Understand Polynomials

Key Ideas

- A polynomial is an expression with more than one term—any combination of coefficients, variables, exponents, and constants.
- Specific types of polynomials include monomials, binomials, and trinomials.

GED® TEST TIP

If there is no coefficient next to a variable, assume the coefficient is 1.

The GED® *Mathematical Reasoning Test* will expect you to understand polynomials. A **polynomial** is made up of terms, and since the prefix *poly-* means "many," a polynomial is a term in which many parts are being combined. Each term in a polynomial may be made up of a combination of coefficients, variables, exponents, and constants.

Coefficients include numbers such as 4, -25, or $\frac{1}{2}$ and will come before the variables. Coefficients can be negative, like -25. If there is no coefficient next to a variable, assume the coefficient is 1. In the expression $x + y + z$, the coefficient in front of each variable is equal to 1.

Variables are letters such as x and y in polynomial terms. In the polynomial $45a^2b$, the variables are a and b.

Exponents, or powers, are a way of showing repeated multiplication. This is also known as raising a number or variable to a power. Polynomials must be raised to whole number exponents. There are no negative or fractional powers in polynomials. In the polynomial $6x^3y^4$, the exponents are 3 and 4. The **degree** of a polynomial with only one variable is the largest exponent of that variable. For example, in the expression $4x^3 + 3x^2 - 5$, the degree is 3 (the largest exponent of x.)

A **constant** is a number on its own that will always remain the same. It is not next to a variable. A constant can be either positive or negative, depending on whether it is being added or subtracted. In the expression $4x^3 + 3x^2 + 5$, the constant is 5. No matter what value you substitute for x, 5 will always be just 5.

Polynomials are built out of all of the components above, and there are different types of polynomials, including monomials, binomials, and trinomials. These are important to know.

- A **monomial** is a polynomial with only one term. An example of a monomial is $4xy^2$. This term is made up of a coefficient (4), two variables (x and y), and an exponent (the power of 2).
- A **binomial** is a polynomial with two terms. An example of a binomial is $3x + 2$. The first term is made up of a coefficient (3) and a variable (x), and the second term consists of a constant (2).
- A **trinomial** is a polynomial with three terms. An example of a trinomial is $4x + 3y^2 - 4$. The first term is made up of a coefficient (4) and a variable (x). The second term is made up of a coefficient (3), a variable (y), and an exponent (the power of 2). The third term is made up of a constant, and this time it is a negative number (-4).

A few types of numbers are not categorized as polynomials and cannot be present in polynomial terms. These include the following:

- Division by a variable, such as $\frac{3}{x+3}$ or $\frac{1}{x}$
- Negative exponents, such as $4xy^{-2}$ (exponents can only be 0, 1, 2, etc.)
- Fractional exponents, such as $3a^{\frac{1}{2}}$
- Variables inside radicals, such as $\sqrt{x}$

In a polynomial term, however, you can divide by a constant or have numbers inside a radical. For example, $\frac{4x}{12}$ is allowed, as is $\sqrt{10}$.

PRACTICE 8

A. **Identify whether each of the following is a monomial, binomial, or trinomial.**

1. $25a$
2. $2xy^2z$
3. $x - 4$
4. $x^3 + y - 1$
5. $7y - 1$
6. $2x^4 + 3x + 4$
7. $\frac{x}{3}$
8. y^2
9. $g^2h^2i^2j^2$
10. $x^2 + x^2$
11. $x^2 + 14x + 3$
12. $x^2y + x^2 + y$
13. $g + h$
14. $\sqrt{49}$
15. $3x^2 - 5$

B. **For each polynomial, identify the terms. Remember that a coefficient can be positive or negative.**

16. $3x^4 - 2x^2 + 3$
17. $12a^2bc$
18. $3g - 4h$
19. $x^2 + y$
20. $-4a - 3b^2 + c$
21. 25
22. $x^2 + 3x - 7$
23. $\frac{3x}{8}$
24. $\sqrt{25}$
25. $\frac{x^2}{9}$
26. $49x^2y^2z^2$
27. $18y^2 - 4y^2 + 8$
28. $3h - 4$
29. $x^2 - x + y^2 - 2$
30. $ab + ab^2 + b^2 - 4$

C. **Choose the <u>one best answer</u> to each question.**

31. What is the sum of the exponents in the expression $4x^3 + 3x^2 + 5$?
    A. 3
    B. 5
    C. 12
    D. 17

32. What is the sum of the coefficients in the expression $a - 3b - c + 2d$?
    A. -3
    B. -1
    C. 2
    D. 3

Answers and explanations begin on page 673.

Simplify Polynomials

Key Ideas

- Like terms can be combined; unlike terms cannot.
- Like terms are either numbers—positive or negative—or variables with the same degree of exponents.
- Polynomials with unlike terms cannot be simplified.

Combine Like Terms

When working with polynomials, you must work to combine like terms. **Like terms** have variables that are the same or are terms that have the same variable raised to the same power, or exponent. For example, $8x^3$ and $4x^3$ are like terms, and $5y$ and $3y$ are also like terms.

Unlike terms have variables that are different or have the same variable raised to different powers. For example, $3a$ and $5b$ are unlike terms. The terms $9x$ and $7x^2$ are also unlike terms because the variables are raised to different powers.

You can only combine like terms when simplifying polynomials. Unlike terms cannot be combined. For example, in the polynomial $7x^2 + 4x^3$, you cannot combine the terms because x is raised to different powers in each term. However, in the polynomial $7x^2 + 4x^2$, you can combine terms that all have the same variable and the same exponent: $7x^2 + 4x^2 = 11x^2$.

Example 1: Combine like terms in the polynomial $4y^3 + 2x^3 - 7y^3$.

1. First, identify and group the like terms. $4y^3 - 7y^3 + 2x^3$

The two y^3 terms are alike; the x^3 term cannot be combined with them.

2. Combine the like terms. $-3y^3 + 2x^3$

Simplify Polynomials

Some polynomials may contain both like and unlike terms. The following example shows how like and unlike terms are handled when you have to simplify a polynomial. Start by combining like terms from the exponent with the largest number, or degree, and proceed to the exponent with the smallest number.

GED® TEST TIP

When working with polynomials, treat the minus sign before a coefficient or constant as an indicator of a negative number. This will help you to combine like terms using the rules of signed numbers that you learned on pages 316–318.

Example 2: Simplify the polynomial $7x^4 - 13x^2 + x^4 + 2x^2 + 5x + 3 + x$.

1. Combine like terms with the largest degree, which is 4. Add: $7x^4 + x^4 = 8x^4$.
2. You now have $8x^4 - 13x^2 + 2x^2 + 5x + 3 + x$. Combine like terms from the exponent with the next largest degree, which is 2. Add: $-13x^2 + 2x^2 = -11x^2$.
3. You now have $8x^4 - 11x^2 + 5x + 3 + x$. Finally, combine like terms with the next largest degree, which is 1. Add: $5x + x = 6x$.
4. You now have $\mathbf{8x^4 - 11x^2 + 6x + 3}$.

This is as much as you can simplify this polynomial by combining like terms. The simplified polynomial has no more like terms and must remain as is.

PRACTICE 9

A. For each pair of terms, indicate if they are like terms or unlike terms.

1. $4x^3, x^3$ ________ terms
2. $3x, x$ ________ terms
3. b, b^2 ________ terms
4. $-2x, 7y$ ________ terms
5. $-2x, 7x$ ________ terms
6. $4a, 4$ ________ terms
7. g^2, g^2hi ________ terms
8. $2x^2y, 8x^2y$ ________ terms
9. $-5m, -5m^2$ ________ terms
10. x^2y, xy^2 ________ terms
11. $ab, 8ab$ ________ terms
12. $3y, 3y^2$ ________ terms
13. $14g, \frac{1}{3}g$ ________ terms
14. $12x^2, x^2$ ________ terms
15. a, ab^2 ________ terms
16. $15x^2, -x^2$ ________ terms
17. $10y, 11yz$ ________ terms
18. $x^2, \frac{x^2}{2}$ ________ terms
19. $g, -g^2$ ________ terms
20. $\frac{x}{8}, x^2$ ________ terms

B. Simplify each expression by combining like terms.

21. $3x^2y + 4x^2y$ = 3x
22. $3b + b$
23. $a - 7a + 3$
24. $14ab + ab^2 + 2ab + 3$
25. $x^2 - 3x^2 + 7$
26. $ab + 2ab + ab$
27. $7g + 7gh + 7g + 7gh$
28. $g^2 + h^2 - 4g^2h^2 + g^2 + h^2 - 4$
29. $9y + y - y^2 - y$
30. $x^2 - 8x^2 + y - 3$
31. $11y + 11y - 7$
32. $9x^2y - 3x^2y + 4y^2 - 21 + y^2 - 2$
33. $8x^2 - 4x + 7 + 4x^3 - x^2 + 7x - 2$
34. $-3x^2 + 6x - 2x^2 - 10x + 5$
35. $9x^2 - 3x^3 + x - 2x^2$

C. Choose the <u>one best answer</u> to each question.

36. What is the simplified form of the expression $3a^2b + 4ab + 3a^2b + 5ab$?
    A. $6a^3b^2 + 8a^3b^2$
    B. $6a^2b + 9ab$
    C. $6a^2b + 8ab$
    D. $15a^6 + b^4$

37. When simplified, how many terms are in the polynomial $j^2 + k^3 - 2j^3 + 5k^3 - 2j^2$?
    A. 2
    B. 3
    C. 4
    D. 5

Answers and explanations begin on page 673.

LESSON

10

Add and Subtract Polynomials

Add Polynomials with One Variable

On the *Mathematical Reasoning Test*, you may be required to add polynomials that have one variable. You can add polynomials when they have like terms, including variables and exponents. When you see polynomials within parentheses linked by an addition sign, add them by combining the like terms across the polynomials.

Example 1: Add $(3x^2 + x + 4) + (2x^2 + 2x - 16)$.

1. To add these polynomials, simply combine like terms. Add: $3x^2 + 2x^2 = 5x^2$.
2. Add: $x + 2x = 3x$.
3. Add: $4 + (-16) = -12$.
4. Combine the results into one polynomial: $\mathbf{5x^2 + 3x - 12}$.

Sometimes, an expression needs to be simplified before you add and combine like terms.

Example 2: Add $(5x^2 + 8x - 4) + (2x^2 - 6x + 14x)$.

1. Simplify within the second parentheses. Add: $-6x + 14x = 8x$.
2. Combine like terms across the polynomials. Add: $5x^2 + 2x^2 = 7x^2$.
3. Use the simplified expression from step 1. Add: $8x + 8x = 16x$.
4. Combine the results into one polynomial: $\mathbf{7x^2 + 16x - 4}$.

Subtract Polynomials with One Variable

The only difference between adding and subtracting polynomials is the minus sign between the parentheses. You must distribute the negative sign across the second polynomial. To distribute the negative sign across the second polynomial, simply reverse the signs of each term. Then drop the parentheses and combine like terms across both polynomials.

Example 3: Subtract $(3x^2 + x + 4) - (7x^2 + 2x - 16)$.

1. Distribute the negative sign to everything in the second set of parentheses by reversing the sign of each term: $-7x^2 - 2x + 16$.
2. Drop the parentheses from the entire expression and combine like terms:
   $3x^2 + x + 4 - 7x^2 - 2x + 16$
   $-4x^2 + x + 4 - 2x + 16$
   $-4x^2 - x + 4 + 16$
   $-4x^2 - x + 20$

Note that subtracting a negative constant or coefficient is the same as adding a positive. In the next example, watch what happens to the negative coefficient inside the second set of parentheses when the sign is distributed.

Key Ideas

- Monomials, binomials, or trinomials can be added or subtracted.
- In some cases, a polynomial needs to be simplified before it can be added or subtracted.
- When subtracting polynomials, distribute the second polynomial, changing the signs for coefficients, and then combine all the like terms.

GED® TEST TIP

When you subtract, you are simply reversing the signs within the second polynomial. Be careful to make all negative signs positive. Study Examples 3 and 4 carefully to avoid common errors.

Example 4: Subtract $(x^2 + 2x + 3) - (4x^2 - x + 6)$.

1. Distribute the negative sign to everything in the second set of brackets by reversing the sign of each term:
   $-4x^2 + x - 6$
2. Drop the brackets and combine like terms:
   $x^2 + 2x + 3 - 4x^2 + x - 6$
   $x^2 - 4x^2 + 2x + x + 3 - 6$
   $\mathbf{-3x^2 + 3x - 3}$

PRACTICE 10.1

A. Add the following polynomials.

1. $(3x + 4) + (2x + 2)$
2. $(17y - y + 3) + (4y + 3y + 3)$
3. $(5x^2 - 3x - 4) + (3x^2 - 2x + 6)$
4. $(-a^2 + 2a) + (16a^2 + 6a)$
5. $(9x^2 - 3x - 2) + (2x^2 + 5x + 5)$
6. $(6a + 6) + (-5a - 5)$
7. $(-8g^2 + 7g + 6) + (8g^2 - 7g - 5)$
8. $(2x^2 + 5x) + (2x^2 + 4x + 7x - 9)$
9. $(13y + 4y + 4) + (7y - 7)$
10. $(-a^2 - a^2 - a - 4) + (-a^2 - a^2 - a - 5)$

B. Subtract the following polynomials.

11. $(3y - 4) - (2y - 2)$
12. $(x + 16) - (4x + 3)$
13. $(2a + 1) - (-a - 1)$
14. $(5x^2 + 2x + 4) - (2x^2 + x + 2)$
15. $(7y + 5y + 5) - (2y - 2)$
16. $(9x^2 + 4x + 5x + 4) - (7x^2 + 6x - 9)$
17. $(-g + g - 1) - (-g - g - 2)$
18. $(17a^2 - 4a - 4) - (16a^2 + 6a - 6)$
19. $(7b^2 + b - 8) - (7b^2 + b + 8)$
20. $(21x^2 + 3x^2 - 2x - 4 - 1) -$
    $(3x^2 + x^2 + x + 2x - 5 - 1)$

C. Choose the <u>one best answer</u> to each question.

21. What is the sum of the polynomials $(2xy + 3xy^2 - 4x^2y) + (5x^2y - 3xy^2 + 2xy)$?
    A. $x^2y + 4xy$
    B. $4xy + 3x + y$
    C. $7x - 2y$
    D. $7x^2y - 2xy$

22. Which of the following equals $(5x^2 - 2x + 1) - (3x^2 - 3x - 2)$?
    A. $x - 5x - 1$
    B. $2x^2 - x + 3$
    C. $2x^2 + x + 3$
    D. $2x^2 - 5x + 3$

Answers and explanations begin on page 673.

LESSON

7

Expressions and Calculator Skills

Key Ideas

- On the *Mathematical Reasoning Test*, you will use a scientific calculator that follows the order of operations.
- Use the *change sign* key to enter a negative number.
- Use the *grouping symbol* keys to change the order of operations.

GED® TEST TIP

To check your work when using a calculator, enter each calculation twice. If the results are the same, you probably pressed the keys that you intended.

When you take the GED® *Mathematical Reasoning Test*, you may use either a hand-held or an online version of a scientific calculator (TI-30XS MultiView™) to use on the second part of the test items. This calculator, like most scientific calculators, uses algebraic logic, which means that it follows the order of operations that you saw on page 324.

You need to practice using a scientific calculator with algebraic logic. You can find out whether your calculator uses algebraic logic by running this simple test.

Press: 4 × 3 x^2 enter. (Your calculator may have an equal sign instead of an *enter* button.)

If the display reads **36**, your calculator uses algebraic logic.

If the display reads **144**, your calculator does not use algebraic logic. You should find another calculator to practice for the *Mathematical Reasoning Test.*

You can use a calculator to evaluate an expression that contains several operations.

Example 1: Find the value of the expression $2x^2 + 3x - 5$ when $x = -4$.

When you come to the variable x, enter −4 by pressing (−) 4. The (−) key is called the **change sign key**.

Press: 2 ((−) 4) x^2 + 3 × (−) 4 − 5 enter.

The right side of the display reads 15.

The value of the expression is **15**.

Expressions sometimes contain grouping symbols to show a different order of operations. You can enter grouping symbols on a scientific calculator. On the TI-30XS MultiView™, the grouping symbols (and) are found above the 8 and 9, respectively. When you enter the left, or open, parenthesis, (, the calculator waits until you enter the right, or closing, parenthesis,), before it calculates what is inside the symbols.

Example 2: Find the value of the expression $2(x+4) + \frac{5x}{3}$ when $x = 6$.

Press: 2 × (6 + 4) + 5 × 6 ÷ 3 enter.

The right side of the display reads **30**.

The value of the expression is **30**.

You can also use your calculator for only part of an expression.

Example 3: Find the value of the expression $\frac{3x+6}{2} + \sqrt{225}$ when $x = 4$.

Substitute 4 for x in the first part of the expression and calculate the results by hand or using your calculator: $\frac{3(4)+6}{2} = \frac{12+6}{2} = \frac{18}{2} = 9$.

Now use your calculator to find the square root of 225.

Press: 2nd x^2 225 enter. The right side of the display reads 15.

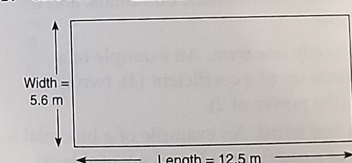
Add the results of the two steps: $9 + 15 = $ **24**.

PRACTICE 7

A. Use a calculator as needed to find the value of the expressions as directed.

1. What is the value of $5x^2 - 3x + 5$ when $x = 2$?
2. Find the value of $\sqrt{7x} + 2x$ to the nearest tenth when $x = 5$.
3. If $x = -3$, what is the value of $7x^2 + 2x - 6$?
4. What is the value of $\frac{1}{2}x + 15$ when $x = 3$?
5. Find the value of $3(2x + 3 + y) - 14$ when $x = -2$ and $y = 9$.
6. If $y = -3$, what is the value of $4y^3 + 2(y^2 - 4)$?
7. What is the value of $2(x^2 + 6) + 3(x - 1)$ when $x = 5$? 5
8. If $x = 4$ and $y = -4$, what is the value of the expression $6x^2 + 3y^2 + 2$?
9. Find the value of $-2(x^3 + 3) + 16x + 2$ when $x = 2$.
10. If $x = 7$, what is the value of the expression $7 + 3(x - 2) - 2x^2$?
11. Find the value of $-(x + y) + 3(2z - y)$ if $x = -5$, $y = -7$, and $z = 4$.
12. What is the value of the expression $x^2 - 7(3 - y) + 4$ when $x = 5$ and $y = 4$?
13. If $x = -2$ and $y = 8$, what is the value of $x^2 + y - 6(y + 3)$?
14. What is the value of $\sqrt{760 - 4x^2}$ to the nearest tenth when $x = 6$?
15. If $x = 20$, what is the value of the expression $4(x + 7) - 3(x - 2)$?
16. Find the value of $\frac{x(25 + 2x - y)}{-z}$ when $x = -3$, $y = 4$, and $z = -1$.
17. What is the value of $5y^2 + 4x^2 - 6(x - y)$ when $x = 5$ and $y = -2$?
18. Find the value of $3x^2 \cdot \frac{2(x - 3y)}{6}$ when $x = 6$ and $y = 1$.
19. What is the value of the expression $(x^2 + 5)(x^2 - x + 2) - 3$ when $x = -4$?
20. Find the value of $(x + y)(x - y)(2x + y)$ when $x = 9$ and $y = 4$.

B. Choose the one best answer to each question.



21. Jake has to buy enough fencing to enclose the rectangular garden shown above. The formula for finding the perimeter of (or distance around) a rectangle is $P = 2l + 2w$, where P = perimeter, l = length, and w = width. Using the values from the drawing, what is the perimeter in meters of the garden?
    A. 18.1
    B. 36.2
    C. 70
    D. 140
22. If $x = -5$ and $y = 2$, which of the following expressions has the greatest value?
    A. $x + y$
    B. $-x + y$
    C. xy
    D. $-2xy$
23. What is the value of the expression $2 \div x^{-4}$ when $x = 2$?
    A. 32
    B. 16
    C. 8
    D. −16

Answers and explanations begin on page 673.