

Key Ideas

- The prime factorization of an integer is the prime numbers that can be multiplied together to equal that number.
- The greatest common factor of two integers is the largest integer number that will divide evenly into both.
- The least common multiple of two integers is the smallest number that both integers will divide evenly into.

GED® TEST TIP

When you need to add or subtract fractions with different denominators and the denominators are large, instead of multiplying the denominators to find a common denominator, use prime factorization to find their least common multiple.

Factors and Multiples

When you multiply two numbers to create a product, the numbers you multiply are factors. The pairs of factors that can be multiplied to produce a whole number can be easily listed in a T-chart.

Example 1: What are all the factors of 10?

1. Make a T-chart:

10	
1	10
2	5
-1	-10
-2	-5

$1 \times 10 = 10$
 $2 \times 5 = 10$
 $-1 \times -10 = 10$
 $-2 \times -5 = 10$

2. Count the factors in the T-chart. The number 10 has eight distinct factors. In order from least to greatest, they are $-10, -5, -2, -1, 1, 2, 5, 10$.

Prime numbers are positive whole numbers with only two positive factors, 1 and the number itself. Examples of prime numbers are 2, 3, 5, 7, 11, and 13. (Note that 1 is *not* a prime number.)

For example, 10 is not a prime number because, as just discovered, it has four positive factors: 1, 2, 5, and 10. However, the numbers 2 and 5 are prime numbers. The only positive factors of 2 are 1 and 2, and the only positive factors of 5 are 1 and 5. Thus, the **prime factorization** of 10 is 2×5 . A number's prime factorization may include more than two factors, as in the next example.

Example 2: What is the prime factorization of 12?

1. Break 12 into its prime factors.

$$12 = 4 \times 3 = 2 \times 2 \times 3$$

Prime factors can be used to find two values that may be useful in simplifying expressions and solving equations.

The **greatest common factor** (GCF) of two integers is the largest number that divides evenly into both integers. To find the GCF, break down both integers into their prime factorizations, find the prime factors they have in common, and multiply those prime factors. This is handy when using the distributive property to rewrite two or more terms as one.

Example 3: What is the greatest common factor of 40 and 140?

1. Identify the prime factors of each number.

$$40 = 4 \times 10 = (2 \times 2) \times (2 \times 5) = 2 \times 2 \times 2 \times 5$$

$$140 = 10 \times 14 = (2 \times 5) \times (2 \times 7) = 2 \times 2 \times 5 \times 7$$

2. See what prime factors the two numbers have in common.

Both integers share two 2s and one 5.

3. Multiply these common factors.

The greatest common factor is $2 \times 2 \times 5 = 20$. This is the largest number that divides evenly into both 40 and 140. Note that you could rewrite the expression $40 + 140$ as $20(2 + 7)$.

The **least common multiple** (LCM) of two integers is the smallest number that both integers divide into. To find the LCM, break down both integers into their prime factorizations, select each prime factor the most times it occurs, and multiply all of the prime factors you have selected.

Example 4: What is the least common multiple of 20 and 30?

1. Identify the prime factors of each number

$$20 = 2 \times 2 \times 5 \quad 30 = 2 \times 3 \times 5$$

2. Select each factor the most times it occurs. The factor 2 appears twice in 20, so select 2×2 . The factor 3 appears once in 30, so select 3. And the factor 5 appears once in both 20 and 30, so select it once.

3. Multiply these factors.

The least common multiple is $2 \times 2 \times 3 \times 5 = 60$. This is the smallest number that both 20 and 30 will divide into. Note that if you needed to solve $\frac{7}{20} + \frac{7}{30}$, you could use a common denominator of 60: $\frac{21}{60} + \frac{14}{60} = \frac{35}{60} = \frac{7}{12}$.

PRACTICE 6.3

A. Find the prime factorization of the following numbers.

1. 6 2. 21 3. 36 4. 54 5. 70

B. For each pair of numbers below, find the greatest common factor and the least common multiple.

6. 6, 9 7. 45, 75 8. 50, 125

C. Solve.

9. $30 + 75 =$

- A. $5(3 + 7)$
- B. $10(6 + 5)$
- C. $15(2 + 5)$
- D. $150(5 + 2)$

10. $\frac{11}{75} - \frac{2}{45} =$

- A. $\frac{23}{225}$
- B. $\frac{43}{225}$
- C. $\frac{3}{10}$
- D. $\frac{3}{5}$

Answers and explanations begin on page 673.

LESSON 12

ALGEBRA BASICS, EXPRESSIONS, AND POLYNOMIALS

Divide Polynomials

Divide Polynomials by a Number

The Mathematical Reasoning Test will sometimes ask you to simplify a polynomial that is written as a fraction. To simplify, divide the numerator and denominator by a common term. A common term can be the following:

- a number
- a variable with a number as a coefficient
- a more complicated expression

You may need to factor the common term out of the numerator or denominator before you can do the division.

Example 1: Simplify $\frac{3x+6}{3}$.

$$\frac{3x+6}{3} = \frac{3(x+2)}{3} = x+2$$

Divide Polynomials by an Expression

Dividing a polynomial by an expression is really no different from dividing by a number. You need a common term in the numerator and denominator, and you may have to factor out that common term (including any variables) before you can do the division.

Example 2: Simplify $\frac{25x^3 - 45x^2}{5x}$.

$$\frac{25x^3 - 45x^2}{5x} = \frac{5x(5x^2 - 9x)}{5x} = 5x^2 - 9x$$

When the numerator has two added or subtracted expressions that both include the common term, it may be easier to split the polynomial into two separate fractions and simplify them separately. Doing it this way will help you keep straight exactly which terms you can cancel.

Example 3: Simplify $\frac{x(x+7) - 3(x+7)}{x+7}$.

$$\begin{aligned} x(x+7) - 3(x+7) &= \frac{x(x+7)}{x+7} - \frac{3(x+7)}{x+7} \\ &= \frac{x(x+7)}{x+7} - \frac{3(x+7)}{x+7} = x-3 \end{aligned}$$

Undefined Expressions

When the denominator of an expression equals zero, the expression is undefined.

Example 4: What is the value of $\frac{x^2 + 2xy + y^2}{y-x}$ when $x = y$?

Because $x = y$, $y - x = y - y$. Because $y - y = 0$, this expression is undefined.

GED® TEST TIP

Remember that you will be provided with a wipe-off board during the Mathematical Reasoning Test. You will be able to use that board to carefully work through the steps needed to solve these types of problems.

PRACTICE 12

A. To simplify, divide each polynomial by a number. (You may need to factor the numerator and/or denominator to find the common term.)

$$1. \frac{2y+30}{2} \quad y+15$$

$$2. \frac{7x+21}{7} \quad x+3$$

$$3. \frac{4x+20}{4} \quad x+5$$

$$4. \frac{3a+3b}{3} \quad a+b$$

$$5. \frac{11x^2+22x}{11x^2} \quad x+2x$$

$$6. \frac{26x^2+39x+13}{13} \quad 2x^2+3x+1$$

$$7. \frac{5x+10y}{5} \quad x+2y$$

$$8. \frac{21a+14b}{7} \quad 3a+2b$$

$$9. \frac{48x+32y}{32} \quad 12x+y$$

$$10. \frac{9a-15b}{12} \quad \frac{3a-5b}{4}$$

$$11. \frac{6b+24c}{18} \quad b+4c$$

$$12. \frac{25a+5b+15c}{10} \quad \frac{5a+b+3c}{2}$$

B. Divide each polynomial. Decide whether it is easier for you to divide the original numerator by the common term (as in Example 2) or whether to split the numerator first (as in Example 3).

$$13. \frac{18x^2+6x}{3x} \quad 6x+2$$

$$14. \frac{10x^2+6x}{2x} \quad 5x+3$$

$$15. \frac{40y^2+10y}{5y} \quad 8y+2$$

$$16. \frac{42xy+49x}{7x} \quad 6y+7$$

$$17. \frac{38xy+38x}{19x} \quad 2y+2$$

$$18. \frac{3x+18}{x+6} \quad 3$$

$$19. \frac{x(x+4)-6(x+4)}{x+4} \quad x+4$$

$$20. \frac{y(y+3)+7(y+3)}{y+3} \quad y+3$$

$$21. \frac{z(z+2)-5(z+2)}{z+2} \quad z+2$$

$$22. \frac{22x^2+66x}{11x} \quad 2x+6$$

$$23. \frac{x(y-2)-6(y-2)}{y-2} \quad x-6$$

$$24. \frac{a(b+3)+3(b+3)}{b+3} \quad a+3$$

C. Choose the one best answer to each question.

25. Which of the following is equal to $\frac{21a+14b}{7}$?

- A. $5a+5b$
- B. $3a+14b$
- C. $3a+2b$
- D. $21a+2b$

26. James has 27 apples, Rachel has 51 oranges, and Glen has 60 peaches.

Which of the following expressions represents the average number of pieces of fruit that James, Rachel, and Glen have in their baskets?

- A. $9x + 51y + 60z$
- B. $46x + 46y + 46z$
- C. $9x + 17y + 60z$
- D. $9x + 17y + 20z$

27. Which of the following expressions is equal to $\frac{z^2 - 12z + 2(z+4) - 8}{2z}$?

- A. $z - 5$
- B. $z(z+2) - 5$
- C. $z(z+2) - 5(z+2)$
- D. $z - 3$

28. For what values of a is the expression

$$\frac{a(4a-16a)}{a \times |a| + a^2}$$

- A. $a = 4$
- B. $a \leq 0$
- C. $a = 16$
- D. $a \geq 0$

Answers and explanations begin on page 674.

ALGEBRA BASICS, EXPRESSIONS, AND POLYNOMIALS

Algebraic Expressions

Writing Algebraic Expressions

An algebraic expression uses numbers, operations, and variables to show number relationships. Variables are letters (such as x and y) that represent unknown numbers. Each time a letter is used within the same expression, it represents the same number.

To solve algebra problems, you will need to be able to translate number relationships described in words into algebraic expressions. Study the following examples.

Algebraic Expressions in Words	In Symbols
the product of 5 and a number	$5x$
a number decreased by 12	$x - 12$
the sum of 3 and the square of a number	$3 + x^2$
6 less than the quotient of a number and 2	$\frac{x}{2} - 6$
one-half a number increased by 15	$\frac{1}{2}x + 15$
4 times the difference of -3 and a number	$4(-3 - x)$
a number less another number	$x - y$
10 less the square root of a number plus 3	$10 - \sqrt{x+3}$

To do well on algebra questions on the GED® Mathematical Reasoning Test, you must be able to translate a common life situation into mathematical symbols. You will use this skill to write equations and functions, to apply formulas, and to solve word problems.

Example 1: Kyle processes sales for an online bookstore. The shipping and handling on an order is equal to 4% of the total cost of the order plus \$0.95 per book. If c represents total cost and n represents the number of books in an order, which of the following expressions could be used to find the shipping and handling for an order?

- (1) $\frac{4}{100}nc + 0.95$
- (2) $(0.04 + 0.95)n + c$
- (3) $0.04c + 0.95n$

This kind of problem is called a **setup problem**. You need to recognize the correct way to find the shipping and handling based on the total cost and number of items. The relationship is described in the second sentence.

$$\text{shipping and handling} = 4\% \text{ of total cost } (c) \text{ plus } \$0.95 \text{ per book } (n)$$

$$= 0.04c + 0.95n$$

The correct answer is option (3), $0.04c + 0.95n$.

Key Ideas

- Algebraic expressions show mathematical relationships using numbers, symbols, and variables.
- Variables are letters that take the place of unknown numbers.

GED® TEST TIP

To check whether the expression you have chosen is correct, substitute easy numbers into the expression and complete the operations. Then see if the result is reasonable for the situation.

PRACTICE 6.1

- A. Write an algebraic expression for each description. Use the variables x and y .
1. a number decreased by 7
 2. the product of 3 and the square of a number increased by that number
 3. the product of 8 and a number less 10
 4. the difference of -3 multiplied by a number and the product of 2 and another number
 5. 5 less than the quotient of 10 and a number
 6. the sum of -8 and the product of 7 and a number
 7. the sum of 16 times a number and the number less another number times 3
 8. a number squared plus the number raised to the fourth power
 9. the sum of the square of a number and 4 divided by 7

- B. Choose the one best answer to each question.

19. A minor-league baseball team is giving a local charity the sum of \$1500 and \$0.50 for each ticket over 2000 sold for one game. Let x represent the number of tickets sold. If the team sells more than 2000 tickets, which of the following expressions could be used to find the amount of the donation?

- A. $\$1500 + \$0.50x$
- B. $\$1500 + \$0.50(2000 - x)$
- C. $\$1500 + \$0.50(x - 2000)$
- D. $\$1500(2000 - x)(\$0.50)$

20. The sum of 3 times a number and 4 times another number is divided by the sum of 2 and a third number. Which of the following expressions represents this series of operations?

- A. $(3x + 4y) \div (2 + z)$
- B. $3x + 4y \div (2 + z)$
- C. $3x + 4y \div 2 + z$
- D. $(3x + 4y) \div 2z$

Question 21 refers to the following information.

Appliance City employees earn an hourly wage plus commission. Wage options are shown below.

Option	Hourly Wage	Commission on Sales
A	\$7.50	1%
B	\$6.00	3%

21. Chandra is paid under Option B. If h represents the number of hours worked and s represents Chandra's total sales, which of the following expressions could be used to find her weekly pay?

- A. $6 + h + 0.03s$
- B. $6h + 0.03s$
- C. $6s + 0.03h$
- D. $0.03(h)s$

Answers and explanations begin on page 672.

ALGEBRA BASICS, EXPRESSIONS, AND POLYNOMIALS

Multiply Polynomials

Multiply a Monomial by a Monomial

To multiply two monomials, multiply each component separately. Start by multiplying the coefficients (or numbers). If there is no coefficient given, assume that the coefficient is 1 (for example, $xy = 1xy$). Then, multiply each variable separately by adding the exponents of like variables. Remember that a variable with no exponent is a variable raised to the first power (for example, $x = x^1$). So $x(x^3) = x^1(x^3) = x^4$.

Example 1: Multiply $(2ab)(4b)$.

$$2 \times 4 \times a \times b^1 \times b^1 = 8ab^2$$

Multiply a Monomial by a Binomial

To multiply a monomial by a binomial, multiply the monomial by the first term in the binomial, then multiply the monomial by the second term in the binomial. Finally, add the resulting terms.

Example 2: Multiply $-2z^2(z + 3yz)$.

- Distribute the negative sign:

$$-2z^2(z + 3yz) = 2z^2(-z - 3yz).$$

- Multiply the monomial by each term in the binomial:

$$2z^2(-z - 3yz) = [2 \times (-1) \times z^2 \times z^1] + [2 \times (-3) \times z^2 \times y^1 z^1].$$

- Add: $-2z^3 + (-6yz^3) = -2z^3 - 6yz^3$.

Multiply Two Binomials

To multiply two binomials, multiply the first terms, then the outer terms, then the inner terms, and, finally, the last terms. Add the results and combine like terms. You can remember this process by the acronym FOIL, which stands for First, Outer, Inner, Last.

Example 3: Multiply $(2g^2 + 9)(3g^2 - 4)$.

- Multiply using FOIL:

$$(2g^2 + 9)(3g^2 - 4) = \overbrace{(2g^2)(3g^2)} + \overbrace{(2g^2)(-4)} + \overbrace{(9)(3g^2)} + \overbrace{(9)(-4)}.$$

- Add: $6g^4 - 8g^2 + 27g^2 - 36$.

- Combine like terms:

$$6g^4 - 8g^2 + 27g^2 - 36 = 6g^4 + 19g^2 - 36.$$

GED® TEST TIP

Always finish the FOIL method by combining like terms after you have multiplied.

PRACTICE 11

A. Multiply the monomials.

- $(6x)(5x)$
- $(2xy)(3y)$
- $(7abc)(4bc)$
- $(12y)(z)$
- $(a)(9bc)$
- $(5xyz)(2xy^2z^4)$
- $(4ab)(bc^2)$
- $(17f^2gh^3)(2fh^4)$

B. Multiply the binomials by the monomials.

- $3z^2(6xy + 4z)$
- $6x(7x - 6z)$
- $-5ab(3b + 11c)$
- $-3f^3(6h - 8fg^2)$
- $10z(7x^7 - 5z)$
- $-z(-z - 6xy)$
- $8b(9ab + 8a)$
- $-9(-2x^4 + 3xy^2)$

C. Multiply the binomials by using FOIL. Remember to complete by combining like terms wherever possible.

- $(x + 5)(x - 6)$
- $(x + y)(x + y)$
- $(z + 9)(z - 9)$
- $(yz^2 + x)(yz^2 - 3x)$
- $(3x + 3)(3x + 5)$
- $(x + y)(x - y)$
- $(y^2 - 6)(y^4 + 10)$
- $(ab + 3)(ab - 4)$

D. Choose the one best answer to each question.

- Which of the following expressions is equal to $(4a^3b^2)(3a^2c)$?
 - $12a^5b^2c$
 - $7a^5b^2c$
 - $12a^6bc$
 - $7a^6b^2c$
- Which of the following expressions is equal to $(4ab + 2)(3ab - 7)$?
 - $7a^2b^2 + 22ab + 5$
 - $12a^2b^2 - 22ab - 14$
 - $12a^2b^2 + 22ab + 14$
 - $7a^2b^2 - 22ab - 5$

- Martha has $2pc^2$ board games, and John has $6p^2c$ board games. Which of the following expressions represents the product of the number of board games that Martha and John have?
 - $8p^3c^3$
 - $8p^2c^2$
 - $12p^3c^2$
 - $12p^3c^3$

Answers and explanations begin on page 674.

Simplifying and Evaluating Expressions

Simplifying an expression means to perform all the operations you can within an expression. When working with variables, you must remember an important rule: you can add or subtract like terms only.

A **term** is a number, a variable, or the product or quotient of numbers and variables. A term cannot include a sum or a difference.

Examples: $5x \quad 3y^2 \quad 13 \quad x^3 \quad \frac{x}{2}$

Like terms have the same variable raised to the same power. For example, $3x^2$ and $5x^2$ are like terms. $8y$ and $4y$ are also like terms. However, $6x$ and $2x^2$ are not like terms because the variables are not raised to the same power.

To simplify an expression, combine like terms.

Example 2: Simplify $2x - 5 + 4x^2 - 8 + 6x$.

$$\begin{aligned} 2x - 5 + 4x^2 - 8 + 6x \\ = (2x + 6x) + (-5 - 8) + 4x^2 \\ = 8x + (-13) + 4x^2 \\ = 4x^2 + 8x - 13 \end{aligned}$$

Combine like terms. It is customary to write the term with the greatest exponent first and to continue in descending order.

The **distributive property** allows you to remove grouping symbols to simplify expressions. We can state the distributive property using symbols.

$$a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac$$

In other words, each term inside the parentheses is multiplied by the term outside the parentheses, and the results are added or subtracted depending on the operation inside the parentheses. Example 3 applies the distributive property.

Example 3: Simplify $4x - 3(x + 9) + 15$.

1. Change subtracting to adding a negative number.
2. Use the distributive property. Multiply -3 by each term in the parentheses.
3. Combine like terms.
(NOTE: $1x$ means x .)

$$\begin{aligned} 4x - 3(x + 9) + 15 \\ = 4x - 3(x + 9) + 15 \\ = 4x + (-3x) + (-3)(9) + 15 \\ = 4x + (-3x) + (-27) + 15 \\ = (4x - 3x) + (-27 + 15) \\ = x - 12 \end{aligned}$$

Evaluating an expression means finding its value. To evaluate an expression, substitute a given number for each variable. Follow the order of operations.

Example 3: Find the value of the expression $\frac{3x + 2y}{4}$ when $x = 6$ and $y = 5$.

1. Replace the variables with the values given in the problem.

$$\frac{3x + 2y}{4} = \frac{3(6) + 2(5)}{4} = \frac{18 + 10}{4} = \frac{28}{4} = 7$$

2. Perform the operations above the fraction bar. Then divide.

NOTE: To remove parentheses from an operation that follows a minus sign, imagine that the parentheses are preceded by 1. Then use the distributive property.

$$\begin{aligned} -(2x + 3) \\ = -1(2x + 3) \\ = -1(2x) + (-1)(3) \\ = -2x + (-3) \text{ or } -2x - 3 \end{aligned}$$

PRACTICE 6.2

A. Simplify.

1. $5 + x^2 - 3 + 3x$
2. $2y + 5 + 17y + 8$
3. $3x - 6(x - 9)$
4. $6x^3 + 4 + 2x^2(15) + x^2$
5. $4(y + 8) + 3(y - 6)$

6. $5 - (x - 3) + 4x$

7. $16x + 6(x - 2)$

8. $5y^2 + 4 - 3y^2 + 5 + y$

9. $-3(x + 3) - 2(x + 4)$

10. $5x - (x + 4) - 3$

B. Evaluate each expression as directed.

11. Find the value of $6(x + 2) + 7$ when $x = 2$.
12. Find the value of $3x^2 + 3(x + 4)$ when $x = 3$.
13. Find the value of $\frac{(x+y)^2}{2} - 10$ when $x = 2$ and $y = 4$.
14. Find the value of $y^2 + 16 - (y - 5)^2$ when $y = 3$.
15. Find the value of $8x + 9y - (2x + y)$ when $x = 4$ and $y = 6$.
16. Find the value of $x^2 + 3y - 4 + 2(x - z)$ when $x = 7$, $y = 5$, and $z = -3$.
17. Find the value of $(14 - x)^2 + 20\sqrt{x}$ when $x = 9$.
18. Find the value of $\frac{3(2x-y)}{3} + 6(y-5)$ when $x = -2$ and $y = 3$.
19. Find the value of $x^2 - (x^3 + 3)$ when $x = -2$.
20. Find the value of $30x + 2 + 2y^2 - 3(x - 2)^2$ when $x = 1$ and $y = 4$.

C. Choose the one best answer to each question.

21. Which of the following expressions is equal to $3x^2 + 3(x - 3) + x + 10$?

- A. $x^2 + 9x + 1$
- B. $3x^2 + 4x + 19$
- C. $3x^2 - 2x + 19$
- D. $3x^2 + 4x + 1$

22. Given the expression $4x^2 - 3(y + 6)$, which of the following values for x and y will result in a value of -11 ?

- A. $x = 2, y = 3$
- B. $x = -2, y = 4$
- C. $x = -1, y = 2$
- D. $x = 1, y = 0$

Question 23 refers to the following information.

Temperature Conversion Formulas

To convert Fahrenheit (F) to Celsius (C)

$$C = \frac{5}{9}(F - 32)$$

To convert Celsius (C) to Fahrenheit (F)

$$F = \frac{9}{5}C + 32$$

23. If the temperature is 68° Fahrenheit, what is the temperature in Celsius?

- A. 20°
- B. 36°
- C. 154.4°
- D. 180°

Answers and explanations begin on page 672.