LESSON

7

EQUATIONS, INEQUALITIES, AND FUNCTIONS

Graphing a Line

Key Ideas

- A linear equation has two variables, *x* and *y*.
- When the solutions to a linear equation are graphed on a coordinate grid, the graph forms a line.
- To find a point on the line, substitute a value for *x* and solve for *y*.
- You must solve for at least two points in order to draw the line.

Using the coordinate system, we can graph equations. When an equation has only two variables, x and y, and neither is raised to a power, the graph of the equation will be a line. When the graph of an equation is a straight line, the equation is a **linear equation.**

To graph an equation, you need to solve for two points on the line.

Example 1: Graph the equation $y = 3x - 4$.

1. Choose any value for x and solve for y. Let $x = 1$.

$y = 3(1) - 4$
$y = 3 - 4$
$y = -1$

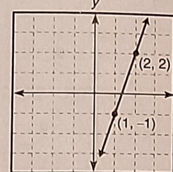
If $x = 1$, then $y = -1$. The ordered pair for the first point is $(1, -1)$.

2. Choose another value for x and solve for y. Let $x = 2$.

$y = 3(2) - 4$
$y = 6 - 4$
$y = 2$

If $x = 2$, then $y = 2$. The ordered pair for the second point is $(2, 2)$.

3. Plot the points on a coordinate grid and draw a line through them.



The line is the graph of all the possible solutions for the equation $y = 3x - 4$. Arrows at both ends of the line indicate that the line continues in both directions. From this, you can see that there is an infinite number of solutions to a linear equation.

GED® TEST TIP

If a linear equation is not written with y on one side of the equation, use inverse operations to isolate y. Example: $2x + y = 15$. Subtract 2x from each side. $y = -2x + 15$.

Some linear equation problems don't require you to draw a graph.

Example 2: Point A lies at $(5, -6)$ on a coordinate grid. The graph of which of the following equations passes through point A?

(1) $y = -5x + 18$
(2) $y = -4x + 14$
(3) $y = -2x - 13$

Use the ordered pair given in the problem. Substitute the x-coordinate, 5, for x in each equation and solve for y. If $y = -6$, the value of the y-coordinate from the ordered pair, you have found the correct equation.

Option (2) is correct.

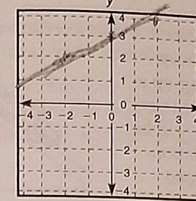
$y = -4x + 14$
$y = -4(5) + 14$
$y = -20 + 14 = -6$

PRACTICE 7

A. Fill in the y column in each table and graph the equation.

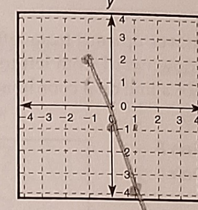
1. $y = \frac{1}{2}x + 3$

If $x =$	then $y =$
–2	2
0	3
2	4



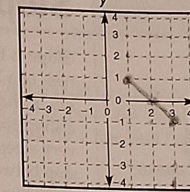
2. $y + 3x = -1$

If $x =$	then $y =$
–1	2
0	–1
1	–4



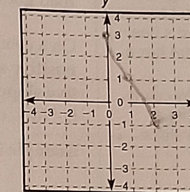
3. $-2 + y = -x$

If $x =$	then $y =$
1	1
2	0
3	–1



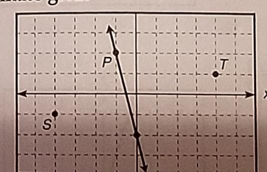
4. $y = 3 - 2x$

If $x =$	then $y =$
0	3
1	1
2	–1



B. Choose the one best answer to each question.

Questions 5 and 6 refer to the following coordinate grid.



5. The graph of the equation $y = \frac{1}{4}x$ will pass through which of the following pairs of points?
   A. point S and $(-1, 2)$
   B. point S and $(0, -2)$
   C. point T and $(0, 0)$
   D. point T and $(0, -2)$

6. Line P is the graph of which of the following equations?
   A. $y = 4x + 1$
   B. $y = -4x - 1$
   C. $y = 4x + 2$
   D. $y = -4x - 2$

7. Point C is located at $(-3, 5)$. A graph of which of the following equations would pass through point C?
   A. $3x + 2y = 5$
   B. $2x + 3y = 9$
   C. $4x - 2y = 8$
   D. $3x - 3y = 6$

Answers and explanations begin on page 678.

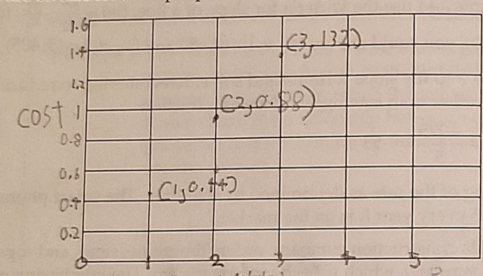
PRACTICE 9.2

A. Use the slope formula to find the rates that correspond to the numbers in these scenarios.

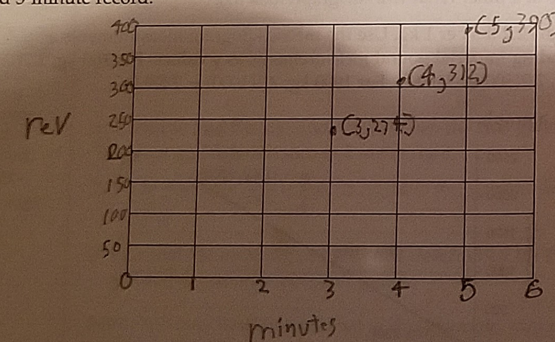
1. A catering company orders 150 lb of fresh clams for a banquet serving 100 people, and 30 lb of mussels for a dinner party of 20 people. Determine the standard number of clams per serving.
2. A painter uses 15 gallons of interior paint to cover 300 square yards of wall and ceiling space. On a different job, he uses 8 gallons to cover 160 square yards. No paint is left over on either job. How many gallons of paint are needed to cover 1 square yard?
3. A scientist is looking for the relationship between the x and y variables in an experiment, and she collects the following data: (3, 2.4), (8, 6.4), (9, 7.2). Determine the rate of quantity y per unit of quantity x.

B. Given the rate, draw a graph showing the relationship between the following quantities.

4. Oranges are on sale for $0.44 per pound. Draw a graph of cost (y) per weight of oranges (x).

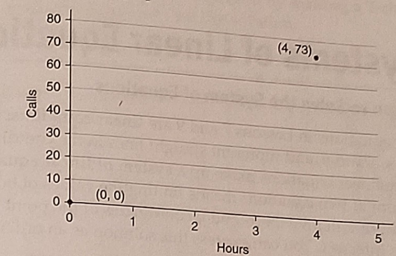


5. A vintage vinyl record player spins at a rate of 78 rpm (revolutions per minute). Draw a graph of number of revolutions (y) per time (x), plotting points for a 3-minute, 4-minute, and 5-minute record.



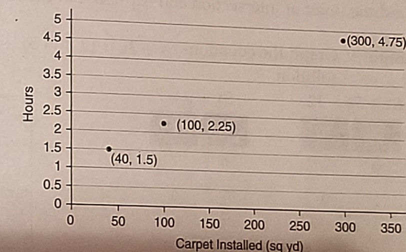
C. Given the following graphs, determine the rates.

Question 6 refers to the following graph.



6. A telemarketing company tracks its employees' efficiency. Use the graph to find the rate of calls made per hour of work.
   A. 0.055 calls per hr
   B. 17.25 calls per hr
   C. 18.25 calls per hr
   D. 69 calls per hr

Question 7 refers to the following graph.



7. On three carpet installation jobs, a contractor records the values above with the goal of analyzing whether her rates are high enough given the time it takes her to install carpet. It takes her 1 hour to collect tools and drive to a worksite. The x-axis shows square yards of carpet, and the y-axis shows hours spent. What is her rate of time per area installed?
   A. 0.0125 hr per sq yd
   B. 0.01625 hr per sq yd
   C. 0.0375 hr per sq yd
   D. 80 hr per sq yd

Answers and explanations begin on page 679.

LESSON

9

Slope and Equations

Key Ideas

- There are two ways to find an equation if you are given the slope of a line and a point.
- One method is the slope-intercept form.
- The other method is the point-slope form.

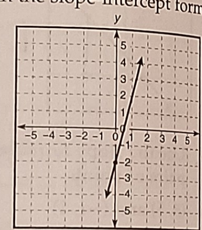
Use the Slope-Intercept Form to Find an Equation

Lesson 8 showed how to find the slope of a line and graph it. The examples in this lesson show how you can use the slope and a point on the line to find the equation of a line in two different forms. (You might also need to find the equation of a line from two points. In this case, you would calculate the slope first, then use the slope and one of the points to find the equation.)

The first of these forms is the **slope-intercept form:** $y = mx + b$. In this form of the equation, the variable m stands for the **slope** of the line. The variable b stands for the **y-intercept**, which is the y-value at the point where the line crosses the y-axis. The variables x and y are the x- and **y-coordinates** of any point on the line and are usually written as an ordered pair.

Follow these steps to find the equation of a line in the slope-intercept form.

1. Substitute the values that you are given for the slope (m) and the x- and y-coordinates (x, y) into the slope-intercept equation. Be careful not to mix up x and y.
2. Use inverse operations to isolate b.
3. Rewrite the equation in slope-intercept form, leaving x and y as variables and substituting values for m and b.



Example 1: Use the slope-intercept form to find the equation of a line that has the slope $m = 4$ and passes through the point $(-1, -6)$.

1. $-6 = (4) \times (-1) + b$
   $-6 = -4 + b$
2. $-2 = b$
3. $y = 4x - 2$

GED® TEST TIP

You can access both the slope-intercept form and the point-slope form from the online GED® Formula Sheet. Just click on the word formula on the computer toolbar, and the sheet will become available.

Use the Point-Slope Form to Find an Equation

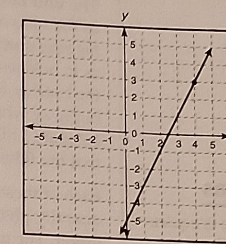
The second important form that is used to describe a line on the *Mathematical Reasoning Test* is the **point-slope form:** $y - y_1 = m(x - x_1)$. In this form, (x_1, y_1) is an ordered pair that corresponds to a point on the line. As in the slope-intercept form, m stands for the slope. If you simplify an equation in point-slope form by solving for y, you will get the slope-intercept form of the equation.

Follow these steps to find the equation of a line in the point-slope form.

1. Call the point that you are given (x_1, y_1).
2. Put the x_1 and y_1 values into the point-slope equation.
3. Put the slope value in the point-slope equation for m.

Example 2: Use the point-slope form to find the equation of the line that passes through the point (4, 3) and has a slope of 2.

1. $(x_1, y_1) = (4, 3)$
2. $y - 3 = m(x - 4)$
3. $y - 3 = 2(x - 4)$



Notice that you can simplify the equation to find the slope-intercept form:

$y - 3 = 2(x - 4)$
$y - 3 = 2x - 8$
$y = 2x - 5$

PRACTICE 9.1

A. **Use the slope-intercept form, $y = mx + b$, to find the equation of the line that passes through the given point and has the given slope.**

1. $(1, -2)$; $m = -4$
2. $(-1, -4)$; $m = 2$
3. $(-4, 2)$; $m = -\frac{1}{3}$

B. **Use the point-slope form, $y - y_1 = m(x - x_1)$, to find the equation of the line that passes through the given point and has the given slope.**

4. $(2, 1)$; $m = 3$
5. $(2, 0)$; $m = -\frac{1}{3}$
6. $(1, -2)$; $m = 1$

C. **Find the equation of the line that passes through the given points. Write your answer in slope-intercept form.**

7. $(-5, 3)$, $(1, 1)$
8. $(-3, 0)$, $(-2, 4)$
9. $(-3, -4)$, $(7, 1)$

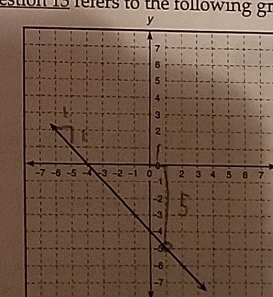
D. **Choose the one best answer for each question.**

10. Which of the following is an equation for the line that passes through $(-1, 0)$ and $(2, -3)$?
    A. $y = 3x + 3$
    B. $y = -3x + 9$
    C. $y = -x - 1$
    D. $y = x - 5$

11. Which of the following is an equation for the line that passes through $(-3, 4)$ and $(1, 4)$?
    A. $y = 0$
    B. $y = 4$
    C. $y = x + 3$
    D. $y = x + 7$

12. Which of the following equations describes the same line as $y - 2 = \frac{1}{2}(x - 6)$?
    A. $y = 3x + 2$
    B. $y = \frac{1}{2}x - 4$
    C. $y = -x + 12$
    D. $y = \frac{1}{2}x - 1$

Question 13 refers to the following graph.



13. Which of the following equations correctly describes the line on the graph?
    A. $y = x - 4$
    B. $y = -x - 4$
    C. $y = x + 4$
    D. $y = -x + 4$

Answers and explanations start on page 679.

LESSON

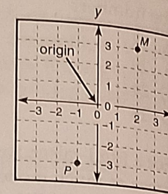
6

The Coordinate Plane

Key Ideas

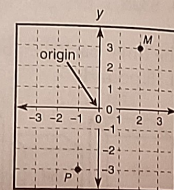
- A coordinate grid is formed by two intersecting axes, or number lines.
- The x-axis is horizontal, and the y-axis is vertical.
- The location of a point is shown by two numbers called an ordered pair: (x, y).

A **coordinate grid** is a way to locate points that lie in a **plane**, or flat surface. The grid is formed by two intersecting lines, an x-axis and a y-axis. The x-axis is actually a horizontal number line, and the y-axis is a vertical number line. The point at which the two axes intersect is called the **origin.**



Each point on the grid can be named using two numbers called an **ordered pair.** The first number is the distance from the origin along the x-axis. The second number is the distance from the origin along the y-axis. The numbers are written in parentheses and are separated by a comma: (x, y).

Example 1: Write the ordered pairs for points M and P.



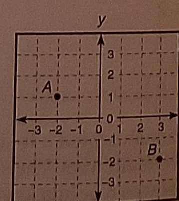
1. Point M lies 2 spaces to the right of the origin along the x-axis and 3 spaces above the origin along the y-axis. The coordinates are **(2, 3).**
2. Point P lies 1 space to the left along the x-axis and 3 spaces down along the y-axis. The coordinates are $(-1, -3)$.

GED® TEST TIP

If you are given a coordinate grid that is not labeled, remember that the origin is at (0,0). Starting at the origin, count either right or left first, then up or down.

To plot points on the grid, use the number lines located at the axes. Remember that right and up are the directions for positive numbers and left and down are the directions for negative numbers.

Example 2: Point A is located at $(-2, 1)$, and point B is located at $(3, -2)$. Plot these points on a coordinate grid.

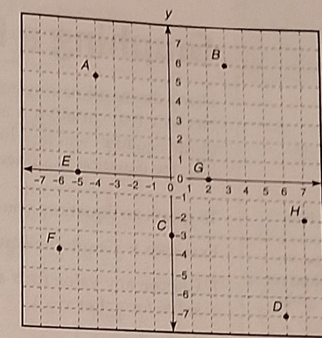


1. To plot point A, start at the origin. Count 2 spaces left along the x-axis. Count 1 space up along the y-axis.
2. To plot point B, start at the origin. Count 3 spaces right along the x-axis. Count 2 spaces down along the y-axis.

PRACTICE 6

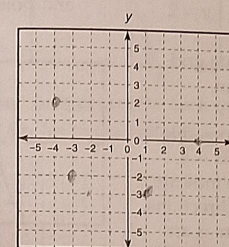
A. **Write the ordered pair for each point.**

1. Point A
2. Point B
3. Point C
4. Point D
5. Point E
6. Point F
7. Point G
8. Point H

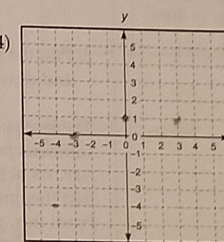


B. **Plot the points on the coordinate grid.**

9. Plot the following points:
   J at $(-3, -2)$
   K at $(4, 0)$
   L at $(1, -3)$
   M at $(-4, 2)$

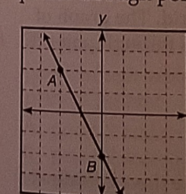


10. Plot the following points:
    N at $(0, -1)$
    O at $(-4, -4)$
    P at $(3, 1)$
    Q at $(-3, 0)$



C. **Choose the one best answer to each question.**

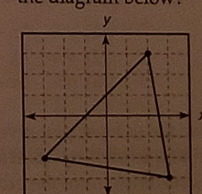
11. On the coordinate grid below, a line passes through points A and B.



Which of the following ordered pairs also lies on the line?

A. (1, 0)
B. (1, −1)
C. (0, −1)
D. (−1, 0)

12. Two of the corners of a triangle are located at (3, −3) and (2, 3). What is the location of the third corner as shown in the diagram below?



A. (−3, −2)
B. (−3, 2)
C. (−2, −3)
D. (3, −2)

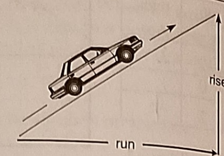
Answers and explanations begin on page 678.

Slope of a Line

Key Ideas

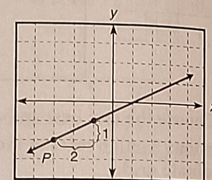
- Slope is the ratio of rise to run.
- Moving from left to right, a line that goes upward has a positive slope, and a line that moves downward has a negative slope.
- You can find slope by counting spaces and writing a ratio or by using the slope formula.

Slope is the measurement of the steepness of a line. Imagine a road going up a hill. If the road must climb upward over a short forward distance, the road will be very steep. Slope measures the relationship between **rise** (how high the road must climb) and **run** (the distance the road goes forward).



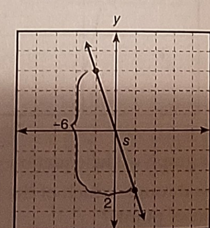
On a coordinate grid, a line that moves upward from left to right has a **positive slope.** A line that moves downward from left to right has a **negative slope.** You can find the slope of a line on a coordinate grid by writing the ratio of rise to run.

Example 1: What is the slope of line P shown on the coordinate grid?



1. Find two points on line P. Count to find the rise and run. The line moves up 1 space for every 2 spaces it goes to the right.
2. Write the ratio: $\frac{\text{rise}}{\text{run}} = \frac{1}{2}$. The slope is $\frac{1}{2}$.

Example 2: What is the slope of line S shown on the coordinate grid?



1. Find any two points on line S. The line moves down 6 spaces (a negative direction) and 2 spaces to the right.
2. Write the ratio: $\frac{\text{rise}}{\text{run}} = \frac{-6}{2} = -3$.

The slope of line S is -3.

GED® TEST TIP

Think of the slope formula this way: write the difference of the y's over the difference of the x's. Just make sure that you always subtract the coordinates in the same order.

You can also find slope using the slope formula on the GED® Formula Sheet. The formula will appear as follows:

slope of a line $(m) = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are two points on a line.

Example 3: A line passes through points at coordinates (1, 4) and (−5, 2). What is the slope of the line?

1. Choose one point to be (x_1, y_1). The other will be (x_2, y_2). It doesn't matter which you choose. For this example, $(x_1, y_1) = (1, 4)$ and $(x_2, y_2) = (-5, 2)$.
2. Substitute the values into the slope formula and solve:

$$\frac{y_2 - y_1}{x_2 - x_1} \quad \frac{2-4}{-5-1} = \frac{-2}{-6} = \frac{1}{3} \quad \text{The slope is } \frac{1}{3}.$$

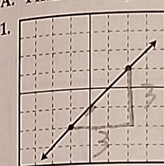
Since the slope is positive, you know that the line rises from left to right. You also know that it goes up 1 space for every 3 spaces it moves to the right.

In working with slope, there are a few special circumstances that you should memorize. A horizontal line, just like a flat stretch of roadway, has a **slope of 0.** The slope of a vertical line is **undefined;** in other words, our definition of slope will not work for a line that has no run at all.

PRACTICE 8

A. **Find the slope of each line.**

1.



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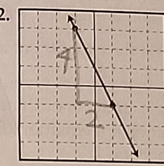
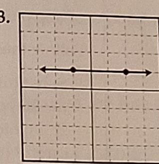
2.



-2

3.



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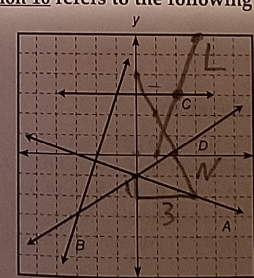
B. **Use the slope formula to find the slope of a line that passes through the following pairs of points.**

4. (3, 5) and (−1, 2)
5. (0, 2) and (4, 0)
6. (4, 2) and (2, 2)
7. (6, 1) and (0, 3)
8. (1, 4) and (−2, −2)
9. (4, −2) and (2, 4)

C. **Choose the one best answer for each question.**

Question 10 refers to the following graph.



10. Which of the following lines shown on the graph has a slope of $-\frac{1}{3}$?
    A. line A
    B. line B
    C. line C
    D. line D

11. Line N passes through the following points: (0, 4), (1, 2), (2, 0), and (3, −2). What is the slope of line N?
    A. −4
    B. −2
    C. 2
    D. 4

12. Line L passes through point (1, 0) and has a slope of 3. Which of the following points also lies on line L?
    A. (0, 3)
    B. (1, 3)
    C. (2, 3)
    D. (2, 5)

Answers and explanations begin on page 678.

LESSON

10

Systems of Linear Equations

Key Ideas

- There are several ways to solve a system of linear equations.
- One method is to graph the equations to see the point of intersection.
- Another method is to find one value (x or y) and substitute it into the other equation to find the solution.

GED® TEST TIP

You may be asked whether a point is a solution to a system of equations, and you can use either of the methods shown here to find the answer.

Graph to Solve the System of Equations

The equations in Lessons 7 and 9 are **linear equations:** they have two variables, x and y, and represent straight lines in the coordinate plane. Two or more linear equations make up a **system of linear equations**. Solving a system of two equations means finding the values of both variables. The **solution** will give the x- and y-coordinates of the point at which the two lines intersect. You can express this solution as an ordered pair: (x, y).

One way to solve a system of equations is by graphing each equation using a **T-chart**—like those that you filled in for the Lesson 7 practice. Graphing a system of linear equations provides a picture of the intersection. Follow these steps to solve a system of equations by graphing.

1. Set up an x and y T-chart for each equation. Find two ordered pairs for each equation: use $x = 0$ and find y and then use $y = 0$ and find x.
2. Graph both lines, using the ordered pairs that you generated in step 1.
3. Find the point of intersection and express it in the form (x, y).

Example 1: Graph the equations $6x + 3y = 12$ and $5x + y = 7$ to find the solution.

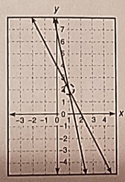
1. $6x + 3y = 12$

X	Y
0	4
2	0

$5x + y = 7$

X	Y
0	7
$\frac{7}{5}$	0

2.



3. The point of intersection—that is, the solution—is **(1, 2)**.

Substitute to Solve the System of Equations

You can also use **substitution** to solve a system of linear equations. Follow these steps to solve a system of linear equations by substitution.

Example 2: Solve the equations $6x + 3y = 12$ and $5x + y = 7$ by substitution.

1. Solve the first equation so that y is expressed in terms of x.

$$6x + 3y = 12$$
$$3y = 12 - 6x$$
$$y = 4 - 2x$$

2. Substitute that value of y into the second equation and solve for x.

$$5x + (4 - 2x) = 7$$
$$3x + 4 = 7$$
$$3x = 3$$
$$x = 1$$

3. Substitute that value of x into the first equation and solve for y.

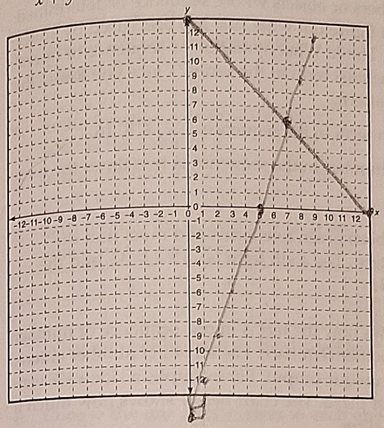
$$6(1) + 3y = 12$$
$$6 + 3y = 12$$
$$3y = 6$$
$$y = 2$$

The solution is **(1, 2)**.

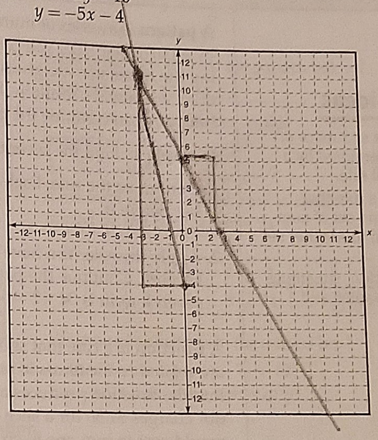
PRACTICE 10

A. Find two pairs of coordinates for each equation by making a T-chart. Use the coordinates to graph the lines and find the solution.

1. $y = 3x - 15$
   $x + y = 13$



2. $4x + 2y = 10$
   $y = -5x - 4$



B. Find the solution for the two equations by substitution. Express as an ordered pair in the form (x, y).

3. $7x - y = 22$
   $4x + 2y = 10$
4. $x + y = 9$
   $2x - 3y = 8$
5. $y = 3x + 15$
   $5x - 2y = -26$
6. $10x - y = -1$
   $y = 12x$

C. Choose the <u>one best answer</u> for each question.

7. Where does the line with the equation $x - 2y = 4$ intersect with the line with the equation $6y + 5x = 4$?
   A. $(2, -1)$
   B. $(3, -2)$
   C. $(-2, 1)$
   D. $(-1, 2)$

8. Where does the line with the equation $y = x$ intersect with the line with the equation $y = -x$?
   A. $(1, -1)$
   B. $(-1, 1)$
   C. $(0, 0)$
   D. $(1, 0)$

9. Where does the line with the equation $y = -2$ intersect with the line with the equation $3y = 2x + 3$?
   A. $(-\frac{2}{9}, -2)$
   B. $(-\frac{1}{3}, -2)$
   C. $(-\frac{9}{2}, -2)$
   D. $(\frac{9}{2}, 2)$

10. Which of the following is the equation of a line that intersects $y = 4x + 2$?
    A. $2y = 8x + 2$
    B. $y = 4x - 2$
    C. $y = -4x + 2$
    D. $\frac{1}{2}y = 2x - 7$

Answers and explanations start on page 679.