Assignments 3.3-Solution

一、阅读（Reading）

1. 阅读教材.

2. 课外阅读：

Predicate Logic（3）-by Gerard O'Regan.pdf.pdf

二、问题解答（Problems）

1. 教材 P53：题 14.

设个体域为全总个体域。令 a：小王；Y(x)：x 是一年级生；L(x)：x 是理科生；

W(x)：x 是文科生；F(x,y)：y 是 x 的辅导员，则推理可以形式化为

前提：

$\forall x(Y(x)\wedge\neg W(x)\rightarrow\exists yF(x,y))$,

$Y(a)$, $E(a)$,

$\forall x(F(a, x)\rightarrow L(x))$,

$\forall x(L(x)\rightarrow\neg W(x))$

结论：

$\exists x\exists y(\neg W(x)\wedge F(x, y))$

2. 教材 P53：题 15（5，6）.

（5）设个体域为全总个体域。令 M(x)：x 是人；C(x)：x 长期吸烟；K(x)：x 长期酗酒；J(x)：x 身体健康；P(x)：x 能参加体育比赛，则推理可以形式化为：

$\forall x((M(x)\wedge(C(x)\vee K(x)))\rightarrow\neg J(x))$, $\forall x((M(x)\wedge\neg J(x))\rightarrow\neg P(x))$, $\exists x(M(x)\wedge P(x))$

$\Rightarrow\exists x(M(x)\wedge\neg K(x))$

(6) 设个体域为全总个体域。

令 $M(x)$：x 是人；$K(x)$：x 是科学工作者；$Q(x)$：x 勤奋；$T(x)$：x 聪明；$S(x)$：x 将获得成功；a：王大志，则推理可以形式化为：

$\forall x((M(x)\wedge K(x))\rightarrow Q(x)), \forall x((M(x)\wedge Q(x)\wedge T(x))\rightarrow S(x)), M(a)\wedge K(a)\wedge T(a)\Rightarrow S(a)$

3. Formalize and prove the following three statements(the domain of discourse is universe，论域为全总域).

(1) Every computer science major is a logical thinker.

John is a computer science major.

Therefore, there is some logical thinker.

Let $C(x)$ mean "x is a computer science major," let $L(x)$ mean "x is a logical thinker," and let the constant b mean "John."

$\forall x\,(C(x) \rightarrow L(x)) \wedge C(b) \rightarrow \exists x\, L(x).$

(2)All computer science majors are people.

Some computer science majors are logical thinkers.

Therefore, some people are logical thinkers.

$\forall x\,(C(x) \rightarrow P(x)) \wedge \exists x\,(C(x) \wedge L(x)) \rightarrow \exists x\,(P(x) \wedge L(x)).$

(3) Socrates is a philosopher.

All philosophers are human.

All humans are mortal.

Therefore Socrates is mortal.

$\forall x(P(x)\rightarrow H(x))\wedge\forall x(H(x)\rightarrow M(x))\wedge P(s)\rightarrow M(s).$

4. Consider the following problem. We know that horses are faster than dogs and that there is a greyhound that is faster than every rabbit. We know that Harry is a horse and that Ralph is a rabbit. Our job is to derive the fact that Harry is faster than Ralph.

Problem translated in FOPL:

$\forall x \ \forall y \ ((Horse(x) \wedge Dog(y)) \rightarrow Faster(x,y))$

$\exists y \ (Greyhound(y) \wedge (\forall z \ Rabbit(z) \rightarrow Faster(y,z)))$

Horse(Harry)

Rabbit(Ralph)

Derive the following fact:

Faster(Harry, Ralph)

Added axioms to represent commonsense knowledge:

$\forall y \ (Greyhound(y) \rightarrow Dog(y))$

$\forall x \ \forall y \ \forall z \ ((Faster(x,y) \wedge Faster(y,z)) \rightarrow Faster(x,z))$

Proving using Proof Theory and a set of inference rules

1.	$\forall x \ \forall y \ Horse(x) \wedge Dog(y) \longrightarrow Faster(x,y)$	Premise
2.	$\exists y \ Greyhound(y) \wedge (\forall z \ Rabbit(z) \longrightarrow Faster(y,z))$	Premise
3.	$\forall y \ Greyhound(y) \longrightarrow Dog(y)$	Premise
4.	$\forall x \forall y \forall z \ Faster(x,y) \wedge Faster(y,z) \longrightarrow Faster(x,z)$	Premise
5.	Horse(Harry)	Premise
6.	Rabbit(Ralph)	Premise
7.	$Greyhound(Greg) \wedge (\forall z \ Rabbit(z) \longrightarrow Faster(Greg,z))$	ES (2)
8.	Greyhound(Greg)	T,I (7)
9.	$\forall z \ Rabbit(z) \longrightarrow Faster(Greg,z))$	T,I (7)

10.	Rabbit(Ralph) $\longrightarrow$ Faster(Greg,Ralph)	US (9)
11.	Faster(Greg,Ralph)	T,I (6),(10)
12.	Greyhound(Greg) $\longrightarrow$ Dog(Greg)	US (3)
13.	Dog(Greg)	T,I (12), (8)
14.	Horse(Harry) $\wedge$ Dog(Greg) $\longrightarrow$ Faster(Harry, Greg)	US (1)
15.	Horse(Harry) $\wedge$ Dog(Greg)	T,I (5), (13)
16.	Faster(Harry, Greg)	T,I (14), (15)
17.	Faster(Harry, Greg) $\wedge$ Faster(Greg, Ralph) $\longrightarrow$ Faster(Harry,Ralph)	US (4)
18.	Faster(Harry, Greg) $\wedge$ Faster(Greg, Ralph)	T,I (11), (16)
19.	Faster(Harry,Ralph)	T,I (17), (19)

Using Resolution to determine logical entailment

1.	{¬Horse(x),¬Dog(y),Faster(x,y)}	Premise
2.	{Greyhound(Greg)}	Premise
3.	{¬Rabbit(z),Faster(Greg,z)}	Premise
4.	{¬Greyhound(y),Dog(y)}	Premise
5.	{¬Faster(x,y), ¬Faster(y,z),Faster(x,z)}	Premise
6.	{Horse(Harry)}	Premise
7.	{Rabbit(Ralph)}	Premise
8.	{¬Faster(Harry, Ralph)}	Negated Goal
9.	{Dog(Greg)}	2, 4
10.	{¬Dog(y), Faster(Harry, y)}	6, 1
11.	{Faster(Harry, Greg)}	9, 10
12.	{Faster(Greg, Ralph)}	7, 3
13.	{¬Faster(Greg, z), Faster(Harry, z)}	11, 5
14.	{Faster(Harry, Ralph)}	12, 13
15.	{}	14, 8