

# 高数常用公式

## 基础不定积分公式

$$\int k dx = kx + C$$

$$\int x^\mu dx = \frac{x^{\mu+1}}{\mu+1} + C \quad \mu \neq -1$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int \frac{1}{1+x^2} dx = \arctan x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int e^x dx = e^x + C$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + C$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \tan x dx = -\ln|\cos x| + C$$

$$\int \cot x dx = \ln|\sin x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x dx = \ln|\csc x - \cot x| + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$$

## 重要的极限

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

## 等价无穷小替换

当  $x \rightarrow 0$  时

$$x \sim \sin x \sim \arcsin x$$

$$x \sim \tan x \sim \arctan x$$

$$x \sim e^x - 1 \sim \ln(1+x)$$

$$(1 - \cos x) \sim \frac{1}{2}x^2$$

$$[(1+x)^\alpha - 1] \sim \alpha x$$

$$(a^x - 1) \sim x \ln a$$

$$\log_a(1+x) \sim \frac{1}{\ln a} x$$

## 万能公式

$$\sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$\tan \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 - \tan^2 \frac{\alpha}{2}}$$

## 基础导数公式

$$\frac{d}{dx} x = 1$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$\frac{d}{dx} a^x = a^x \ln a$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x} \quad x > 0$$

$$\frac{d}{dx} \log_a(x) = \frac{1}{x \ln a}$$

## 积化和差公式

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\sin \alpha \sin \beta = -\frac{1}{2} [\cos(\alpha + \beta) - \cos(\alpha - \beta)]$$

## 和差化积公式

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$

## 两角和与差

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

## 倍角公式

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\begin{aligned} \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 \\ &= 1 - 2 \sin^2 \alpha \end{aligned}$$

## 莱布尼茨定理

$$\begin{aligned} (u \cdot v)^n &= u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \cdots + \frac{n(n-1) \cdots (n-k+1)}{k!} u^{n-k} v^k + \cdots + u v^n \\ &= \sum_{k=0}^n C_n^k u^{n-k} v^k \end{aligned}$$

## 整数幂的差

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + a^{n-4}b^3 + \cdots + ab^{n-2} + b^{n-1}) \quad n > 0$$

## 泰勒公式

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \cdots + \frac{f^n(x_0)}{n!}(x - x_0)^n + R_n(x)$$

$$\text{其中 } R_n(x) = \frac{f^{n+1}(\xi)}{(n+1)!}(x - x_0)^{n+1} \quad (\xi \text{ 在 } x_0 \text{ 与 } x \text{ 之间})$$

## 麦克劳林公式

带有拉格朗日型余项

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^n(0)}{n!}x^n + \frac{f^{n+1}(\theta x)}{(n+1)!}x^{n+1} \quad (0 < \theta < 1)$$

带有皮亚诺型余项

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^n(0)}{n!}x^n + o(x^n)$$

常用函数的麦克劳林公式

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + o(x^n)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1})$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n})$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots + (-1)^{n-1} \frac{1}{n}x^n + o(x^n)$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + o(x^n)$$

$$\begin{aligned} (1+x)^m &= 1 + mx + \frac{m(m-1)}{2!}x^2 + \cdots \\ &\quad \cdots + \frac{m(m-1) \cdots (m-n+1)}{n!}x^n + o(x^n) \end{aligned}$$

## 二项式定理

对任何正整数  $n$

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{1 \cdot 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}a^{n-3}b^3 + \cdots + nab^{n-1} + b^n$$