. To avoid confusion, mathematicians have agreed to perform operations in a certain order.

Key Ideas

- To evaluate an expression correctly, you must follow the order of operations.
- If an expression uses more than one set of grouping symbols, start with the inside set and work to the outside.
- The division bar may be used as a grouping symbol.

GED® TEST TIP

You can use the parentheses keys on your online calculator to help you follow the order of operations.

The Order of Operations

1. Parentheses or any other grouping symbols that enclose operations
2. Exponents and roots
3. Multiplication and division, working from left to right
4. Addition and subtraction, working from left to right

Study the following example to see how to apply the order of operations. Notice that parentheses are used in two places in the expression; however, only the first set of parentheses encloses an operation.

Example 1: Evaluate the expression $\frac{(5+3)^2}{4}+3(-1)$.

1. Perform the addition operation in parentheses. $\frac{(8)^2}{4}+3(-1)$
2. Raise 8 to the second power. $\frac{64}{4}+3(-1)$
3. Divide, then multiply. $16+(-3)$
4. Add. 13

The value of the expression $\frac{(5+3)^2}{4}+3(-1)$ is 13.

In more complicated expressions, one set of grouping symbols may be nested within another set. To avoid confusion, you can also use brackets [ ] or braces { } to group operations. To evaluate an expression with more than one set of grouping symbols, work from the inside to the outside.

Example 2: Evaluate the expression $4[5(-4)+3)+2]$.

1. Perform the operation in the inner set of grouping symbols: $(-4+3)$. $4[5(-4+3)+2]$ $4[5(-1)+2]$
2. Do the operations inside the brackets. Since multiplication comes before addition in the order of operations, multiply 5 and −1 and then add 2. $4[-5+2]$ $4[-3]$
3. Multiply 4 and −3. -12

The division bar is also a grouping symbol. Before you divide, perform any operations shown above and below the bar.

Example 3: Evaluate the expression $\frac{15+25}{2(5)}+6$.

1. Perform the operations above and below the fraction bar. $\frac{15+25}{2(5)}+6$
2. Divide, then add. $\frac{40}{10}+6$

$4+6=10$

PRACTICE 4

A. Solve. You MAY NOT use a calculator.

1. $4(3)-2\div(6+4\cdot2)$
2. $16\div(10-6)^2$
3. $5^2-(5-7)(2)$
4. $3(-3)+(7+4)$
5. $\frac{3^3}{5-2}-\frac{(4-2)^2}{2}$
6. $\frac{25}{(4+1)}\cdot3+(6-1)$
7. $2^3+(8-5)^2-3$
8. $(4-12)(-6)+(10-3)$
9. $30\div3(5-4)$
10. $15+(4)(3)-2^2$
11. $(4+2)^2+(7-2)^3$
12. $7^2\div(11-4)+(9+14)$
13. $2\left[(17-11)^2\cdot\frac{(15-5)}{2}\right]$
14. $(5^2+6-3)\div(16-3^2)$
15. $150-4\left[\frac{3+9}{4-1}\cdot(14-11)^2\right]$

B. Choose the <u>one best answer</u> to each question.

<u>Question 16</u> refers to the following information.

Susan is in charge of planning Midvale Hospital's parent education classes. She uses the table below to determine the cost of each class to the hospital.

Midvale Hospital Parenting Workshops

Type of Workshop	Cost per Participant
Childbirth Classes	$35 per couple
Infant Care	$30 per person
Teaching Your Child to Read	$60 per person

16. A local foundation has offered to pay 75% of the cost of infant care classes. The hospital will cover any remaining costs. There are 28 parents enrolled in the upcoming class. Which of the following expressions could be used to find the amount the hospital will pay?
   A. (75)(28)(30)
   B. (28)(30) − (0.75)(30)
   C. (1 − 0.75)(28)(30)
   D. (1 − 0.75)(30) + 28

17. In the expression

$$5+2\left[7\left(\frac{10^2}{10}\right)+(6-2)(3)\right],$$

what is the last operation you should perform to find the value of the expression?
   A. Subtract 2 from 6.
   B. Add 5.
   C. Multiply by 2.
   D. Find the square of 10.

18. Find the value of the expression $22+6[(14-5)\div3(17-14)]$.
   A. 2.73
   B. 28
   C. 76
   D. 97

Answers and explanations begin on page 672.