Assignments 10.2

一、阅读（Reading)

1. 阅读教材.

2. 课外阅读：

Abstract Algebra-Morphisms（by James L. Hein）.pdf

二、问题解答（Problems)

1. 教材第七章习题：题 12、13、15、19.

2. Find the three morphisms(定义 3 个同态映射) that exist from the algebra $<N_3; +_3>$ to the algebra $<N_6; +_6>$.

3 个同态映射分别为 f, g, and h:

$f(0) = 0, f(1) = 0, f(2) = 0;$

$g(0) = 0, g(1) = 2, g(2) = 4;$

$h(0) = 0, h(1) = 4, h(2) = 2.$

3. Show that there is an epimorphism(满同态) between the set B of binary numerals(二进制数) with the usual binary addition(一般二进制加法) defined on B and the set N of natural numbers with the usual addition on N.

Tips：设$+_{bi}$、+分别为 B、N 上二进制加法与普通加法运算，显然，容易证明运算满足封闭性. 进一步可以定义 f_{two} 为 B 到 N 的映射: $f_{two}(b_k b_{k-1} \ldots b_1 b_0) = 2^k b_k + 2^{k-1} b_{k-1} + \ldots + 2^1 b_1 + b_0$.，可以证明 f_{two} 为满射，且满足同态方程: $f_{two}(x +_{bi} y) = f_{two}(x) + f_{two}(y)$. 故代数结构 $<B; +_{bi}>$到 $<N; +>$存在满同态关系.

4. Suppose we define f : Z → Q by $f(n) = 2^n$.

+ is usual addition operation on Z and and ○ is usual multiplication on Q; - is negation operation(求负数运算) on Z and *inv* is inverse (求倒数).

Show that

a. f is a monomorphism(单同态) from the algebra <Z; +> to the algebra <Q; ○>.

b. f is a monomorphism from <Z; +, - > to <Q; ○, *inv*>.

Tips：注意到 f 是 Z 到 Q 的单射，但不是满射，且：

$f(n + m) = 2^{n+m} = 2^n \circ 2^m = f(n) \circ f(m)$,

$f(-n) = 2^{-n} = (2^n)^{-1} = inv(f(n))$.

5.

a. Show that $<N_k; +_k>$ is a semigroup(半群) .

b. Let ○ be the binary operation over {a, b, c} defined by the following table. Show that ○ is associative by finding an isomorphism(同构) of the two algebras <{a, b, c}; ○> and $<N_3; +_3>$.

○	a	b	c
a	c	a	b
b	a	b	c
c	b	c	a

Tips：

a. 封闭性是显然的。$\forall a,b,c \in N_k$

$(a+_k b)+_k c=(a+b-n_1k)+_k c=(a+b-n_1k+c)-n_2k=(a+b+c)-(n_1+n_2)k$

(其中，n_1 满足 $0 \le (a+b)-n_1k<k$，进而 n_2 满足 $0 \le (a+b-n_1k+c)-n_2k<k$)

$=(a+b+c)(\bmod k)$.

类似地，

$a+_k(b+_kc)=a+_k(b+c-m_1k)=(a+b-m_1k+c)-m_2k=(a+b+c)-(m_1+m_2)k$

(其中，m_1 满足 $0\le(b+c)-m_1k<k$，进 m_2 满足 $0\le(a+b-m_1k+c)-m_2k<k$)

$=(a+b+c)(\bmod k)$.

从而，$(a+_kb)+_kc=a+_k(b+_kc)$，N_k 上运算 $+_k$ 满足结合律。

b. 定义 f: {a, b, c} → N_3，其中 f(b)=0,f(a)=1, f(c)=2，或 f(b)=0,f(a)=2, f(c)=1，可以证明 f 为<{a, b, c}; ○>到<N_3; $+_3$>的同构映射. 根据(a)，N_3 上运算 $+_3$ 满足结合律，故{a, b, c}上运算○也满足结合律.

三、项目实践（Programming）（Optional）

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