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4º sinal

$T_0 = 1$

$\omega_0 = \frac{2\pi}{1} = 2\pi$

$$a_0 = \frac{1}{1}\int_{-1/8}^{7/8} x(t)\,dt = \int_{-1/8}^{1/8} \underbrace{x(t)}_{1}\,dt = t\Big|_{-1/8}^{1/8} = \frac{2}{8} = \frac{1}{4}$$

$$a_k = 2\int_{-1/8}^{7/8} x(t)\cos(k2\pi t)\,dt = 2\int_{-1/8}^{1/8} 1.\cos(k2\pi t)\,dt$$

$$\rightarrow = \frac{2}{2k\pi}(\text{sen}\,k2\pi t)\Big|_{-1/8}^{1/8}$$

$$= \frac{1}{k\pi}\left[\text{sen}\left(\frac{k\pi}{4}\right) + \text{sen}\left(\frac{k\pi}{4}\right)\right]$$

$$= \frac{2}{k\pi}\,\text{sen}\left(\frac{k\pi}{4}\right)$$

$\rightarrow$ Como o sinal possui simetria par, $b_k = 0$:

$$b_k = 2\int_{-1/8}^{7/8} x(t)\,\text{sen}(k2\pi t)\,dt = 2\int_{-1/8}^{1/8} 1.\,\text{sen}(k2\pi t)\,dt$$

$$\rightarrow = \frac{2}{2\pi k}\left[-\cos(k2\pi t)\right]\Big|_{-1/8}^{1/8}$$

$$= \frac{1}{\pi k}\left[-\cos\left(\frac{k\pi}{4}\right) + \cos\left(\frac{-k\pi}{4}\right)\right] = 0$$

Aplicando em $x(t)$:

$$x(t) = \frac{1}{4} + \sum_{k=1}^{\infty}\left[\frac{2}{k\pi}\,\text{sen}\left(\frac{k\pi}{4}\right)\cos(k2\pi t)\right]$$

$$x(t) = \frac{1}{4} + \frac{\sqrt{2}}{\pi}\cos(2\pi t) + \frac{1}{\pi}\cos(4\pi t)$$ $\rightarrow$ 2 termos

$$x(t) = \frac{1}{4} + \frac{\sqrt{2}}{\pi}\cos(2\pi t) + \frac{1}{\pi}\cos(4\pi t) + \frac{\sqrt{2}}{3\pi}\cos(6\pi t)$$ $\rightarrow$ 3 termos

$$\boxed{x(t) = \frac{1}{4} + \frac{\sqrt{2}}{\pi}\cos(2\pi t) + \frac{1}{\pi}\cos(4\pi t) + \frac{\sqrt{2}}{3\pi}\cos(6\pi t) - \frac{\sqrt{2}}{5\pi}\cos(10\pi t)}$$

$\rightarrow$ 5 termos

$$\boxed{\begin{aligned} x(t) = {} & \frac{1}{4} + \frac{\sqrt{2}}{\pi}\cos(2\pi t) + \frac{1}{\pi}\cos(4\pi t) + \frac{\sqrt{2}}{3\pi}\cos(6\pi t) - \frac{\sqrt{2}}{5\pi}\cos(10\pi t) \\ & - \frac{1}{3\pi}\cos(12\pi t) - \frac{\sqrt{2}}{7\pi}\cos(14\pi t) + \frac{\sqrt{2}}{9\pi}\cos(18\pi t) + \frac{1}{5\pi}\cos(20\pi t) \end{aligned}}$$

$\rightarrow$ 10 termos

5º sinal

$T_0 = 1,\ \omega_0 = 2\pi$

$$a_0 = \frac{1}{1}\int_{-1/4}^{1/4} x(t)\,dt = t\Big|_{-1/4}^{1/4} = \frac{2}{4} = \frac{1}{2}$$

$$a_k = 2\int_{-1/4}^{1/4} x(t)\cos(k2\pi t)\,dt$$

$$= 2\int_{-1/4}^{1/4} 1.\cos(k2\pi t)\,dt$$

$$= \frac{2}{2k\pi}\left(\operatorname{sen}(k2\pi t)\right)\Big|_{-1/4}^{1/4}$$

$$\to = \frac{1}{k\pi}\left[\operatorname{sen}\frac{1\pi k}{2} + \operatorname{sen}\frac{1\pi k}{2}\right]$$

$$= \frac{2}{k\pi}\operatorname{sen}\left(\frac{\pi k}{2}\right)$$

$$b_k = 2\int_{-1/4}^{1/4} x(t)\operatorname{sen}(k2\pi t)\,dt$$

$$= 2\int_{-1/4}^{1/4} 1.\operatorname{sen}(k2\pi t)\,dt$$

$$= \frac{2}{2k\pi}\left(-\cos(k2\pi t)\right)\Big|_{-1/4}^{3/4}$$

$$\to = \frac{1}{k\pi}\left[-\cos\left(\frac{1}{2}\pi k\right) + \cos\left(\frac{-1}{2}\pi k\right)\right]$$

$$= 0$$

$$x(t) = \frac{1}{2} + \sum_{k=1}^{\infty}\left[\frac{2}{k\pi}\operatorname{sen}\left(\frac{\pi k}{2}\right)\cos(k2\pi t)\right]$$

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi t) \rightsquigarrow k = 1, 2$$

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi t) - \frac{2}{3\pi}\cos(6\pi t) \rightsquigarrow k = 1, 2, 3$$

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi t) - \frac{2}{3\pi}\cos(6\pi t) + \frac{2}{5\pi}\cos(10\pi t) \to k = 1, 2, 3, 4, 5$$

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi t) - \frac{2}{3\pi}\cos(6\pi t) + \frac{2}{5\pi}\cos(10\pi t) - \frac{2}{7\pi}\cos(14\pi t) + \frac{2}{9\pi}\cos(18\pi t)$$