课后作业（Assignments）

一、阅读（Reading）

1. 阅读教材.

2. 课外阅读：

Propositional Logic（3）-by Gerard O’ Regan.pdf

二、问题解答（Problems）

1. 教材 P30：题 13（2）

2. 教材 P30：题 14（1）

3. 教材 P30：题 15（5）

4. 教材 P30：17 题

5. Prove that the following rule, called the Destructive Dilemma rule, can be derived from the original and derived proof rules.

Premises: $\neg C \vee \neg D$, $A \rightarrow C$, $B \rightarrow D$

Conclusion: $\neg A \vee \neg B$.

Textbook: (假言易位) $A \rightarrow C \equiv \neg C \rightarrow \neg A$, $B \rightarrow D \equiv \neg D \rightarrow \neg B$, ...

Or：按照补充形式逻辑系统——

1.	$\neg C \vee \neg D$	P
2.	$A \to C$	P
3.	$B \to D$	P
4.	$\quad \neg C$	P [for $\neg C \to \neg A \vee \neg B$]
5.	$\quad \neg A$	2, 4, MT
6.	$\quad \neg A \vee \neg B$	5, Add
7.	$\neg C \to \neg A \vee \neg B$	4–6, CP
8.	$\quad \neg D$	P [for $\neg D \to \neg A \vee \neg B$]
9.	$\quad \neg B$	3, 8, MT
10.	$\quad \neg A \vee \neg B$	5, Add
11.	$\neg D \to \neg A \vee \neg B$	8–10, CP
12.	$\neg A \vee \neg B$	1, 7, 11, Cases
	QED	1–3, 7, 11, 12, CP.

6. Two students came up with the following different wffs to formalize the statement "If A then B else C."

$(A \wedge B) \vee (\neg A \wedge C)$.

$(A \to B) \wedge (\neg A \to C)$.

Prove that the two wffs are equivalent by finding formal proofs for the following two statements.

a. $((A \wedge B) \vee (\neg A \wedge C)) \to ((A \to B) \wedge (\neg A \to C))$.

b. $((A \to B) \wedge (\neg A \to C)) \to ((A \wedge B) \vee (\neg A \wedge C))$.

a. CP?

b. 变换右边?

7. Consider, for example, the following argument that aims to prove that Superman does not exist.

If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil he would be impotent; if he were unwilling to prevent evil he would be malevolent; Superman does not prevent *evil; If superman exists he is neither malevolent nor impotent. Therefore Superman does not exist.*

（参考阅读材料）

First, letters are employed to represent the propositions as follows:

a: Superman is able to prevent evil
w: Superman is willing to prevent evil
i: Superman is impotent
m: Superman is malevolent
p: Superman prevents evil
e: Superman exists

Then, the argument above is formalized in propositional logic as follows:

Premises	
P_1	$(a \wedge w) \rightarrow p$
P_2	$(\neg a \rightarrow i) \wedge (\neg w \rightarrow m)$
P_3	$\neg p$
P_4	$e \rightarrow \neg i \wedge \neg m$
Conclusion	$P_1 \wedge P_2 \wedge P_3 \wedge P_4 \Rightarrow \neg e$

直接证明/间接证明（反证法）

Proof that Superman does not exist

1.	$a \wedge w \rightarrow p$	*Premise* 1
2.	$(\neg a \rightarrow i) \wedge (\neg w \rightarrow m)$	*Premise* 2
3.	$\neg p$	*Premise* 3
4.	$e \rightarrow (\neg i \wedge \neg m)$	*Premise* 4
5.	$\neg p \rightarrow \neg(a \wedge w)$	1, *Contrapositive*
6.	$\neg(a \wedge w)$	*3, 5 Modus Ponens*
7.	$\neg a \vee \neg w$	*6, De Morgan's Law*
8.	$\neg (\neg i \wedge \neg m) \rightarrow \neg e$	*4, Contrapositive*
9.	$i \vee m \rightarrow \neg e$	*8, De Morgan's Law*
10.	$(\neg a \rightarrow i)$	*2, $\wedge$ Elimination*
11.	$(\neg w \rightarrow m)$	*2, $\wedge$ Elimination*
12.	$\neg \neg a \vee i$	10, A→ B equivalent to ¬A∨ B
13.	$\neg \neg a \vee i \vee m$	11, ∨ Introduction
14.	$\neg \neg a \vee (i \vee m)$	
15.	$\neg a \rightarrow (i \vee m)$	14, A → B equivalent to ¬A∨ B
16.	$\neg \neg w \vee m$	11, A → B equivalent to ¬A∨ B
17.	$\neg \neg w \vee (i \vee m)$	
18.	$\neg w \rightarrow (i \vee m)$	17, A → B equivalent to ¬A∨ B
19.	$(i \vee m)$	7, 15, 18 ∨*Elimination*
20.	$\neg e$	*9, 19 Modus Ponens*

Second Proof

1.	$\neg p$	P_3
2.	$\neg(a \wedge w) \vee p$	P_1 $(A \rightarrow B \equiv \neg A \vee B)$
3.	$\neg(a \wedge w)$	1, 2 $A \vee B, \neg B \vdash A$
4.	$\neg a \vee \neg w$	3, De Morgan's Law
5.	$(\neg a \rightarrow i)$	P_2 ($\wedge$-Elimination)
6.	$\neg a \rightarrow i \vee m$	5, $x \rightarrow y \vdash x \rightarrow y \vee z$
7.	$(\neg w \rightarrow m)$	P_2 ($\wedge$-Elimination)
8.	$\neg w \rightarrow i \vee m$	7, $x \rightarrow y \vdash x \rightarrow y \vee z$
9.	$(\neg a \vee \neg w) \rightarrow (i \vee m)$	8, $x \rightarrow z, y \rightarrow z \vdash x \vee y \rightarrow z$
10.	$(i \vee m)$	4, 9 Modus Ponens
11.	$e \rightarrow \neg(i \vee m)$	P_4 (De Morgan's Law)
12.	$\neg e \vee \neg (i \vee m)$	11, $(A \rightarrow B \equiv \neg A \vee B)$
13.	$\neg e$	10, 12 $A \vee B, \neg B \vdash A$

Therefore, the conclusion that Superman does not exist is a valid deduction from the given premises.

三、项目实践（Programming）(Optional)

编程实现一个命题逻辑推理系统：输入形式化的推理描述，计算机自动判定该推理是否有效.