Course Project Reprot: Goofspiel

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1 Overview

In this report, I'll show the flow of my mind on solving Goofspiel, the game of pure strategy.

Main contribution of this report includes: (a) a reasonable good strategy; (b) a general way to refine any given strategy.

I am not going to explain rules of the Game, as you can always find better explanation online when searching for "Goofspiel".

Theoretical approach of solving the problem will be explored first. Then followed empirical approach based on the concept of "doing the best you can" and "If you find something is good to do, do it now".

If you are not interested in my thought flow but only the final solution, please jump to the last page directly and read the conclusion.

To narrow down my topic, a bunch of assumptions are states as follow.

2 Assumptions

1. Try to win at the end of all rounds is equal to try to win at each round. In another word, I treat opponent in each round as a new one, and delete all memory from round I played before, so I am not going to consider how to guess opponent's strategies across all rounds.
2. There are only two players in each round, me and the opponent.
3. Since my strategies do not make any difference on carryover or nocarryover case, I'll just discuss my strategies and attach simulation results on both cases.

3 Theoretical Approaches

In the approach, I am not going to guess or crack the strategy my opponent chooses, but try to find a better strategy that beats most strategies known with theoretical proof as support.

A. antiRandom

To start with, let's first assume that the opponent is play in a random way as given in the software.

Ross proved that one can win with a expect value of 28 point against opponent playing randomly by play card equal to every upcard (Ross, 1971).

This algorithm is implemented in my software with source file antiRandom.cpp. To call it, set the strategy name as "antiRandom".

Let Alice use random, and Bob use antiRandom strategy, run 100 rounds, we got the results:

```
Up Card: 8H (value = 8)
Calling gxkStrategy001
Calling antiRandom
Alice played: QC
Bob played: 8D
        Alice gets 8 points
        Alice's points at round 100: 25 (Total for all rounds: 2833)
        Bob's points at round 100: 66 (Total for all rounds: 5707)

Alice's total points: 2833
Bob's total points: 5707
```

Figure 1: carryover case

```
Up Card: KH (value = 13)
Calling gxkStrategy001
Calling antiRandom
Alice played: AC
Bob played: KD
        Bob gets 13 points
        Alice's points at round 100: 32 (Total for all rounds: 2715)
        Bob's points at round 100: 59 (Total for all rounds: 5555)

Alice's total points: 2715
Bob's total points: 5555
```

Figure 2: nocarryovercase

As we can see, Bob won an average of around 28 points in each round on both cases, which verifies Ross's proof.

B. antiRandomKiller

An easy way to against antiRandom is play (upcard+1) if upcard is not equal to 13 and play 1 when the upcard is equal to 13. So you'll only lose 13 and win 1 to 12, which sum up to 78. Thus the expect value of win is 65 points.

This algorithm is implemented in my software with source file antiRandomKiller.cpp. To call it, set the strategy name as "antiRandomKiller".

Let Alice use antiRandom while Bob uses antiRandomKiller, we got the following result for 100 rounds:

```
Up Card: AH (value = 1)
Calling antiRandom
Calling antiRandomKiller
Alice played: AC
Bob played: 2D
        Bob gets 1 points
        Alice's points at round 100: 13 (Total for all rounds: 1300)
        Bob's points at round 100: 78 (Total for all rounds: 7800)

Alice's total points: 1300
Bob's total points: 7800
```

Figure 3: carryover case

```
Up Card: AH (value = 1)
Calling antiRandom
Calling antiRandomKiller
Alice played: AC
Bob played: 2D
        Bob gets 1 points
        Alice's points at round 100: 13 (Total for all rounds: 1300)
        Bob's points at round 100: 78 (Total for all rounds: 7800)

Alice's total points: 1300
Bob's total points: 7800
```

Figure 4: nocarryover case

As we can see Bob won 65 points on average of each round on both cases.

Addtionally, I am also interested the performance of antiRandomKiller against random players. So Iet Alice use random and Bob use antiRandomKiller, we got the following result for 100 rounds:

```
Up Card: 7H (value = 7)
Calling gxkStrategy001
Calling antiRandomKiller
Alice played: 8C
Bob played: 8D
        Played the same card - no points awarded
        Alice's points at round 100: 33 (Total for all rounds: 3407)
        Bob's points at round 100: 51 (Total for all rounds: 5030)

Alice's total points: 3407
Bob's total points: 5030
```

Figure 5: carryover case

```
Up Card: KH (value = 13)
Calling gxkStrategy001
Calling antiRandomKiller
Alice played: 8C
Bob played: AD
        Alice gets 13 points
        Alice's points at round 100: 40 (Total for all rounds: 3320)
        Bob's points at round 100: 51 (Total for all rounds: 5021)

Alice's total points: 3320
Bob's total points: 5021
```

Figure 6: nocarryover case

As we can see, antiRandomKiller is also far way better than random strategy.

In all, we can say that theoretically, antiRandomKiller is better than random and antiRandom Strategy.

C. antiRandomKillerKiller

Same way as how I against antiRandom by antiRandomKiller, I can also win antiRandomKiller by antiRandomKillerKiller, which play (upcard+2)mod 13 when upcard is not equal to 11 and play 13 when upcard is equal to 13.

Let Alice use antiRandomKiller and Bob use antiRandomKillerKiller, we got the result for 100 rounds:

```
Up Card: 7H (value = 7)
Calling antiRandomKiller
Calling antiRandomKillerKiller
Alice played: 8C
Bob played: 9D
        Bob gets 7 points
        Alice's points at round 100: 12 (Total for all rounds: 1200)
        Bob's points at round 100: 79 (Total for all rounds: 7900)

Alice's total points: 1200
Bob's total points: 7900
```

Figure 7: carryover case

```
Up Card: 4H (value = 4)
Calling antiRandomKiller
Calling antiRandomKillerKiller
Alice played: 5C
Bob played: 6D
        Bob gets 4 points
        Alice's points at round 100: 12 (Total for all rounds: 1200)
        Bob's points at round 100: 79 (Total for all rounds: 7900)

Alice's total points: 1200
Bob's total points: 7900
```

Figure 8: nocarryover case

We can see that antiRandomKillerKiller won an average of 67 points each round, this is because antiRandomKillerKiller only loses when upcard is equal to 12.

Remember that antiRandomKillerKiller should beat all strategies before to be a better strategy.

Let Alice use antiRandom and Bob use antiRandomKillerKiller, we got the following results for 100 rounds:

```
Up Card: 6H (value = 6)
Calling antiRandom
Calling antiRandomKillerKiller
Alice played: 6C
Bob played: 8D
        Bob gets 6 points
        Alice's points at round 100: 25 (Total for all rounds: 2500)
        Bob's points at round 100: 66 (Total for all rounds: 6600)

Alice's total points: 2500
Bob's total points: 6600
```

Figure 9: carryover case

```
Up Card: 5H (value = 5)
Calling antiRandom
Calling antiRandomKillerKiller
Alice played: 5C
Bob played: 7D
        Bob gets 5 points
        Alice's points at round 100: 25 (Total for all rounds: 2500)
        Bob's points at round 100: 66 (Total for all rounds: 6600)

Alice's total points: 2500
Bob's total points: 6600
```

Figure 10: nocarryover case

We can see that antiRandomKillerKiller won an average of 41 points each round since it only loses when upcard=12 and upcard=13.

Let Alice use random strategy and Bob use antiRandomKillerKiller, we got the following results for 100 rounds:

```
Up Card: TH (value = 10)
Calling gxkStrategy001
Calling antiRandomKillerKiller
Alice played: 4C
Bob played: QD
        Bob gets 10 points
        Alice's points at round 100: 44 (Total for all rounds: 3957)
        Bob's points at round 100: 47 (Total for all rounds: 4406)

Alice's total points: 3957
Bob's total points: 4406
```

Figure 11: carryover case

```
Up Card: AH (value = 1)
Calling gxkStrategy001
Calling antiRandomKillerKiller
Alice played: 9C
Bob played: 3D
        Alice gets 1 points
        Alice's points at round 100: 43 (Total for all rounds: 3825)
        Bob's points at round 100: 38 (Total for all rounds: 4552)

Alice's total points: 3825
Bob's total points: 4552
```

Figure 12: nocarryover case

We can see antiRandomKillerKiller is still better than random strategy.

So we can say that so far antiRandomKillerKiller is better than all strategies ahead.

D. A **worse case: antiRandomKillerKillerKiller**

Similarly, I consider strategy antiRandomKillerKillerKiller, which play (upcard+3)mod 13 when upcard is not equal to 10 and play 13 when the upcard is equal to 10.

To save the length of report, let's do some math instead of take a picture of the results.

antiRandomKillerKillerKiller win antiRandomKillerKiller at expect value of 69 points since it loses only when upcard=11.

antiRandomKillerKillerKiller win antiRandomKiller at expect value of 45 points since it loses only when upcard=11 and upcard=12.

antiRandomKillerKillerKiller win antiRandom at expect value of 19 points since it loses only when upcard=11, 12 or 13.

But how does antiRandomKillerKillerKiller against random strategy?

Let Alice use random strategy and Bob use antiRandomKillerKillerKiller, we got the result for 100 rounds:

```
Up Card: 5H (value = 5)
Calling gxkStrategy001
Calling antiRandomKillerKillerKiller
Alice played: QC
Bob played: 8D
        Alice gets 5 points
        Alice's points at round 100: 53 (Total for all rounds: 4493)
        Bob's points at round 100: 30 (Total for all rounds: 4028)

Alice's total points: 4493
Bob's total points: 4028
```

Figure 13: carryover case

```
Up Card: JH (value = 11)
Calling gxkStrategy001
Calling antiRandomKillerKillerKiller
Alice played: JC
Bob played: AD
        Alice gets 11 points
        Alice's points at round 100: 41 (Total for all rounds: 4432)
        Bob's points at round 100: 40 (Total for all rounds: 4083)

Alice's total points: 4432
Bob's total points: 4083
```

Figure 14: nocarryover case

As we can see, win is not guarantee now. So we can't say antiRandomKillerKillerKiller is better.

E. Conclusion

For theoretical approach, antiRandomKillerKiller is the best.

4 Empirical Approaches

In this approach, I am going to somehow crack the strategy my opponent chosen and react accordingly.

A. antiTheoretical

To crack all theoretical strategies above, one should notice that, given a short sequence (say sequence length equal to 3 or 4), it's hard to find if it is random, but it's reasonable easy to find it is correlate to some other known sequence.

Let playedCardOpp donates the card my opponent played given an upcard, and playedCardMe donates the card I play given a upcard. To beat any strategy mentioned in "Theoretical way", just play

```
If (playedCardOpp+1) is not equal to 13
        playedCardMe  = (playedCardOpp+1)mod 13
Else
        playedCardMe=13
```

Algorithm 1

In order to guess what card the opponent will play given a certain upcard, we need to guess the strategy the opponent uses.

To guess the strategy of the opponent, when the opponent only take strategies among those in "theoretical way", just calculate the correlation between the played cards sequence of opponent and should played cards sequence corresponding to each strategies.

In all, the antiTheoretical strategy plays like this, play 1, 2, 3 regardless of the value of first 3 upcards.

Use algorithm 1 to play after find the strategy of the opponent. If calculated playedCardMe is equal to some card already been played, choose the closet but large unplayed card instead.

Further, to beat random strategy, just let antiTheoretical play upcard when no correlation has been found between opponent played cards sequence and any antiRandom strategies should have played cards.

While the longer the sequence I choose to calculate correlation the accurate my calculation will be, but the reason to choose only first 3 is because I'll lose 11+12+13=36 points at first in the worst case. But I'll still win in the end. If I choose first 4, then I'll lose 10+11+12+13=46 point at first in the worst case, then I got no chance to win afterwards.

Let Alice use antiRandom, antiRandomKiller, antiRandomKillerKiller, and random strategy, and Bob use antiTheoretical, we got following results for 100 rounds:

```
Up Card: 8H (value = 8)
Calling antiRandom
Calling antiTheoretical
Alice played: 8C
Bob played: QD
        Bob gets 8 points
        Alice's points at round 100: 24 (Total for all rounds: 2863)
        Bob's points at round 100: 57 (Total for all rounds: 5953)

Alice's total points: 2863
Bob's total points: 5953
```

Figure 15: carryover case

```
Up Card: JH (value = 11)
Calling antiRandom
Calling antiTheoretical
Alice played: JC
Bob played: KD
        Bob gets 11 points
        Alice's points at round 100: 38 (Total for all rounds: 2947)
        Bob's points at round 100: 53 (Total for all rounds: 5995)

Alice's total points: 2947
Bob's total points: 5995
```

Figure 16: nocarryover case

```
Up Card: 3H (value = 3)
Calling antiRandomKiller
Calling antiTheoretical
Alice played: 4C
Bob played: 3D
        Alice gets 3 points
        Alice's points at round 100: 55 (Total for all rounds: 3464)
        Bob's points at round 100: 25 (Total for all rounds: 5329)

Alice's total points: 3464
Bob's total points: 5329
```

Figure 17: carryover case

```
Up Card: 7H (value = 7)
Calling antiRandomKiller
Calling antiTheoretical
Alice played: 8C
Bob played: TD
        Bob gets 7 points
        Alice's points at round 100: 26 (Total for all rounds: 3725)
        Bob's points at round 100: 63 (Total for all rounds: 5207)

Alice's total points: 3725
Bob's total points: 5207
```

Figure 18: nocarryover case

```
Up Card: 2H (value = 2)
Calling antiRandomKillerKiller
Calling antiTheoretical
Alice played: 4C
Bob played: 9D
        Bob gets 2 points
        Alice's points at round 100: 20 (Total for all rounds: 4025)
        Bob's points at round 100: 71 (Total for all rounds: 4498)

Alice's total points: 4025
Bob's total points: 4498
```

Figure 19: carryover case

```
Up Card: 4H (value = 4)
Calling antiRandomKillerKiller
Calling antiTheoretical
Alice played: 6C
Bob played: 9D
        Bob gets 4 points
        Alice's points at round 100: 17 (Total for all rounds: 4171)
        Bob's points at round 100: 61 (Total for all rounds: 4489)

Alice's total points: 4171
Bob's total points: 4489
```

Figure 20: nocarryover case

```
Up Card: 7H (value = 7)
Calling gxkStrategy001
Calling antiTheoretical
Alice played: 6C
Bob played: JD
        Bob gets 7 points
        Alice's points at round 100: 18 (Total for all rounds: 3603)
        Bob's points at round 100: 63 (Total for all rounds: 4801)

Alice's total points: 3603
Bob's total points: 4801
```

Figure 21: carryover case

```
Up Card: JH (value = 11)
Calling gxkStrategy001
Calling antiTheoretical
Alice played: 2C
Bob played: JD
        Bob gets 11 points
        Alice's points at round 100: 40 (Total for all rounds: 3536)
        Bob's points at round 100: 51 (Total for all rounds: 4899)

Alice's total points: 3536
Bob's total points: 4899
```

Figure 22: nocarryover case

So far, we can see that antiTheoretical is better than random strategy, antiRandom, antiRandomKiller and antiRandomKillerKiller.

B. probabilityTransferMatrix

When searching on the internet, I found Glenn and Laurent claims that the solved the Goofspiel by some probablityTransferMatrix.

The matrix is listed below (The entry in row i column j is the probability with which you should play card i when card j is the initial upturned card.):

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	0	.0519	.0309	0	0	0	0	0	0	.0145	0	.0100
2	.4144	.2266	0	.0204	.0558	.0727	0	.0338	.0473	.0361	0	.0304	0
3	.0897	.0216	.1784	.0952	.0357	.0025	.0694	.0215	.0015	0	.0410	0	.0370
4	.4959	.2991	.0340	.0610	.0873	.0985	0	.0536	.0648	.0673	0	.0561	0
5	0	.0976	.2301	.1343	.0667	0	.1237	.0395	0	0	.0803	0	.0655
6	0	.3551	.0929	.1073	.1242	.1846	0	.0767	.1198	.0985	0	.0819	0
7	0	0	.2740	.1754	.1014	.0018	.1675	.0602	.0012	0	.1016	.0080	.0875
8	0	0	.1386	.1544	.1654	.2184	.0209	.1028	.1422	.1384	.0156	.0987	0
9	0	0	0	.2210	.1481	.0451	.2021	.0917	.0295	.0151	.1229	.0280	.1262
10	0	0	0	0	.2155	.2664	0	.1436	.1766	.1696	0	.1243	.1288
11	0	0	0	0	0	.1100	.3967	.1506	0	.0635	.2530	.0646	0
12	0	0	0	0	0	0	0	.2262	.4171	.2410	.0228	0	0
13	0	0	0	0	0	0	0	0	0	.1704	.3482	.5081	.6610

What strategy probabilityTransferMatrix does is playing the unplayed card with highest probability according to the matrix when given an upcard.

However, simulation shows this strategy is not so good or not even close.

Let Alice use random strategy, and Bob use probabilityTransferMatrix. We got the results for 100 rounds:

```
Up Card: KH (value = 13)
Calling gxkStrategy001
Calling probablityTransferMatrix
Alice played: 2C
Bob played: AD
        Alice gets 13 points
        Alice's points at round 100: 43 (Total for all rounds: 3970)
        Bob's points at round 100: 39 (Total for all rounds: 4409)

Alice's total points: 3970
Bob's total points: 4409
```

Figure 23: carryover case

```
Up Card: 9H (value = 9)
Calling gxkStrategy001
Calling probablityTransferMatrix
Alice played: 2C
Bob played: Error in get_suit_descriptionError in get_suit_description
        Alice gets 9 points
        Alice's points at round 100: 48 (Total for all rounds: 4022)
        Bob's points at round 100: 31 (Total for all rounds: 4350)

Alice's total points: 4022
Bob's total points: 4350
```

Figure 24: nocarryover case

As shown in the figure above, probabilityTransferMatrix slightly won random strategy.

Let Alice use antiRandom and Bob use probabilityTransferMatrix. We got the results for 100 rounds:

```
Up Card: 8H (value = 8)
Calling antiRandom
Calling probablityTransferMatrix
Alice played: 8C
Bob played: Error in get_suit_descriptionError in get_suit_description
        Alice gets 8 points
        Alice's points at round 100: 42 (Total for all rounds: 4930)
        Bob's points at round 100: 49 (Total for all rounds: 3885)

Alice's total points: 4930
Bob's total points: 3885
```

Figure 25: carryover case

```
Up Card: 5H (value = 5)
Calling antiRandom
Calling probablityTransferMatrix
Alice played: 5C
Bob played: 2D
        Alice gets 5 points
        Alice's points at round 100: 48 (Total for all rounds: 4832)
        Bob's points at round 100: 43 (Total for all rounds: 4020)

Alice's total points: 4832
Bob's total points: 4020
```

Figure 26: nocarryover case

As shown in the figure above, probabilityTransferMatrix can't beat antiRandom, which clarifies that probabilityTransferMatrix is not a better strategy.

C. furtherRefined

So far, the best strategy is antiTheoretical. But how do we further refine it?

Notice that there are some conditions that we should win some cards.

For example, if the current upcard is larger than any remained upcard, while your unplayed card is no less than opponent's unplayed card, you should play your largest unplayed card to win the current upcard.

Now, an intuitive way of refining antiTheoretical is to add some detectors, which detects if some conditions meet, then do corresponding actions. Example above is one of these conditions.

Add this condition and reaction to antiTheoretical and name the new strategy furtherRefined.

Let Alice use furtherRefined and Bob use antiTheoretical. We got the result for 100 rounds:

```
Up Card: 9H (value = 9)
Calling antiTheoretical
Calling furtherRefined
Alice played: 9C
Bob played: 9D
        Played the same card - no points awarded
        Alice's points at round 100: 0 (Total for all rounds: 94)
        Bob's points at round 100: 0 (Total for all rounds: 201)

Alice's total points: 94
Bob's total points: 201
```

Figure 27: carryover case

```
Up Card: 5H (value = 5)
Calling antiTheoretical
Calling furtherRefined
Alice played: 9C
Bob played: 9D
        Played the same card - no points awarded
        Alice's points at round 100: 13 (Total for all rounds: 1252)
        Bob's points at round 100: 36 (Total for all rounds: 1632)

Alice's total points: 1252
Bob's total points: 1632
```

Figure 28: nocarryover case

The result demonstrates that furtherRefined is better than antiTheoretical.

To ensure that furtherRefined is so far the best one, run it against random, antiRandom, antiRandomKiller, antiRandomKillerKiller and probabilityTransferMatrix. We found furtherRefined beat all other. Thus, furtherRefined is the best strategy I know so far.

5 Conclusion

Strategy furtherRefined is the best.

6 Further work

The way I refined antiTheoretical to furtherRefined, is a general method to refine any given strategy, including strategy furtherRefined.

Thus, the further work could be discover more qualified conditions, similar to the example, and add it the existing best strategy, then a better strategy is developed.

Reference

ROSS, S. M. (1971). Goofspiel: The Game of Pure Strategy. Journal of Applied Probability. 8,.

GLENN C. RHOADS AND LAURENT BARTHOLDI (2012), http://arxiv.org/pdf/1202.0695.pdf