课后作业 3.1 (Assignments)

一、阅读 (Reading)

- 1. 阅读教材.
- 2. 课外阅读:
- Predicate Logic (1) -by Gerard O' Regan.pdf.pdf

二、问题解答 (Problems)

- 1. 教材 P51: 题 1 (1, 3, 5, 7)
- 2. 教材 P51: 题 5;
- 3. 教材 P52: 题 7;
- 4. 教材 P52: 题 10;
- 5. 教材 P52: 题 11 (1, 3, 5);
- 6. 教材 P52: 题 12 (1, 3, 5);
- 7. Given the wff W = $\exists x p(x) \rightarrow \forall x p(x)$.
- a. Find all possible interpretations of W over the domain $D = \{a\}$. Also give the truth value of W over each of the interpretations.
- b. Find all possible interpretations of W over the domain D = $\{a, b\}$. Also give the truth value of W over each of the interpretations.
- 8. Find a model for each of the following wffs.
- a. $p(c) \land \exists x \neg p(x)$.
- b. $\exists x p(x) \rightarrow \forall x p(x)$.
- c. $\exists y \forall x p(x, y) \rightarrow \forall x \exists y p(x, y)$.

d. $\forall x \exists y p(x, y) \rightarrow \exists y \forall x p(x, y)$.

e.
$$\forall x (p(x, f(x)) \rightarrow p(x, y)).$$

- 9. Given the wff W = $\forall x p(x, x) \rightarrow \forall x \forall y \forall z (p(x, y) \lor p(x, z) \lor p(y,z))$.
- a. Show that W is true for any interpretation whose domain is a singleton.
- b. Show that W is true for any interpretation whose domain has two elements.
- c. Show that W is not valid.
- d. Find an example of a wff that is true for any interpretation that has a domain with three or fewer elements but is not valid.
- 10. Prove that each of the following wffs is valid, unsatisfiable, or invalid.
- a. $\forall x (p(x) \rightarrow p(x))$.
- b. $\exists x (p(x) \land \neg p(x)).$
- c. $\exists x \forall y (p(x, y) \land \neg p(x, y)).$
- d. $\forall x \ A(x) \ \lor \ \forall x \ B(x) \ \rightarrow \ \forall x \ (A(x) \ \lor \ B(x)).$
- e. $\forall x (A(x) \rightarrow B(x)) \rightarrow (\forall x A(x) \rightarrow \exists x B(x)).$

三、项目实践 (Programming) (Optional)

无.