LESSON

4

GEOMETRY

Perimeter and Area

Perimeter is the distance around a figure. To find perimeter, simply add the lengths of the sides. For common figures, you can apply a formula to find the perimeter. You need to memorize these formulas.

square	Perimeter $= 4 \times$ side	$P = 4s$
rectangle	Perimeter $= 2 \times$ length $+ 2 \times$ width	$P = 2l + 2w$
triangle	Perimeter $=$ side$_1$ + side$_2$ + side$_3$	$P = a + b + c$

Example 1: A rectangle is 16 inches long and 9 inches wide. What is the perimeter of the rectangle?

Use the formula: Perimeter $= 2 \times$ length $+ 2 \times$ width
$= 2 \times 16 + 2 \times 9$
$= 32 + 18$
$=$ **50 in.**

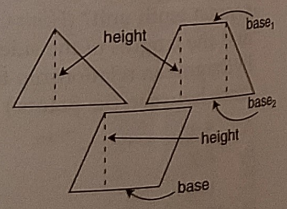
Area is the measure of the space inside a flat figure. Area is measured in square units. For example, if the sides of a figure are measured in inches, its area will be measured in square inches. The formulas for finding area are shown below.

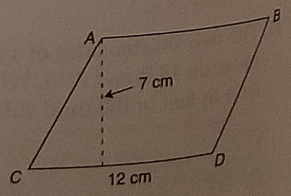
square	Area $=$ side^2	$A = s^2$
rectangle	Area $=$ length $\times$ width	$A = lw$
parallelogram	Area $=$ base $\times$ height	$A = bh$
triangle	Area $= \frac{1}{2} \times$ base $\times$ height	$A = \frac{1}{2}bh$
trapezoid	Area $= \frac{1}{2} \times$ height (base$_1$ + base$_2$)	$A = \frac{1}{2}h(b_1 + b_2)$

Three of the formulas mention two new measures: base and height. The **base** is one side of the figure. The **height** is the length from the vertex to the base, forming a right angle to the base.

Example 2: Find the area of figure *ABCD*.

1. Identify the figure. *ABCD* is a parallelogram.
2. Find the facts you need. To use the formula for finding the area of a parallelogram, you need to know the height and the length of the base. Ignore the length of side *BD*.
3. Use the formula Area $=$ base $\times$ height.
   Area $= 12 \times 7$
   $=$ **84 sq cm or 84 cm^2**





Key Ideas

- Perimeter is the distance around a figure.
- Area is the measure of the space inside a flat figure. It is measured in square units.
- You need to memorize the formulas on this page except those for area of a parallelogram and area of a trapezoid.

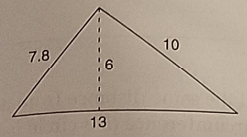
GED® TEST TIP

Several area formulas appear on the GED® formula sheet. You can check there if you don't remember a specific formula on test day. However, memorizing the most common ones (rectangle, triangle, and circle) will likely save you time as you're testing.

PRACTICE 4

A. Find the area and perimeter of each figure.

1.



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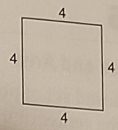

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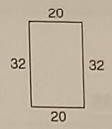

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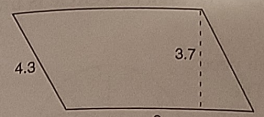

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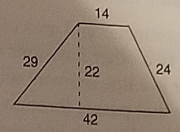

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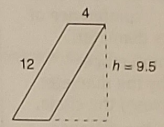
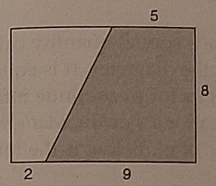


B. Choose the one best answer to each question.

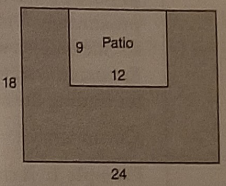
Question 7 refers to the following figure.



7. What is the area in square inches of the shaded portion of the rectangle?
   A. 38
   B. 40
   C. 56
   D. 88

8. The four sides of a retangle measure 9 feet, 6 feet, 9 feet, and 6 feet. What is the area of the retangle in square feet?
   A. 30
   B. 36
   C. 54
   D. 81

9. Martin is building a rectangular patio centered on one side of his yard. The rest of his yard, shown in the diagram, is planted in grass.



If the measurements in the diagram are in feet, what is the square footage of the grass portion of Martin's yard?
   A. 108
   B. 162
   C. 324
   D. 432

10. A square measures 6 centimeters on one side. What is the perimeter of the square in centimeters?
   A. 12
   B. 24
   C. 36
   D. 216

Answers and explanations begin on page 683.

Triangles

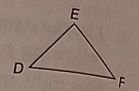
Key Ideas

- Triangles are named by their vertices.
- They are classified in two ways: by their side lengths and by their angle measures.
- The sum of the interior angles of any triangle is 180°.

The Properties of Triangles

A **triangle** is a closed three-sided plane figure. From the definition, we can infer other properties. Since a triangle has three sides, it must also have three interior angles and three vertices.

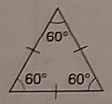
A triangle is named by writing its vertices in any order. The triangle shown at right could be named ΔDEF. Its sides are DE, EF, and DF.



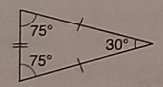
Triangles can be classified by the lengths of their sides and by the measures of their angles. In the figures below, sides with the same number of marks are equal.

Classified by Side Lengths

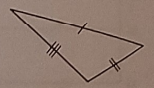
equilateral triangle
All sides are equal in length. Note that the angles also are equal.



isosceles triangle
Exactly two sides are equal in length. Note that the two angles opposite these sides are equal.

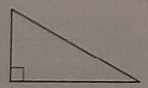


scalene triangle
No sides are equal in length, and no angles are equal.

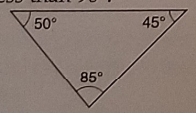


Classified by Angle Measures

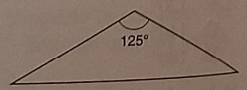
right triangle
One angle measures 90°.



acute triangle
All angles measure less than 90°.



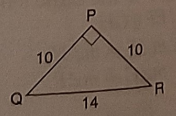
obtuse triangle
One angle is greater than 90°.



Each triangle can be classified in two ways.

Example 1: What kind of triangle is ΔPQR?

1. Classify by its sides: Two sides have the same length, so ΔPQR is an isosceles triangle.
2. Classify by its angles: $\angle P$ is a right angle, so ΔPQR is a right triangle.



ΔPQR is a **right isosceles triangle**.

GED® TEST TIP

Classify triangles by their properties, not by how they look. For example, the triangle in Example 1 may not immediately look like a right triangle because the right angle is at the top.

The sum of the measures of the interior angles of any triangle is 180°. We can use this fact to solve for a missing angle.

Example 2: In ΔABC, $\angle A$ measures 55° and $\angle B$ measures 100°. What is the measure of $\angle C$?

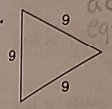
Write an equation and solve.

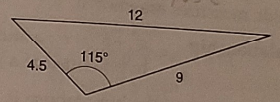
$$55° + 100° + \angle C = 180°$$
$$155° + \angle C = 180°$$
$$\angle C = 25°$$

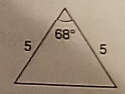
The measure of $\angle C$ is **25°**.

PRACTICE 2

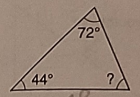
A. Classify each triangle in two ways.

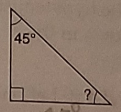
1. 

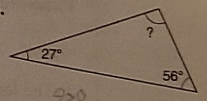
2. 

3. 

B. Find the measure of the unknown angle in each triangle.

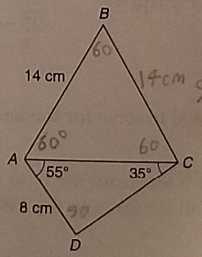
4. 

5. 

6. 

C. Choose the one best answer to each question.

Questions 7 and 8 refer to the following figure.



7. If $\angle DAB$ measures 115° and $\angle DCB$ measures 95°, what is the length of side AC in centimeters? (*Hint*: Use the facts in the problem to find $m\angle BAC$ and $m\angle BCA$.)
   A. 6
   B. 8
   C. 14
   D. 22

8. What kind of triangle is ΔACD?
   A. isosceles
   B. acute
   C. right
   D. obtuse

9. One angle in a scalene triangle measures 38°, and another angle measures 56°. What is the measure of the third angle?
   A. 38°
   B. 56°
   C. 86°
   D. 124°

Answers and explanations begin on page 682.

LESSON 3

GEOMETRY

Pythagorean Relationship

Key Ideas

- The hypotenuse is the longest side of a right triangle. It is found opposite the right angle.
- The sum of the squares of the legs of a right triangle equals the square of the hypotenuse.

GED® TEST TIP

Watch for right triangles with sides in a 3-4-5 or 5-12-13 ratio. These triangles often appear on math tests because the measures of the sides are all whole numbers.

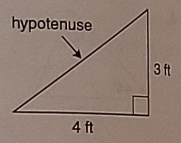
As you know, a right triangle has one right angle. The side directly across from the right angle, called the **hypotenuse**, is the longest side of the right triangle. The remaining sides, the rays of the right angle, are the **legs** of the triangle.

Thousands of years ago, people found a special relationship, called the **Pythagorean relationship**, among the sides of a right triangle. You can use this relationship to find the measure of any side of a right triangle if the other two side measures are known.

Pythagorean relationship $a^2 + b^2 = c^2$; a and b are legs, and c is the hypotenuse of a right triangle

In other words, the square of the hypotenuse is equal to the sum of the squares of the two legs of the right triangle.

Example 1: What is the length of the hypotenuse of the right triangle shown in the diagram?



1. The lengths of the legs are 3 ft and 4 ft. Let one leg equal a and the other equal b.

2. Solve for c. Substitute the values.

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$

3. When one side of an equation equals a squared variable, isolate the variable by finding the square root of both sides.

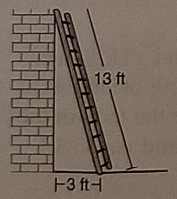
$$25 = c^2$$
$$\sqrt{25} = c$$
$$5 = c$$

The length of the hypotenuse is **5 feet**.

The Pythagorean relationship can also be used to solve for the length of a leg.

Example 2: If John places a 13-foot ladder 3 feet from the base of a wall, how far up the wall will the ladder reach to the nearest tenth of a foot?

The wall, ground, and ladder form a right triangle. The hypotenuse is 13 ft in length. One leg is 3 ft. You need to find the length of the other leg.



$$a^2 + b^2 = c^2$$
$$3^2 + b^2 = 13^2$$
$$9 + b^2 = 169$$
$$b^2 = 160$$
$$b = \sqrt{160}$$
$$b \approx 12.6$$

The ladder will extend **12.6 feet** up the wall.

NOTE: Most of the time, you will need to use your calculator for the final step when using the Pythagorean relationship. To find the square root of 160 on the TI-30XS MultiView™ calculator, press: 2nd x^2 160 enter (use ◀▶ to convert the result from a radical to a decimal format).

Some right triangles display special proportions, which are worth memorizing.

- A right triangle whose angles are 45°, 45°, and 90° displays the following relationship: leg : leg : hypotenuse = $x : x : x\sqrt{2}$.
- A right triangle whose angles are 30°, 60°, and 90° displays the following relationship: leg : leg : hypotenuse = $x : x\sqrt{3} : 2x$.

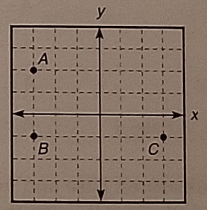
PRACTICE 3

A. The lengths of two sides of a right triangle are given. Find the length of the remaining side to the nearest tenth unit. You MAY use a calculator.

1. leg a: 8 in.
   leg b: 8 in.
   hypotenuse c: ? in.
2. leg a: 9 yd
   leg b: 12 yd
   hypotenuse c: ? yd
3. leg a: 1.5 cm
   leg b: 2 cm
   hypotenuse c: ? cm
4. leg a: ? m
   leg b: 3 m
   hypotenuse c: 6 m
5. leg a: 6 mm
   leg b: ? mm
   hypotenuse c: 10 mm
6. leg a: ? ft
   leg b: 5 ft
   hypotenuse c: 18 ft
7. leg a: 7 cm
   leg b: 10 cm
   hypotenuse c: ? cm
8. leg a: 15 in.
   leg b: ? in.
   hypotenuse c: 30 in.
9. leg a: 4 km
   leg b: 5 km
   hypotenuse c: ? km

B. Choose the one best answer to each question.

10. On a coordinate plane, points A, B, and C can be connected to form a right triangle.



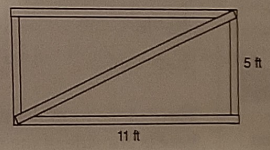
What is the distance from A to C, to the nearest tenth unit? (*Hint:* Count units to find the lengths of the sides and use the Pythagorean relationship to find the distance between the points.)

A. 5.2
B. 6.7
C. 8.4
D. 10.1

11. The two shorter sides of a right triangle measure 18 ft and 24 ft. What is the measure in feet of the third side?

A. 25
B. 28
C. 30
D. 42

12. Jan has built a rectangular frame out of wood to use for the bottom of a platform. He wants to add a diagonal brace as shown in the drawing below.



What will the length of the brace be, to the nearest tenth of a foot?

A. 16.0
B. 13.7
C. 12.8
D. 12.1

13. The hypotenuse of a right triangle measures 39 inches. If one leg measures 15 inches, what is the measure of the other leg, in inches?

A. 36
B. 24
C. 18
D. 12

Answers and explanations begin on page 683.

LESSON

1

GEOMETRY

Plane Figures

Key Ideas

- Plane figures are classified by the properties of their sides and angles.
- The sum of the interior angles of any four-sided plane figure is 360°.
- By using the properties of any four-sided plane figure and algebraic reasoning, you can find missing angle measures.

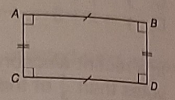
GED® TEST TIP

Don't rely on sight alone to identify a geometric figure. Read carefully to see which properties are given. Then identify the figure based on the properties.

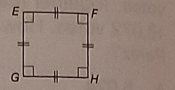
Four-Sided Plane Figures

A **plane figure** is a set of line segments, all lying on a single plane. To prepare for the GED® *Mathematical Reasoning Test*, learn the properties of each shape. You will need to identify the characteristics of different types of four-sided plane figures and draw conclusions about their angles and sides.

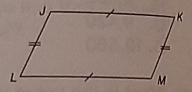
You are already familiar with rectangles and squares. A **rectangle** is a four-sided figure with four right angles. The opposite sides (sides across from each other) are the same length, and they are parallel.



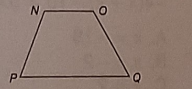
A **square** is actually a kind of rectangle. It, too, has four right angles with parallel opposite sides. However, a square has one additional property: its four sides are all the same length.



A **parallelogram** is a four-sided figure whose opposite sides are parallel and the same length. In addition, its opposite angles (the angles diagonally across from each other) are also equal in measure. A special parallelogram, called a **rhombus** (not shown), has four sides of equal length.

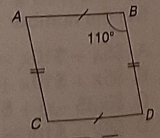


A **trapezoid** is a four-sided figure with exactly one pair of parallel sides. The definition of a trapezoid does not dictate the measure of the angles or the lengths of the sides.



All four-sided plane figures have one important property in common. The sum of the measures of the interior angles is 360°. You can use this fact to find a missing angle measure.

Example: In figure *ABCD*, the opposite sides are parallel. What is the measure of $\angle A$?



$\overline{AC} \parallel \overline{BD}$

$\overline{AB} \parallel \overline{CD}$

1. Identify the figure. The notation on the drawing tells you that the opposite sides are equal in measure. Since they are also parallel, the figure is a parallelogram.

2. Find the measure of $\angle C$. The opposite angles of a parallelogram are equal in measure; therefore, $m\angle C = m\angle B$. Both $\angle B$ and $\angle C$ measure 110°.

3. Find the measure of $\angle A$. You know the measures of $\angle A$ and $\angle D$ are equal and that the sum of all four angles equals 360°. Let $x = m\angle A$. Therefore, $2x$ = the sum of $m\angle A$ and $m\angle D$. Write an equation and solve.

$$2x + 110° + 110° = 360°$$
$$2x + 220° = 360°$$
$$2x = 140°$$
$$x = 70°$$

The measure of $\angle A$ is **70°**.

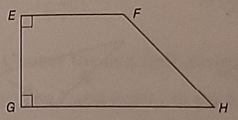
PRACTICE 1

A. List the names of four-sided plane figures introduced on page 388 that can exhibit the following properties. Write *None* if no four-sided plane figure has the given property.

1. four right angles
2. opposite sides are equal in length
3. exactly one pair of parallel sides
4. all angles are equal in measure
5. only three right angles
6. opposite angles are equal in measure
7. all four sides are equal in length
8. sum of interior angles is 360°
9. sides are all of different lengths
10. four equal angles and four equal sides

B. Choose the one best answer to each question.

Questions 11 and 12 refer to the following figure.



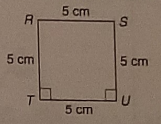
11. Angle *F* is 20° more than three times the measure of $\angle H$. What is the measure of $\angle F$?

A. 40°
B. 120°
C. 140°
D. 180°

12. In order for figure *EFGH* to be a trapezoid, which of the following must be a true statement?

A. *EF* is the same length as *FH*.
B. $EF \parallel GH$.
C. $m\angle F = m\angle H$.
D. $m\angle G = m\angle F$.

Question 13 refers to the following figure.



13. If the opposite sides in figure *RSUT* are parallel, what is the measure of $\angle R$?

A. 5°
B. 20°
C. 90°
D. 270°

14. A four-sided plane figure has sides measuring 10, 15, 10, and 15. The opposite angles are equal, but there are no right angles. What is the figure?

A. rhombus
B. rectangle
C. parallelogram
D. trapezoid

Answers and explanations begin on page 682.

LESSON 5

GEOMETRY

Circles

Key Ideas

- Pi, expressed as 3.14, is the ratio of the circumference of a circle to its diameter.
- To find circumference, multiply pi by the diameter (distance across) the circle.
- The radius is the distance from the center to the edge of the circle. To find area, multiply pi by the square of the radius.

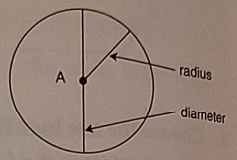
GED® TEST TIP

You will need to memorize the formulas for circumference and area of a circle. They do not appear on the GED® Formulas Sheet.

Circumference and Area

A **circle** is a closed set of points that are all the same distance from a single point, the center of the circle. The **circumference** of a circle is its perimeter, or the distance around the circle. The **area** of a circle is the space inside the circle.

To find perimeter and area of a circle, you need to know two other measures of a circle. The **diameter** is a line segment with endpoints on the circle that passes through the center of the circle. The **radius** is a line segment that connects the center of the circle to any point on the circle. As you can see from the diagram, the radius is one-half the diameter.



The formulas for circumference and area use a special quantity called **pi** (π). Pi is the ratio of the circumference to the diameter. It is equal to approximately 3.14. As far as we know, the digits for pi continue infinitely, so calculations with pi are always approximations. For the *Mathematical Reasoning Test*, you will use 3.14 as the value of pi. Below is the formula for finding the circumference of a circle.

Circumference $= \pi \times$ diameter, or $C = \pi d$

Example 1: A china plate has a gold rim. If the plate's diameter is 10.5 inches, what is the distance around the rim to the nearest tenth of an inch?

Use the formula: $C = \pi d$
$= 3.14(10.5)$
$= 32.97$, which rounds to **33.0 inches**

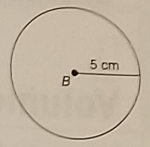
Use this formula to find the area of a circle: Area $= \pi \times$ radius^2, or $A = \pi r^2$.

Example 2: The circular surface of a satellite component must be covered with heat-resistant tiles. If the radius of the component is 4 meters, what is the area in square meters?

Use the formula: $A = \pi r^2$
$= 3.14(4^2)$
$= 3.14(16)$
$=$ **50.24 square meters**

In some situations, you may need to solve for either the diameter or radius. Remember, the diameter is twice the radius ($d = 2r$), and the radius is one-half the diameter: $r = \frac{1}{2}d$.

Example 3: What is the circumference of circle B to the nearest tenth of a centimeter?

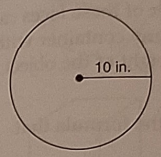


1. The radius of the circle is 5 cm. Therefore, the diameter is 2×5, or 10 cm.
2. Use the formula: $C = \pi d$
$= 3.14(10)$
$=$ **31.4 cm**

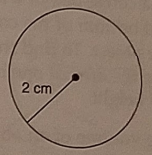
PRACTICE 5

A. Find the circumference and area of each circle. Round answers to the nearest tenth.

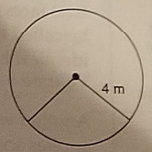
1.



2.

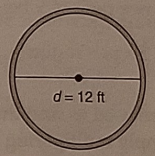


3.



B. Choose the one best answer to each question.

Questions 4 and 5 refer to the following drawing.

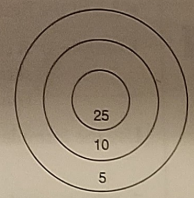


4. If workers lay a tile border around the edge of the fountain shown in the diagram, how many feet long will the border be to the nearest foot?
   A. 19
   B. 36
   (C.) 38
   D. 57

5. Which of the following expressions could be used to find the area of the bottom surface of the fountain?
   A. 3.14×6
   (B.) 3.14×6^2
   C. 3.14×12
   D. 3.14×12^2

6. The radius of a circle is 6.5 cm. What is the diameter of the circle in centimeters?
   A. 3.25
   (B.) 13.0
   C. 33.16625
   D. 132.665

7. On the target below, the 5- and 10-point bands are each 2 inches wide, and the 25-point inner circle has a diameter of 2 inches.



To the nearest inch, what is the outer circumference of the 10-point band?
   A. 6
   B. 13
   (C.) 19
   D. 113

Answers and explanations begin on page 683.