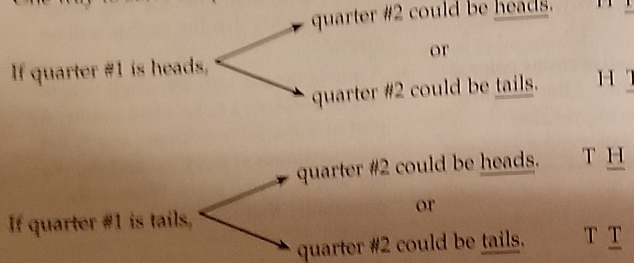
Dependent and Independent Probability

You know how to find the probability of a single event. You can use this knowledge to find the probability of two or more events.

Example 3: Brad tosses two quarters into the air. What is the probability that both will land so that the heads' sides are showing?

One way to solve the problem is to list or diagram all the possible outcomes.



There are four possible outcomes, and only one is favorable (HH). Therefore, the probability of having both land with the heads side up is $\frac{1}{4}$, or **25%**.

You can also use multiplication to find the probability.

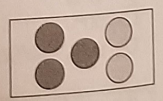
1. Find the probability of the individual events. The probability that one coin will be heads is $\frac{1}{2}$, and the probability that the other will be heads is $\frac{1}{2}$.
2. Multiply to find the probability of both events: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

The two coin tosses in Example 3 are **independent events**. When events are independent, one does not affect the probability of another. In Example 4 below, the events are **dependent**. Once the first event takes place, the probability of the second event is changed.

Example 4: A box contains four blue marbles and two red marbles. If you select two marbles, what is the probability that both will be blue?

(*Hint:* Even though the marbles in the box are taken out at the same time, think of one as the first marble and the other as the second marble.)

1. There are six marbles in the box, and four are blue. The probability that the first marble will be blue is $\frac{4}{6}$, which reduces to $\frac{2}{3}$.



2. Assume the first marble selected is blue. Now there are only five marbles in the box, and three are blue. The probability that the second marble will be blue is $\frac{3}{5}$.
3. Multiply to find the probability of the two events: $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$, or $\frac{2}{5}$.

The probability that both marbles will be blue is **2 out of 5**.

NOTE: The events in Example 4 would not be dependent if the first marble were replaced before the second marble was selected. Always think carefully about the situation to decide whether two events are dependent or independent.

PRACTICE 7.2

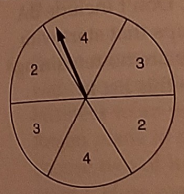
A. Solve as directed. Express answers as fractions.

1. Kim rolls two standard six-sided dice. What is the chance that both will be 4s?
2. Ten cards are numbered from 1 to 10. Toni draws out a card, replaces it, and then draws another card. What is the probability that both cards will be numbers greater than 5?
3. A spinner has four equal sections. Two sections are red, one is green, and one is blue. If the spinner is spun three times, what is the probability that all three spins will be red?
4. Twenty marbles are placed in a bag. Ten are red, and ten are black. One marble is drawn from the bag and set aside. Another marble is drawn from the bag. What is the chance that both marbles will be red?
5. Allison tosses a coin four times. What is the chance that the coin will be heads all four times?
6. If you roll two standard dice, what is the probability that both will be an odd number?

B. Choose the one best answer to each question. You MAY use your calculator.

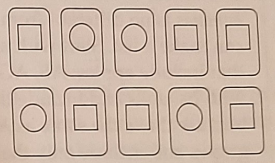
Questions 7 and 8 refer to the following information.

In a game a player rolls a die, numbered from 1 to 6, and spins a spinner. The spinner is shown below.



7. What is the probability of rolling a 5 and then spinning an even number?
   - A. $\frac{1}{9}$
   - B. $\frac{1}{6}$
   - C. $\frac{2}{3}$
   - D. $\frac{5}{6}$
8. What is the chance that a player will get the same number on both the die and the spinner?
   - A. $\frac{5}{6}$
   - B. $\frac{2}{3}$
   - C. $\frac{1}{3}$
   - D. $\frac{1}{6}$

Daniel uses the ten cards below in a magic trick.



9. Daniel shuffles the cards and asks an audience member to choose and hold two cards. If the cards are chosen randomly, what is the chance that both will be marked with a square?
   - A. 8 out of 14
   - B. 3 out of 5
   - C. 1 out of 3
   - D. 1 out of 5
10. There are 15 colored chips in a bag. Eight are green, and seven are white. Five white chips are removed. What is the probability that the next chip selected will be green?
    - A. 100%
    - B. 80%
    - C. 75%
    - D. 25%

Answers and explanations begin on page 669.

LESSON 9

Permutations

Key Ideas

- Permutations are used to find the number of possible outcomes when the order does matter.
- Use the total number of people or items and the number of possible outcomes to find the number of permutations that are possible.

ON THE GED® TEST

When you are reading a question that suggests that you may want to find a combination or a permutation, ask yourself, "Am I finding a number of groups (combinations), or am I finding the number of possible sequences (permutations)?"

On the *Mathematical Reasoning Test*, you may be asked how many possible permutations, or sequences, are possible given a group of items. Consider the following example:

Example 1: Eliza is planning her day off. She wants to visit the art museum, try the new coffee shop, call her mom, and take a walk, but not necessarily in that order. How many possible sequences of those four activities are there?

This question asks you for all possible permutations, or ways to sequence the items. Thus, *coffee-art-mom-walk* and *art-coffee-mom-walk* are two different possible outcomes. Consider how this problem differs from the combinations problems on pages 300–303. In those cases, order did not matter; in a permutations problem, order does matter.

To find how many ways you can sequence all the items in a list, multiply the possibilities like this. Start with how many things could go first: here, there are four possibilities for Eliza's first activity. Once she has done that first activity, three possibilities remain. After she has done a first and second activity, two possibilities remain, and then only one. So you simply multiply: $4 \times 3 \times 2 \times 1$. There are **24** possible permutations.

Some permutations questions may ask you to determine the possible orderings for only some of the items in a list. Consider this example:

Example 2: Ten runners are competing in a race. There are prizes for first, second, and third place. How many possible sequences of the top three prize winners are there?

Notice that you are not simply figuring out how many groups of three can be made out of the ten. Rather, you are figuring out how many sequences of three can be made out of the ten. That can sometimes be represented on the GED® Test with the notation P(10, 3).

Now, multiplying $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ would give you all the possible permutations of all ten runners, so that approach will not work here. Instead, start with how many people could win first place: here, ten. Once someone has won first, there are nine possibilities for second place. Once someone has won first and someone else has won second, there are eight possibilities for third place. And then you stop multiplying: $10 \times 9 \times 8 = 720$. There are **720** possible ways that the runners could be arranged in the top three prize-winning slots. (Note that simply multiplying 10 by 3 will not work.)

PRACTICE 9

A. Read the problems and decide whether you are being asked to find combinations or permutations. Then solve.

1. Noemi is trying to remember the password for her email. She knows that it has the following characters in it: M, Q, $, L, 7. But she can't remember the order they go in. How many possible sequences of those five characters are there?
2. Bilal is going to the museum. She wants to see the contemporary art exhibit, the Impressionists, the Greek vases, and a special Picasso exhibit but not necessarily in that order. How many sequences of those exhibits are possible?
3. Five students in a class have volunteered for a special project. Only three can actually help with the project. How many possible groupings of three out of the five are possible?
4. Tyrell is the curator at a gallery and is deciding how to arrange six paintings that will be displayed in a line along one wall. All six paintings will be included. How many possible sequences could Tyrell choose?

B. Choose the <u>one best answer</u> to each question about combinations or permutations. You MAY use your calculator.

5. Soraya has been given six tasks to do at work, but she has time to complete only four of them. She must decide in what order to do the tasks. How many possible orderings of four tasks are available to Soraya?

   A. 24
   B. 36
   C. 360
   D. 720

6. In a certain public garden, the gardener wanted to show the different visual effects that arranging flowers in different sequences can have. So he chose three kinds of flowers and planted flower beds showing each of the possible sequences of the three kinds. How many such flower beds did the gardener plant?

   A. 3
   B. 6
   C. 9
   D. 12

7. Ten people hope to become extras in a movie. The movie's casting director will choose four people to fill the following specific roles in the movie: Bystander #1, Bystander #2, Bystander #3, and Bystander #4. How many ways could those four roles be filled?

   A. 24
   B. 40
   C. 540
   D. 5,040

<u>Questions 8 and 9</u> **refer to the following information:**

Clark and his daughter are at the amusement park, and Clark is offering his daughter a choice. They don't have time to ride all six rides at the amusement park, but they can ride three of them.

8. How many combinations of three rides could Clark's daughter choose?

   A. 18
   B. 20
   C. 120
   D. 720

9. Clark's daughter can also choose the order in which she wants to enjoy the rides. How many possible orderings of three out of the six rides are possible?

   A. 18
   B. 20
   C. 120
   D. 720

Answers and explanations begin on page 670.

LESSON

10

DATA, STATISTICS, AND PROBABILITY

Compare Proportions in Different Formats

Key Ideas

- A proportion can be represented in multiple ways, but its magnitude does not change.
- It is easiest to compare the magnitude of two values if they are represented in the same way.

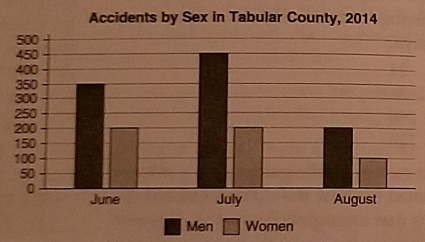
GED® TEST TIP

When a question asks you to compare two proportions presented in different formats, convert one or both proportions so they are in the same format. Usually, proportions in a numerical format are easiest to compare.

The relative magnitude of two quantities can be represented in multiple ways: as simple numbers, on a graph or chart, or as dimensions on a blueprint or diagram. It is harder to compare two quantities when they are presented differently—for example, when one is represented on a bar graph and the other is represented in a table of values.

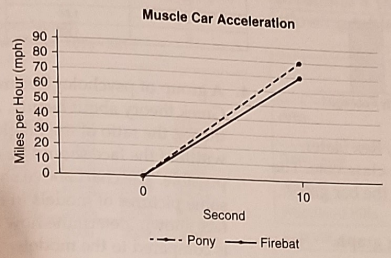
To directly compare quantities represented differently, translate them into the same format. Typically, using a numerical representation is most efficient for both translation and comparison purposes.

Example 1: The bar graph shows the number of accidents for men and women aged 18–35 in Tabular County for the summer of 2014. The following year, the county released a report that the monthly rate of accidents for men in 2015 had been 1.6, 2.3, and 1.8 times that of women for June, July, and August, respectively. Of the six months reported, when was an accident victim more likely to be male?



1. Determine the proportion of 2014 male accidents to female accidents from the bar graph. For June, the ratio is $\frac{350}{200} = 1.75$, for July $\frac{450}{200} = 2.25$, and for August $\frac{200}{100} = 2$.
2. Compare the calculated values to determine the greatest proportion of male to female accidents. The highest value for this ratio is in July 2015, when men were 2.25 times as likely to have an accident as women.

Example 2: The following velocity-time graph shows the rate of acceleration of two classic muscle cars. A newer model, the Hermes, is able to accelerate from 0 to 60 miles per hour (mph) in 8 seconds (s).



How does the Hermes compare to the cars on the graph in terms of acceleration?

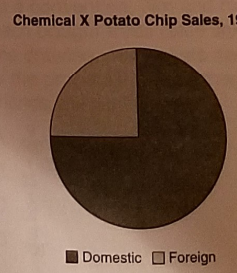
1. Use the formula for slope of a line, $(m) = \frac{\text{rise}}{\text{run}}$, to find the rate of acceleration for the cars on the graph. The Pony's acceleration is $\frac{80-0}{10-0} = 8\text{mph/s}$, and the Firebat's is $\frac{70-0}{10-0} = 7\text{mph/s}$.
2. Write the rate of acceleration of the Hermes the same way, as $\frac{60}{8} = 7.5\text{ mph/s}$.
3. Compare the values to determine the relative speeds of the cars. The Hermes is faster than the Firebat but still slower than the Pony.

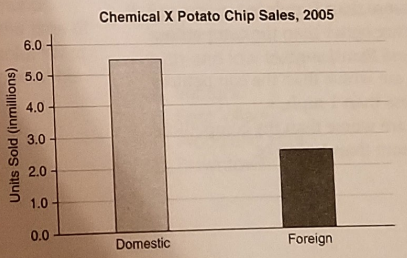
PRACTICE 10

Compare the proportional data in the following scenarios.

Question 1 is based on the following graphs.

1. The two graphs show domestic and foreign sales for a brand of US potato chips. For which year were the foreign sales higher as a percentage of total sales, and what was the difference in that percentage between the years?





Probability

Key Ideas

- Probability is a ratio. It can be expressed as a ratio, fraction, decimal, or percent.
- Theoretical probability is the ratio of favorable outcomes to possible outcomes.
- Experimental probability is the ratio of favorable outcomes to the number of trials in an experiment.

GED® TEST TIP

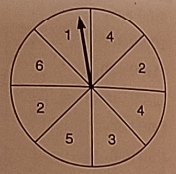
In a multiple-choice probability problem, skim the answer choices to see if they are in fraction, percent, or ratio form. Knowing the answer form will help you decide how to do your calculation.

Simple Probability

Probability tells whether something is likely or unlikely to happen. The probability of any event can be expressed by a number from 0 to 1. If an event has 0 probability, the event is impossible. An event with a probability of 1 is certain to happen. Most events are somewhere in between.

To find the probability of a simple random event, we must identify favorable and possible outcomes. A **favorable outcome** is the event that we are interested in. The **possible outcomes** are all the possible events that could occur. **Theoretical probability** (sometimes called **simple probability**) is the ratio of favorable outcomes to possible outcomes.

Example 1: The spinner is divided into 8 equal sections. What is the probability of spinning a 4 on the spinner?



1. There are two sections labeled 4 on the spinner, and there are 8 sections in all.
2. Use the probability ratio: $\frac{\text{favorable outcomes}}{\text{possible outcomes}} = \frac{2}{8} = \frac{1}{4}$.

The probability of spinning a 4 on the spinner is **1 out of 4, $\frac{1}{4}$, 0.25, or 25%.**

In Example 1, probability was based on what we knew could happen. Another type of probability, called **experimental probability**, is based on what actually happens during the trials of an experiment. The number of trials are the number of times you try the experiment.

Example 2: Ricardo and Scott used the same spinner to play a game. They kept track of the numbers that they got on each spin for 20 spins. The numbers are shown below.

2, 4, 4, 6, 4, 3, 4, 6, 4, 3, 1, 6, 2, 2, 5, 2, 4, 2, 1, 2

Based on their results, what is the experimental probability of spinning a 4?

1. Ricardo and Scott spun a 4 six times out of twenty.
2. Use this ratio: $\frac{\text{favorable outcomes}}{\text{number of trials in experiment}} = \frac{6}{20} = \frac{3}{10}$, **0.3, or 30%.**

Notice that experimental probability is close to, but not necessarily equal to, theoretical probability. Theoretical probability can tell you what will probably happen, but it can't predict what will actually happen.

PRACTICE 7.1

A. Express probability as a fraction, decimal, and percent for questions 1 through 5. Do not use a calculator.

1. A game has 50 wooden tiles. Players draw tiles to spell words. If 20 of the tiles are marked with vowels, what is the probability of drawing a vowel from the tiles?

2. A spinner has five equal sections colored either red, white, or blue. After 40 spins, a player has the following results:

Color	Frequency
red	𝍸
white	𝍸 ||
blue	𝍸 𝍸 𝍸 𝍸 |||

What is the experimental probability of not spinning blue on the spinner?

3. There are four red, four blue, and two green marbles in a bag. If one marble is chosen at random from the bag, what is the probability that the marble will be green?

4. A movie theater sells 180 adult tickets and 60 children's tickets to a movie. As part of a special promotion, one ticket will be chosen at random, and the winner will receive a prize. What is the probability that the winner will be a child?

5. A spinner has six equal sections numbered from 1 to 6. What is the probability of spinning either a 5 or 6?

B. Choose the one best answer to each question. You MAY use your calculator.

Questions 6 and 7 refer to the following information.

A deck of 12 cards is marked with the following symbols.

6. If a card is chosen at random, what is the probability of selecting a diamond (♦)?
   A. 6%
   B. 12%
   C. 50%
   D. 60%

7. If a card is chosen at random, what is the probability of selecting something other than a club (♣)?
   A. $\frac{3}{4}$
   B. $\frac{2}{3}$
   C. $\frac{1}{3}$
   D. $\frac{9}{100}$

Questions 8 and 9 refer to the following information.

Erin flipped a coin 40 times and made this table to show how many outcomes were "heads" and how many were "tails."

heads	𝍸 𝍸 𝍸 𝍸 ||||
tails	𝍸 𝍸 𝍸 |

8. Based on Erin's data, what is the experimental probability of getting tails on a coin flip?
   A. 3 out of 5
   B. 3 out of 4
   C. 2 out of 3
   D. 2 out of 5

9. Based on Erin's data, what is the experimental probability of getting heads on a coin flip?
   A. 3 out of 5
   B. 3 out of 4
   C. 2 out of 3
   D. 1 out of 2

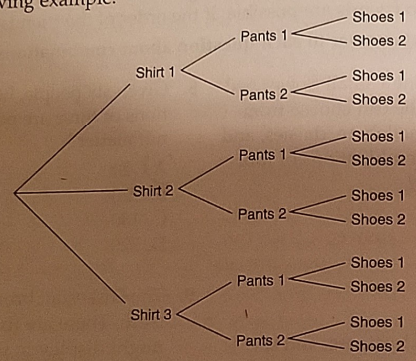
Answers and explanations begin on page 669.

Combinations with More Than One Type of Item

Sometimes you may be asked to count possible combinations with more than one type of item. Consider the following example:

Example 2: Sarah is deciding what to wear. She has three shirts, two pairs of pants, and two pairs of shoes. How many possible outfits does she have, if an outfit is one shirt, one pair of pants, and one pair of shoes?

This question differs from the question about Pablo on page 300. In that question, Pablo was choosing from one overall group of fruits. Here, Sarah is choosing one out of each of three groups—shirts, pants, and shoes. If you are given more than one type of item and must choose one of each type, you can use a tree diagram to figure out all the possible combinations. Study the following example:



Each "branch" of the tree represents a possible combination. The number of branches on the right-hand side represents the possible number of combinations; in this case, there are **12** possible outfits.

However, you can solve the same type of problem (where you have more than one type of item and are choosing one of each type) using the **fundamental counting principle,** which works like this:

Start with shirts: for each shirt, Sarah has two pants options. That's $3 \times 2 = 6$. Additionally, for each of those six options, she has two options for shoes. That's $3 \times 2 \times 2 = \mathbf{12}$.

Use the fundamental counting principle to solve the following problem:

Example 3: Gordon is at a restaurant that serves a three-course meal: an appetizer, an entrée, and a dessert. There are three appetizers, six entrees, and four desserts to choose from. Gordon will order one of each. How many different meals could he order?

Multiply three appetizers by six entrees by four desserts: $3 \times 6 \times 4 = 72$. There are **72** possible ways Gordon could order his meal.

PRACTICE 8.2

A. Use a tree diagram or the fundamental counting principle to solve the following problems.

1. Frank is making a sandwich. His sandwich will have one type of bread, one type of meat, and one condiment. He has four types of bread: white, wheat, rye, and a Kaiser roll. Meats he can choose from are chicken, turkey, and roast beef. For condiments, Frank can choose mayonnaise, mustard, vinaigrette, or horseradish. How many ways can Frank make his sandwich?

2. Van Ahn attended a trivia contest and won three rounds. For the first round she won, she could choose one of three restaurant gift certificates. The prize for another round was one of two T-shirts. Winning the third round allowed her to choose one of four souvenir hats. How many possible combinations of prizes were available to Van Ahn?

3. Henry is wrapping a gift for his daughter. He will use one type of wrapping paper, one type of ribbon, and one bow. He has three different types of wrapping paper, three different types of ribbon, and three different bows. How many ways can Henry wrap the gift?

4. Julio is looking forward to a three-day weekend. On Friday, he will either go to the beach or attend a street festival. On Saturday, he plans to visit a museum, but he is trying to decide among three different museums. On Sunday, he will try one of four new restaurants. How many ways could Julio combine these weekend activities?

B. Choose the one best answer to each question about combinations. Determine whether you are being asked to find possible combinations from one type of item or from multiple types of items. You MAY use your calculator.

5. The Nu Mu Beta fraternity is deciding what secret passphrase it should require its members to say before being admitted to meetings. The first and third words of the passphrase will be the names of Greek letters. There are 24 letters in the Greek alphabet. The second word will be a number from one to nine. How many possible passphrases are there?

   A. 24
   B. 57
   C. 576
   D. 5,184

6. A video game designer is creating a new superhero, who will have three of five potential superpowers. How many combinations of superpowers could the new superhero have?

   A. 10
   B. 15
   C. 100
   D. 125

7. A chef will make a soup with five ingredients: one of four meats, one of four vegetables, one of four kinds of noodles, one of four kinds of broth, and one of four spices. How many possible combinations of ingredients could the chef put into the soup?

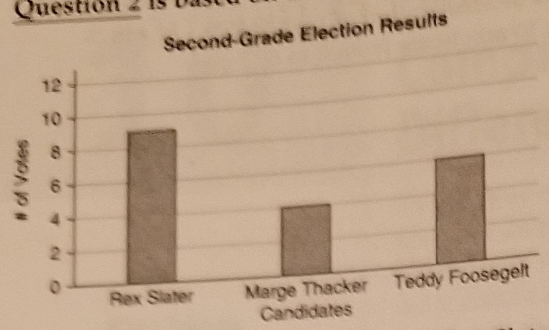
   A. 20
   B. 256
   C. 1,024
   D. 3,125

8. A doctor is deciding how to treat a given disease. The doctor will prescribe one medication, one dietary change, and one type of vitamin supplement. There are five medications, five dietary changes, and five types of vitamins the doctor might prescribe. How many combinations are possible?

   A. 125
   B. 625
   C. 1,250
   D. 3,125

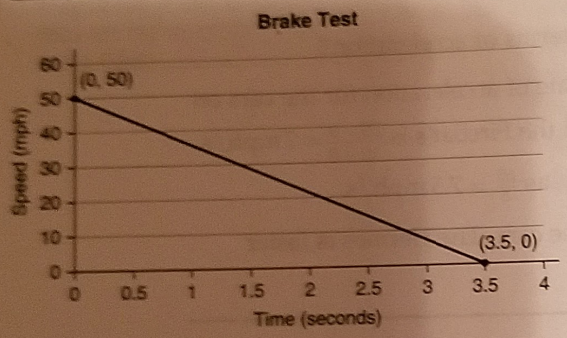
Answers and explanations begin on page 669.

Question 2 is based on the following graph.



2. A second-grade political candidate, Rex Slater, predicted he would win 55% of the class vote. The results of the election are shown in the bar graph. Was he correct?

Question 3 is based on the following graph.

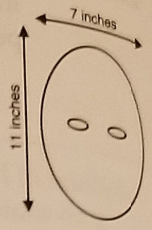


3. Engineers at an automotive parts company are testing brakes to find out whether their brakes beat those of their competitors, which can decelerate the same car at a rate of 15 miles per hour (mph) per second (s). Which statement correctly describes the results of their latest test, shown on the graph?

A. The brakes are better than the competitors' brakes by more than 1 mph/s.
B. The brakes are better than the competitors' brakes by less than 1 mph/s.
C. The brakes are worse than the competitors' brakes by less than 1 mph/s.
D. The brakes are worse than the competitors' brakes by more than 1 mph/s.

Question 4 is based on the following chart and figure.

Model	Ratio
A	1.50
B	1.65
C	1.70
D	1.56
E	1.53



4. A group of psychologists are testing an ancient Greek theory about beauty. According to this theory, the ratio of the height of the face to the width of the face should be as close to 1.6 as possible. The chart above shows the results for some pictures of models in magazines. Given the face shown, determine how beautiful the person is compared to the models.

A. The person is more beautiful than any of the models.
B. The person is more beautiful than four of the models.
C. The person is more beautiful than three of the models.
D. The person is more beautiful than two of the models.

Answers and explanations begin on page 670.

Combinations

Key Ideas

- Combinations are used to select several things out of a larger group when the order doesn't matter.
- If you are asked to find how many ways to combine items from one group, use an organized list or table.
- If you are asked to find how many ways to combine one item from each of a number of different types of items, use a tree diagram or the fundamental counting principle.

ON THE GED® TEST

While you are taking the test on computer, you can use the provided marker and wipe-off board to make a tree diagram or other diagrams to help you visualize problems.

Combinations with One Type of Item

Sometimes the GED® *Mathematical Reasoning Test* will ask you how many ways you can combine a set of items. Sometimes you may have only one type of item to combine, and sometimes you may have more than one type to combine. This difference will determine your problem-solving strategy.

Consider an example with *only one type of item:*

Example 1: Pablo is going shopping at a fruit stand that sells apples, bananas, grapes, and pears. Pablo will buy two different kinds of fruit. How many combinations of two kinds of fruit could Pablo buy?

The question is asking you to list possible groups of two out of the overall set of four. Notice that order doesn't matter—that is, *apples and bananas* is no different from *bananas and apples*. To solve this problem, you can make a list. Start with apples and list all of the **combinations** that include apples: AB, AG, AP. Then make a list of groups that start with bananas and include the remaining fruits: BG, BP. Don't include BA because that's the same as AB. Then make a list of groups that start with grapes and include only the remaining fruits: GP. There are no remaining groups that start with P. You may find it easier to make this list in columns, as follows:

AB

AG BG

AP BP GP

There are 6 possible combinations.

You can also draw a quick table like the following. Include a column for each of the fruits, and let each row represent a possible combination. Place two *x*'s in each row to represent a combination of two fruits.

Apples	Bananas	Grapes	Pears
x	x		
x		x	
x			x
	x	x	
	x		x
		x	x

Notice that you could not draw any more rows with two *x*'s without duplicating some of the existing rows. Count the rows: there are 6 possible combinations of two fruits.

PRACTICE 8.1

A. Use counting, an organized list, or a table to solve the following problems.

1. As a supervisor, Rob is choosing four of his employees to work on a special project. The employees are Angela, Barbara, Colin, David, and Elizabeth. Of those five employees, how many teams of four are possible?
2. Jessica wants to take three books on vacation with her. She has five books to choose from. How many possible combinations of three books could she take on vacation?
3. Grant is cooking a homemade pizza, and he has the following toppings available: anchovies, ham, mushrooms, and sausage. He will choose three of those toppings. How many ways could Grant combine those toppings?
4. Five friends want to play a video game designed for two players. How many different combinations of two friends are possible, if the order of the friends does not matter?

B. Choose the <u>one best answer</u> to each question about combinations.

5. Celia is going to plant a small flower bed with four flowers. She can choose from begonias, fuchsias, hellebore, daisies, and salvia. How many combinations of four flowers are possible?

   A. 5
   B. 20
   C. 24
   D. 120

6. Regan owns a small business, and she wants to send three people from her business to an upcoming trade show. Regan plans to attend, and she will also choose two of her four employees to attend. How many combinations of employees could Regan choose to send to the trade show?

   A. 4
   B. 6
   C. 10
   D. 24

7. A book club, which has two women and three men, can send three of its members to a book signing. How many possible combinations of members could attend the book signing?

   A. 1
   B. 5
   C. 10
   D. 120

8. Given six people, how many combinations of three are possible if order does not matter?

   A. 20
   B. 120
   C. 180
   D. 720

9. The Sarkesian brothers own a hardware store. There are five items the owners might display in the window. Joe Sarkesian thinks that the window should have two items, but his brother Rick thinks the window should have three items. Counting both Joe's ideas and Rick's ideas for the window, how many possible combinations of items might go into the window display?

   A. 10
   B. 20
   C. 30
   D. 120

Answers and explanations begin on page 669.