MARKOV CHAINS

SPICED Academy

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Outline

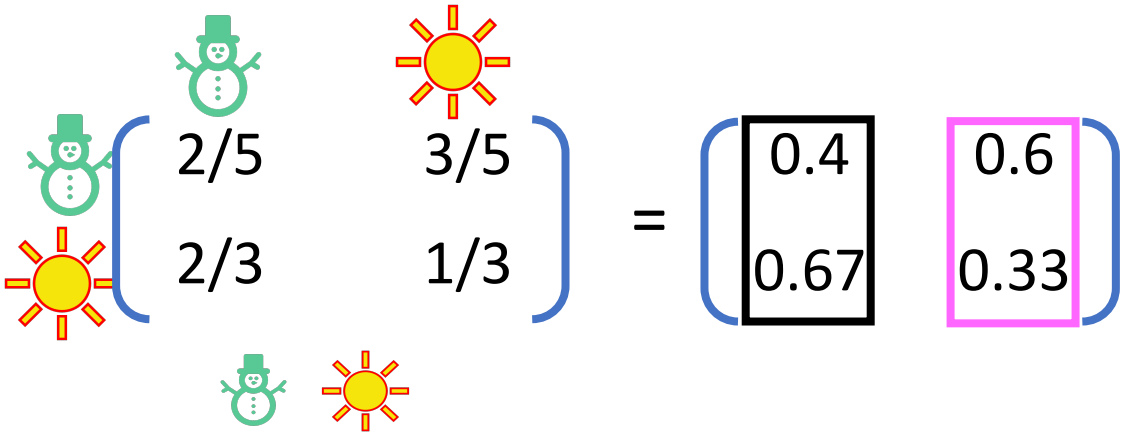
- Weather example
- Conceptual introduction
- Astrophysics example
- Coding example: Generating text
- Market share example
- Key details of MCMC
- Summary

Steady State Weather Forecast

- Training data

	Day 1	Day 2	Day 3	Day 4	Day 4	Day 5	Day 6	Day 8	Day 9
									

- Transition matrix:



- Initial state vector (today) = [1 0]
- Prediction: Initial state vector * Transition Matrix
- Tomorrow = [1 0] * [0.4 / 0.67] [1 0] * [0.6 / 0.33] = [0.4 0.6]

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- Markov Chain: A stochastic (random) process with Markov Property.

Markov Property: Current state is enough to determine the next state of the system (the previous sequence does NOT provide more information).

Markov Chain Monte Carlo in Astrophysics

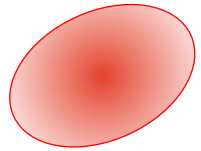
Estimating the mass M of the galaxy in the past before it lost matter to the Milky Way.

Parameter space

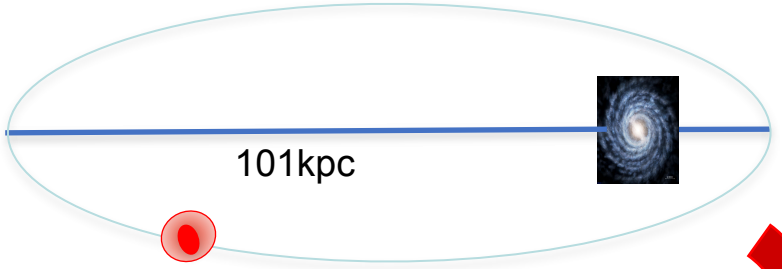
10^6 M < Mass < 10^{10} M

Radius < 5 kpc

Model with M and R



Simulation around the Milky Way



Is my model as destroyed as the real galaxy?

Accept/Reject Model

Random step In parameter space

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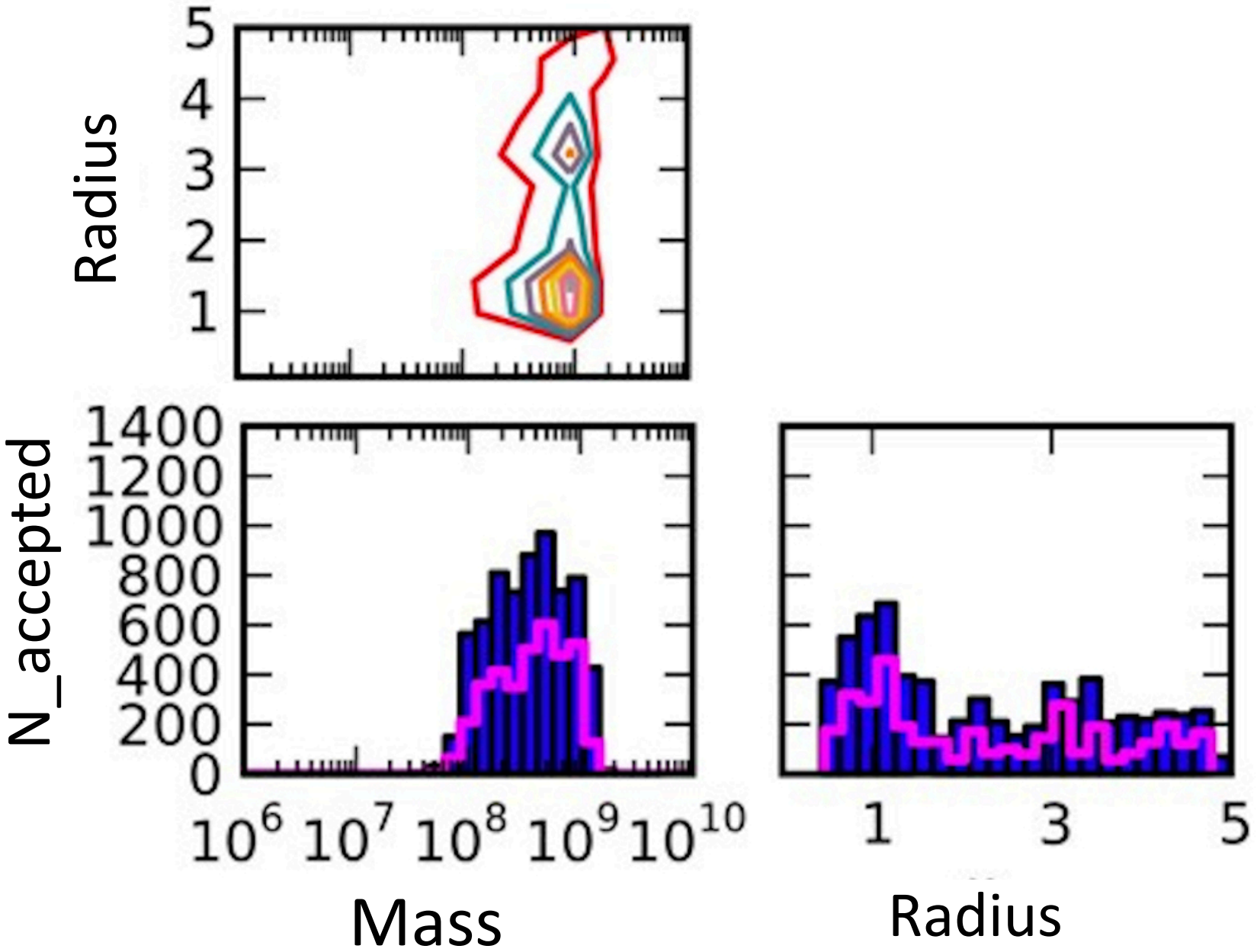

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Why Markov Chains?

- **Advantages:**
  - Prediction of probabilities
  - Simplicity due to Markov property. (Steady state weather)
  - Efficient sampling of the parameter space. (MCMC)
  - Distribution of good models in equilibrium state. (MCMC)

- **Disadvantages**:
  - Too simple for the real world (elections).
  - Computationally expensive.

Practical Example:

Generating Text

Market Share Example

Awareness Consideration Purchase No Purchase

- 1. vector_0 = [1500 2300 2000 3000]

- 2. matrix =

	A	C	P	N
A	0.7	0.1	0.12	0.08
C	0.05	0.3	0.55	0.1
P	0.01	0.05	0.7	0.24
N	0.5	0.2	0.15	0.15

Sum of probabilities =1

Probability of going from N to A

- vector_time_1 = vector_0 * matrix

A	C	P	N
vector_0 * Column A	vector_0 * Column C	vector_0 * Column P	vector_0 * Column N

- vector_time_5 = vector_0 * matrix ^{5}

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Summary

- Markov Chain: Memory-less stochastic process (past and future independent).
- MCMC: A Markov Chain which explores the parameter space with random variables.
- Simple probabilistic outcome prediction. (Too simple?)
- Or random sampling of a parameter space. (Too expensive?)
- Used in research & industry.

Other Examples

- Land-Use change dynamics [Müller & Middleton 1997]
- Election results in Ghana [Nortey et al. 2016]
- Google's PageRank algorithm (until last year).
- Speech recognition (Hidden Markov Model).
- Democratisation of a country.
- Text generation.
- Computer music composition.

Links

- A very nice & simple explanation of financial modelling: http://www.math.chalmers.se/Stat/Grundutb/CTH/mve220/1617/redingprojects16-17/IntroMarkovChainsandApplications.pdf
- Heavier explanation but good if you feel motivated: https://twiecki.io/blog/2015/11/10/mcmc-sampling/
- Musical signature of classical music composers: https://www.worldscientific.com/doi/pdf/10.1142/S2010194512007829
- A Hidden Markov Model with probabilistic states: https://en.wikipedia.org/wiki/Hidden_Markov_model
- Burn-in if you are into statistics: http://users.stat.umn.edu/~geyer/mcmc/burn.html
- Land use change in the Niagara Region https://link.springer.com/article/10.1007%2FBF00124382
- Market share example https://towardsdatascience.com/marketing-analytics-through-markov-chain-a9c7357da2e8
- Democratisation https://en.wikipedia.org/wiki/Markov_chain#Social_sciences