LESSON

11

Patterns and Functions

Key Ideas

- A pattern is a sequence of numbers determined by a mathematical rule.
- A function is a rule that shows how one set of numbers is related to another set of numbers.
- To use a function, substitute values for variables and solve.

GED® TEST TIP

To figure out what rule has been used to form a pattern, begin by finding the difference between each term and the term that follows it in the sequence.

A **pattern** is a series of numbers or objects whose sequence is determined by a particular rule. You can figure out what rule has been used by studying the terms you are given. Think: What operation or sequence of operations will always result in the next term in the series? Once you know the rule, you can continue the pattern.

Example 1: Find the seventh term in the sequence: 1, 2, 4, 8, 16, . . .

1. Determine the rule. Each number in the sequence is two times the number before it.

2. Apply the rule. You have been given five terms and must find the seventh. Continue the pattern. The sixth term is $16 \times 2 = 32$, and the seventh term is $32 \times 2 = $ **64**.

A **function** is a rule that shows how the terms in one sequence of numbers are related to the terms in another sequence. Each distinct number entered into the function produces a unique output. For example, a sidewalk vendor charges $1.50 for a slice of pizza. The chart below shows how much it would cost to buy one to six slices.

Number of Pizza Slices	1	2	3	4	5	6
Cost	$1.50	$3.00	$4.50	$6.00	$7.50	$9.00

Each number in the first row corresponds to a price in the second row. We could say that the amount a customer will pay is a function of (or depends upon) the number of slices the customer orders. This function could be written:

Cost = number of slices × $1.50, or $C = n(\$1.50)$.

If you know the function and a number in the first set of numbers, you can solve for its corresponding number in the second set.

Example 2: Using the function $y = 3x + 5$, what is the value of y when $x = -3$?

1. Substitute the given value of x. $\quad y = 3(-3) + 5$
2. Solve for y. $\quad y = -9 + 5$
$\quad y = -4$

Example 3: Using the function $n = 100 - 4(3 + m)$, what is the value of n when $m = 6$?

1. Substitute the given value of m. $\quad n = 100 - 4(3 + 6)$
2. Solve for n. $\quad n = 100 - 4(9)$
$\quad n = 100 - 36$
$\quad n = 64$

PRACTICE 11

A. Solve.

1. Which number should come next in the following pattern?
$-12, -9, -6, -3,$ ___0___

2. What is the next number in the sequence?
21, 26, 31, 36, ___41___

3. In the function $y = 4x + 10$, if $x = -2$, what is the value of y?

4. In the function $y = 2x(4 + x) - 2$, if $x = 3$, what is the value of y?

5. Each term in the second row is determined by the function $y = 2x - 1$.

x	1	2	3	4	5	. . .	12
y	1	3	5	7	9	. . .	

What number belongs in the shaded box?

6. In the function $y = \frac{x+3}{6} - 8$, if $x = 21$, what is the value of y?

7. What is the next term in the pattern below?
1000, 500, 250, 125, 62.5, ___31.25___

8. What is the next number in the sequence?
$3, -5, 7, -9, 11,$ ___-13___

9. Each term in the second row is determined by the function $y = 3x + 5$.

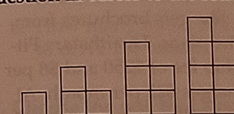
x	−2	−1	0	1	2	. . .	9
y	−1	2	5	8	11	. . .	

What number belongs in the shaded box?

10. In the function $y = (x - 7) + 12$, if $x = -10$, what is the value of y?

B. Choose the one best answer to each question.

Question 11 refers to the following drawing.



1st 2nd 3rd 4th

11. How many blocks would be needed to build the 25th construction in the sequence?
A. 47
B. 49
C. 51
D. 55

12. What is the sixth term in the sequence below?
$-14, -8, -2, 4, \ldots$
A. 10
B. 14
C. 16
D. 22

13. The price per scarf is a function of the number of scarves purchased. The table shows the price per scarf for purchases of up to four scarves.

number (n) of scarves	1	2	3	4
cost (c) per scarf	$5.00	$4.75	$4.50	$4.25

Which of the following functions was used to determine the prices shown in the table?
A. $c = n(\$5.00 - \$0.25)$
B. $c = \$5.00 - \$0.25(n - 1)$
C. $c = \$5.00 - \$0.25n$
D. $c = \$5.00n - \$0.25n$

14. Which of the following sequences of values of y could be created using the function $y = 4x - 3$?
A. 1, 4, 7, 10, 13, . . .
B. 1, 5, 9, 13, 17, . . .
C. 1, 4, 8, 13, 19, . . .
D. $1, -1, -3, -5, -7, \ldots$

Answers and explanations begin on page 680.

LESSON

12 EQUATIONS, INEQUALITIES, AND FUNCTIONS

Function Applications

Key Ideas

- Functions are used to make many common work calculations.
- Functions can be used to make comparisons.
- To use a function, you must know the meaning of the variables. This information should be given in the text of the problem.

Functions are used in many business applications. For instance, they can be used to calculate profit, cost, employee wages, and taxes. On the *Mathematical Reasoning Test*, you will read about common work and life situations. The problems may contain or describe a function that you can use to solve the problem.

Example 1: Celino Advertising is finishing a series of print ads for a client. Finishing the project will cost \$2000 per day for the first seven days and \$3500 per day after seven days. The finishing costs can be found using the function $C = \$2000d + \$1500(d - 7)$, where C = the cost of finishing the project and d = the number of days. If the project takes 12 days to complete, what will the project cost?

Use the function to solve the problem.
$$\begin{aligned} C &= \$2000d + \$1500(d-7) \\ &= \$2000(12) + \$1500(12-7) \\ &= \$24000 + \$1500(5) \\ &= \$24000 + \$7500 \\ &= \mathbf{\$31,500} \end{aligned}$$

You may be asked to use functions to make comparisons.

Example 2: Nita decides to join a health club. She gets brochures from two health clubs and compares the plans. Healthstars Fitness charges a one-time membership fee of \$250 and \$8 per month. Freedom Health Center charges \$25 per month. At both health clubs, the price (P) Nita will pay is a function of the number of months (m) she attends the club. The functions are as follows:

Healthstars Fitness $P = \$250 + \$8m$

Freedom Health Center $P = \$25m$

Nita plans to move in 18 months. If she attends a health club until she moves, which one offers the better price?

1. Find the price at Healthstars Fitness:
$$\begin{aligned} P &= \$250 + \$8m \\ &= \$250 + \$8(18) \\ &= \$250 + \$144 \\ &= \$394 \end{aligned}$$

2. Find the price at Freedom Health Center:
$$\begin{aligned} P &= \$25m \\ &= \$25(18) \\ &= \$450 \end{aligned}$$

3. Compare the results. Even though Nita will have to pay a large amount up front, **Healthstars Fitness** offers the better price.

GED® TEST TIP

There is often more than one way to work a problem, even when a function is given. Solve using the function. If you see another way to solve the problem, use it to check your answer.

PRACTICE 12

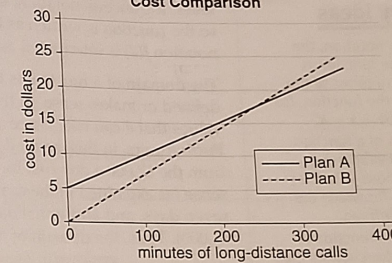
A. Solve. You MAY use a calculator.

1. The Chimney Sweep charges \$25 for a chimney inspection. If the customer purchases additional services, \$15 of the inspection fee is deducted. Let s = the cost of any additional services. The total cost (C) of an inspection and services can be determined by the function $C = \$25 + (s - \$15)$ where s is not 0.
   a. Jan has her chimney inspected and purchases a smoke guard for \$89. How much will she be charged?
   b. After an inspection, Ahmed decides to have a new damper installed for \$255. How much will he pay?

2. Ricardo does a great deal of driving for his work. He generally estimates his driving time in hours (t) using the function $t = \frac{m}{60}$, where m = the number of miles.
   a. How many hours will it take Ricardo to drive 330 miles?
   b. How many hours will it take Ricardo to drive 255 miles?

3. A customer's phone charges are a function of the number of minutes of long-distance calls made. The graph shows a comparison of two plans available.

Long-Distance Plan Cost Comparison



   a. Michelle looks at her previous phone bills and finds that she makes about 350 minutes of long-distance calls per month. Which plan is better for her?
   b. Craig usually makes about 150 minutes of long-distance calls per month. Which plan is better for him?

B. Choose the <u>one best answer</u> to each question. You MAY use a calculator.

<u>Questions 4 and 5</u> refer to the following information.

Alicia is considering three job opportunities. At all three jobs, weekly pay (P) is a function of the number of hours (h) worked during the week. The functions are shown below:

Job 1	$P = \$9.75h$
Job 2	$P = \$70 + \$8.40h$
Job 3	$P = \$380 \times \frac{h}{38}$

4. If Alicia works 30 hours in a week, how much more will she earn at Job 2 than at Job 1?
   A. \$5.33
   B. \$29.50
   C. \$40.50
   D. \$59.00

5. If Alicia works 40 hours per week, which of the following is a true statement?
   A. Alicia will earn the least at Job 3.
   B. Job 1 will pay more than Job 3.
   C. Job 3 will pay more than Job 2.
   D. Alicia will earn the most at Job 2.

6. A company is awarded a \$95,000 job that will cost \$5,400 per day in expenses. Profits (P) can be calculated using $P = \$95,000 - \$5,400d$, where d = days. What is the company's profit if the job takes 14 days to complete?
   A. \$10,800
   B. \$19,400
   C. \$66,100
   D. \$75,600

Answers and explanations begin on page 680.

Function Notation

Key Ideas

- In function notation, the result of the function is expressed in terms of the variable in the function, for example, $f(x) = x + 4$.
- The domain of a function is the set of valid input values, or those values of the variable for which the function is defined.
- The range of a function is the set of output values that it can generate, given the values in its domain.

GED® TEST TIP

Sometimes you may be given a function and an input value and be asked to solve for the output, and sometimes you may be given the output and be asked to solve for the input. In either case, the question will always provide enough information to figure out what the function is.

Functions can be written with **function notation**. Take the function in Example 1 of the previous lesson: $C = \$2000d + \$1500(d - 7)$, where C = the cost of finishing a project and d = the number of days the project takes. In function notation, the cost would be written as $f(d)$ instead of C, so the function is written as follows: $f(d) = \$2000d + \$1500(d - 7)$. The notation $f(d)$ is interpreted as "the function of d" or "f of d."

The **domain** of a function is the set of values for which the function is defined or makes sense. In the above example, the domain is all the values that d can take. Since d is an amount of time, it cannot be negative. Furthermore, in this case if $d < 7$, then the function begins to subtract from the \$2,000 a day cost for finishing the project, which does not make sense (as explained in the example, \$2,000/day is the rate for the first seven days, and additional days cost even more; no days cost less than \$2,000). Thus, the domain of $f(d)$ is $d \geq 7$.

Take a look at the following example involving a quadratic expression.

Example 1: What is the domain of $f(x) = \dfrac{1}{1-x^2}$?

1. The domain is the set of values that x can take on to produce a defined value for $f(x)$. Division by zero is undefined.
2. Calculate the values of x that produce a denominator of zero.
$$1 - x^2 = 0$$
$$(1 + x)(1 - x) = 0$$
$$x = \pm 1$$
3. The domain is all real numbers not equal to −1 or 1.

The **range** of a function is the set of outputs or results of the function. Look again at the example $f(d) = \$2000d + \$1500(d - 7)$. When d is as small as possible—that is, when $d = 7$—then $f(d) = \$2000 \times 7 + \$1500(7 - 7) = \$14{,}000$. There is no upper limit to how many days a job could take, so there is no upper limit to the cost or to $f(d)$. Thus, the range of $f(d)$ is numbers greater than or equal to 14,000.

Example 2: What is the range of $f(x) = x^2$?

1. First, determine the domain of this function. Any number can be squared, so the domain is all values of x.
2. Find the minimum value of $f(x)$. A squared real number is never negative. When $x = 0$, $f(x) = 0$ and that is the function's minimum value.
3. Find the maximum value of $f(x)$. As x moves away from zero, whether in a positive or negative direction, $f(x)$ takes on a greater positive value. There is no limit on how large $f(x)$ can be. Thus, the range of $f(x)$ is numbers greater than or equal to zero.

Note that sometimes graphing a function is very helpful to find its domain and range.

A question could give you the value of a function and ask what value(s) of the variable would produce that result.

Example 3: Rafael enjoys baking cookies with his nieces and nephews. For every cup of oats, they can make 24 oatmeal cookies. If they made 180 oatmeal cookies today, how many cups of oats did they use?

1. Write the function.
$f(c) = 24c$, where c is the number of cups of oats and $f(c)$ is the number of cookies.
2. Substitute the value of the outcome for $f(c)$.
$180 = 24c$
3. Solve for c.
$c = \dfrac{180}{24} = 7.5$

Rafael and his family used 7.5 cups of oats to make the cookies.

PRACTICE 13

A. Find the domain and range of the following functions.

1. $f(x) = x - 2$
2. $f(x) = \dfrac{5}{x+3}$ x
3. $f(x) = x^2 + 4x + 4$
4. $f(b) = -|-6b|$
5. $f(x) = -(x^2 - 1)$
6. $f(z) = \dfrac{z^2 - 9}{z^2 - 6z + 9}$

B. Solve.

7. Carla's daughter is participating in a summer reading program at the library. She is reading two books a week. Which of the following functions expresses her summer reading, where w is the number of weeks in the program?
   A. $f(w) = w + 2$
   B. $f(w) = 2w$
   C. $f(w) = 2b$
   D. $2f(w) = w$
8. Bryan is on a road trip with his best friend, and they like to get up early. Each day, they drive 100 miles before breakfast, and then they drive an average of 60 miles per hour for each hour they are on the road until they stop for the day. They never spend more than 8 hours on the road after breakfast. If h is the number of hours they drive after breakfast, which of the following are the domain and range of the function that represents their daily driving?
   A. Domain is $0 \leq h \leq 8$; range is $100 \leq f(h) \leq 580$.
   B. Domain is $100 \leq h \leq 580$; range is $0 \leq f(h) \leq 8$.
   C. Domain is $h \geq 0$; range is $f(h) \geq 100$.
   D. Domain is $h \leq 8$; range is $60 \leq f(h) \leq 100$.

Answers and explanations begin on page 681.