Chapter 0: Functions

Summary: This chapter sets out to describe mathematical functions. This idea is central to the rest of the book as virtually all concepts will be framed in the context of a function. The important concepts that are covered here are the domain and range of functions, creating new functions by combining functions and representing functions parametrically. Along the way, families of functions such as polynomials, rational functions, exponential functions, logarithmic functions, and trigonometric functions are introduced.

OBJECTIVES: After reading and working through this chapter you should be able to do the following:

1. Identify a function and its domain and range (§0.1).
2. Combine functions arithmetically or through composition to create new functions (§0.2).
3. Modify functions through translation, stretching either the inputs or the outputs or using an appropriate reflection (§0.2).
4. Know the properties of power functions, polynomials, rational functions, exponential functions, logarithmic functions, and trigonometric functions (§0.3, 0.5).
5. Find the inverse of a function if it is possible to do so (§0.4).

0.1 Functions

PURPOSE: To describe what functions are.

In this section, the main idea of a function is developed. Here the main discussion is about the domain and range of a function. In particular, functions are described as having the relationship that for each **input** to a function there is a **unique output** value. A clear distinction is made between the inputs and outputs in a function relationship. Inputs are referred to as the independent variable and outputs are the dependent variable.

input $\rightarrow$ unique output

domain and range

Typically, the **domain** of a function is found by realizing what inputs (or x-values) will give valid outputs (or y-values). For example, since the square root $\sqrt{x}$ does not give a real number value for $x < 0$ then $y = \sqrt{x}$ would only have a domain of $x \geq 0$. The **range** of the function is directly dependent upon the domain (hence y is the dependent variable). Whatever values are obtained using the valid inputs (the domain) results in the appropriate range.

Things to watch out for when inspecting domains are:

1. Division by zero,
2. Functions that only accept certain inputs (i.e., $\sqrt{x}$ or $\ln x$),
3. Any restrictions given explicitly.

vertical line test

The **vertical line test** is used to determine if a relationship between x- and y-values is actually a function. The convention is that the inputs are usually given along the horizontal axis (x-axis) and that the outputs are given along the vertical axis (y-axis). So the vertical line test will indicate when any given input will give you more than one output which would result in a nonfunction. An example is $x^2 + y^2 = 1$. When $x = 1/2$ then $y = \sqrt{3}/2$ or $y = -\sqrt{3}/2$. So this fails the vertical line test and is not a function.

Checklist of Key Ideas:

- ☐ function
- ☐ input x and output y
- ☐ independent variable
- ☐ dependent variable
- ☐ graph
- ☐ zero/root of a graph (also x-intercept)
- ☐ vertical line test
- ☐ absolute value or magnitude
- ☐ piecewise functions
- ☐ domain and range

0.2 New Functions from Old

PURPOSE: To use combinations of "simple" functions to create and graph more complex functions.

The primary focus in this section is upon the combination of functions to create new functions. The arithmetic combination of functions (such as $f(x)+g(x)$ or $f(x)g(x)$) is discussed as well as the **composition** of functions (such as $f \circ g(x) = f(g(x))$). **Translations** and **stretching** of functions are considered where a constant value c is used in some way to modify either the input, x, of a function or the output, $f(x)$, of a function. Pay special attention to how the constant value of c is used. In cases where c is used to modify the input of the function such as $f(cx)$ or $f(x+c)$ you will see the function change in its horizontal behavior. This is because the inputs are given along the horizontal axis. Likewise, when c is applied to the output value such as $cf(x)$ or $f(x)+c$ then this will affect the vertical behavior of the function.

composition: $f(g(x))$
translation: $f(x)+C$ and $f(x+C)$
stretching: $cf(x)$ and $f(cx)$

The behavior of some functions is also described in terms of **symmetry**. The main types of symmetry to be aware of are **even** functions which are symmetric across the y-axis and **odd** functions which are symmetric across the origin. Even functions have the property that $f(x) = f(-x)$ for all x-values while odd functions have the property that $f(-x) = -f(x)$ for all $x-$values.

symmetry, even, odd

The function $y = x^2$ is an example of simple even function and $y = x^3$ is an example of a simple odd function. These are also good functions to experiment with when learning about translations and stretching. For example, using either $y = f(x) = x^2$ or $y = f(x) = x^3$ try plotting the following functions:

1. $f(x+2)$ or $f(x-2)$
2. $f(2x)$ or $f(x/2)$
3. $f(x)+2$ or $f(x)-2$
4. $2f(x)$ or $f(x)/2$

Composition is also easy to learn if you use some simple functions and remember the difference between inputs and outputs. The idea of composition is to use the output of one function as the input for another. Try it with $f(x) = x-2$ and $g(x) = 1/(x+1)$. What are $f \circ g(x)$ and $g \circ f(x)$? How do these compositions change the domains and ranges of the functions?

Checklist of Key Ideas:

☐ composition

☐ "inside" and "outside" functions

☐ vertical and horizontal shifts or translations

☐ vertical and horizontal stretching or compressing

☐ symmetry

☐ even and odd functions

0.3 Families of Functions

PURPOSE: To describe some common groups of functions that are similar to one another.

Many functions have similar definitions and so they will have similar behaviors. A group of functions with similar definitions may be called a family of functions (i.e., they are similar to each other and so they are in the same family). Types of functions that are described here are **power functions** (with both positive and negative exponents), **polynomials**, **rational functions** and some basic trigonometric functions.

power functions: $y = x^n$

Power functions $y = x^n$ where n is an integer, will either be even functions (if n is even) or odd functions (if n is odd). Even power functions will all go through the points $(-1,1)$ and $(1,1)$ (showing symmetry across the y-axis) while odd power functions will all go through the points $(-1,-1)$ and $(1,1)$.

If $n > 0$ then power functions will approach $+\infty$ as x grows large positively. The behavior as x grows large negatively can be determined by their symmetry.

IDEA: As x gets large, polynomials and rational functions also behave like power functions.

rational function = $\dfrac{\text{numerator}}{\text{denominator}}$

Power functions, polynomials and rational functions are very closely related. This is because polynomials are made up of power functions with positive integer powers and rational functions are a ratio of polynomial functions. The dominant term of a polynomial (the one with the highest power of x) will control the behavior of the polynomial as x becomes very large negatively or positively. Likewise, the behavior of a rational function is controlled by its dominant terms in both the numerator and the denominator.

continuity, see §1.5

While polynomials are unbroken curves (or **continuous** as described in Section 1.5), rational functions can have either holes or vertical asymptotes at x-values which cause division by zero.

period, amplitude

The trigonometric functions $y = A\sin(Bx - C)$ and $y = A\cos(Bx - C)$ are periodic functions meaning that they repeat themselves after a certain period of input values. This **period** is equal to $2\pi/|B|$. They also oscillate about the x-axis with an **amplitude** of $|A|$. These graphs are also shifted horizontally to the left or right by C/B units depending upon whether $C/B < 0$ or $C/B > 0$ respectively.

Checklist of Key Ideas:

- ☐ family of functions
- ☐ parameters
- ☐ power functions
- ☐ proportionality constant

- ☐ polynomials
- ☐ coefficients
- ☐ degree
- ☐ linear, quadratic, cubic, quartic and quintic
- ☐ rational functions
- ☐ vertical and horizontal asymptotes
- ☐ algebraic functions
- ☐ amplitude, period and frequency

0.4 Inverse Functions; Inverse Trigonometric Functions

PURPOSE: To describe what an inverse function is and how it relates to a function.

Functions can often have a related function, f^{-1}, called an inverse function. Inverse here is meant to be a function that inverts its outputs back to the original inputs. For example, $f(f^{-1}(x)) = f^{-1}(f(x)) = x$. This is called the cancellation equation. In other words, if $y = f(x)$ goes through the point (a,b) then $y = f^{-1}(x)$ will go through the point (b,a). Another way of describing this property is to say that f and f^{-1} are reflections of each other across the line $y = x$. This kind of reflection does not always result in a function. If the resulting curve is a function then it is called the inverse function and denoted by f^{-1}.

<ins>reflection across $y = x$</ins>
If $y = f(x)$ goes through (a,b)
then $y = f^{-1}(x)$ goes through (b,a).

This section discusses when to expect that a function can have an inverse function and how to find its inverse. One particular strategy that is employed with trigonometric functions is to first restrict the domain of the function so that the resulting reflection across the line $y = x$ will be a function.

Finding $y = f^{-1}(x)$

1. solve for x in $y = f(x)$
   $\rightarrow\ x = g(y)$
2. swap x and $y \rightarrow y = g(x)$
3. then $g(x) = f^{-1}(x)$

<ins>Checklist of Key Ideas:</ins>

- ☐ inverse function
- ☐ cancellation equations
- ☐ domain and range of inverse functions
- ☐ finding the inverse of a function
- ☐ one-to-one or invertible
- ☐ horizontal line test
- ☐ increasing and decreasing functions
- ☐ restricting the domain
- ☐ domains of inverse trigonometric functions

0.5 Exponential and Logarithmic Functions

PURPOSE: To introduce the functions a^x and $\log_a x$.

exponential: $y = a^x$
logarithmic: $y = \log_a(x)$

Note:
$a^x = b \Leftrightarrow \log_a(b) = x$

Exponential and **Logarithmic** functions are introduced in this section. These are two types of functions that arise naturally as inverses of each other. One key idea to remember well is that a logarithm is simply a way of representing an exponent. Also because logarithms and exponential functions are inverses of each other, the cancellation equations result in these functions undoing each other.

$$a^{\log_a(x)} = x \text{ and } \log_a(a^x) = x$$

Checklist of Key Ideas:

- ☐ integer, rational and irrational powers
- ☐ exponential functions
- ☐ the number e and the natural exponential function
- ☐ representing exponents with logarithms
- ☐ logarithm with base b
- ☐ b^x and $\log_b x$ are inverse functions
- ☐ the natural logarithmic function
- ☐ change of base formulas for logarithms
- ☐ exponential and logarithmic growth

Chapter 0 Sample Tests

Section 0.1

1. Answer true or false. If $f(x) = 4x^2 - 1$, then $f(0) = 0$.
2. Answer true or false. If $f(x) = \dfrac{1}{x}$, then $f(0) = 0$.
3. $f(x) = \dfrac{1}{x^2 - 1}$. The natural domain of the function is
   (a) all real numbers
   (b) all real numbers except 1
   (c) all real numbers except -1 and 1
   (d) all real numbers except -1, 0 and 1
4. Use a graphing utility to determine the natural domain of $h(x) = \dfrac{1}{|x| - 1}$.
   (a) all real numbers
   (b) all real numbers except 1
   (c) all real numbers except -1 and 1
   (d) all real numbers except -1, 0 and 1
5. Use a graphing utility to determine the natural domain of $g(x) = \sqrt{9 - x^2}$.
   (a) $\{x : -3 \le x \le 3\}$
   (b) $\{x : -9 \le x \le 3\}$
   (c) $\{x : x \ge -9\}$
   (d) $\{x : x \ge -3\}$
6. Answer true or false. $f(x) = x - |x+1|$ can be represented in the piecewise form by $f(x) = \begin{cases} 2x+1, & \text{if } x \le -1 \\ -1, & \text{if } x > -1 \end{cases}$.
7. Find the x-coordinate of any hole(s) in the graph of $f(x) = \dfrac{(x^2 - 9)(x+4)}{(x+3)(x+4)}$.
   (a) 3
   (b) -3 and -4
   (c) 3 and 4
   (d) -12
8. Answer true or false. $f(x) = \dfrac{x^2 - 4}{x+2}$ and $g(x) = x - 2$ are identical except $f(x)$ has a hole at $x = -2$.
9. Use a graphing device to plot the function, then find the indicated value from observing the graph. A flying object attains a height $h(t)$ over time according to the equation $h(t) = 3\sin t - \cos t$. If $t = \dfrac{\pi}{4}$, then $h\left(\dfrac{\pi}{4}\right)$ is approximately
   (a) 0
   (b) 0.707
   (c) 1.414
   (d) 3
10. A box is made from a piece of sheet metal by cutting a square whose sides measure x from each corner. If the sheet of metal initially measured 20 cm by 10 cm, the volume of the box is given by
    (a) $V(x) = (20 - 2x)(10)x$ cm^3
    (b) $V(x) = (20 - 2x)(10 - x)x$ cm^3
    (c) $V(x) = (20 - 2x)(10 - 2x)x$ cm^3
    (d) $V(x) = (20 - 2x)(10 - 2x)(2x)$ cm^3
11. Answer true or false. At a given location on a certain day the temperature T in $^\circ F$ changed according to the equation $T(t) = t\sin\left(\dfrac{t\pi}{12}\right) + 4$ where t represents time in hours starting at midnight. The temperature at 6 P.M. was approximately $58^\circ F$.
12. The speed of a truck in miles/hour for the first 10 seconds after leaving a red light is given by $f(x) = \dfrac{x^2}{2}$. Find the speed of the truck 6 seconds after leaving the red light.
    (a) 18 miles/hour
    (b) 36 miles/hour
    (c) 3 miles/hour
    (d) 6 miles/hour
13. Find $f(2)$ if
    $f(x) = \left\{\dfrac{4}{x}, \text{ if} x < 2 \text{ and } 3x \text{ if } x \ge 2\right\}$.
    (a) 2
    (b) 4
    (c) 6
    (d) it cannot be determined
14. Determine all $x-$values where there are holes in the graph of $f(x) = \dfrac{x^2 - 4}{(x+2)^2(x-2)}$.
    (a) $-2, 2$
    (b) -2
    (c) 2
    (d) none
15. Assume the hourly temperature in $^\circ F$ starting at midnight is given by $T(x) = 36 - (x - 15)^2 + x$. Find the temperature at 3 P.M.

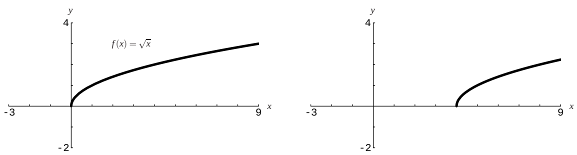
(a) $36^\circ F$

(b) $51^\circ F$

(c) $39^\circ F$

(d) $27^\circ F$

Section 0.2

1. 

The graph on the left is the graph of $f(x)=\sqrt{x}$. The graph on the right is the graph of

(a) $y = f(x+4)$

(b) $y = f(x-4)$

(c) $y = f(x)-4$

(d) $y = f(x)+4$

2. The graph of $y = 1+(x+2)^2$ is obtained from the graph of $y = x^2$ by

(a) translating horizontally 2 units to the right, then translating vertically 1 unit up

(b) translating horizontally 2 units to the left, then translating vertically 1 unit up

(c) translating horizontally 2 units to the right, then translating vertically 1 unit down

(d) translating horizontally 2 units to the left, then translating vertically 1 unit down

3. The graph of $y=\sqrt{x}$ and $y=\sqrt{-x}$ are related. The graph of $y=\sqrt{-x}$ is obtained by

(a) reflecting the graph of $y=\sqrt{x}$ about the x-axis

(b) reflecting the graph of $y=\sqrt{x}$ about the y-axis

(c) reflecting the graph of $y=\sqrt{x}$ about the origin

(d) The equations are not both defined.

4. The graphs of $y = x^3$ and $y = 4-2(x-1)^3$ are related. Of reflection, stretching, vertical translation, and horizontal translation, which should be done first?

(a) reflection

(b) stretching

(c) vertical translation

(d) horizontal translation

5. Answer true or false. $f(x) = x^2$ and $g(x) = x-2$. Then $(f-g)(x) = x^2-x-2$.

6. Answer true or false. $f(x) = x^2-9$ and $g(x) = x^3+8$. f/g has the same domain as g/f.

7. $f(x) = \sqrt{x^2+1}$ and $g(x) = x^2-2$. $f\circ g(x) =$

(a) $\sqrt{x^4-1}$

(b) $\sqrt{x^2-1}$

(c) $\sqrt{x^4-4x^2+5}$

(d) $\sqrt{x^2+2x-1}$

8. $h(x) = |x+3|^3$ can be written as the composition $h(x) = f\circ g(x)$ if

(a) $f(x) = x+3$ and $g(x) = |x^3|$

(b) $f(x) = (x+3)^3$ and $g(x) = |x^3|$

(c) $f(x) = \sqrt[3]{x+3}$ and $g(x) = |x^3|$

(d) $f(x) = x^3$ and $g(x) = |x+3|$

9. $f(x) = \sqrt[3]{x}$. Find $f(3x)$.

(a) $3\sqrt[3]{x}$

(b) $\sqrt[3]{3x}$

(c) $\dfrac{\sqrt[3]{x}}{3}$

(d) $\sqrt[3]{\dfrac{x}{3}}$

10. $f(x) = x^2+1$. Find $f(f(x))$.

(a) x^4+2

(b) $2x^2+2$

(c) x^4+2x^2+2

(d) x^4+2x^2

11. $f(x) = |x+2|$ is

(a) an even function only

(b) an odd function only

(c) both an even and an odd function

(d) neither an even nor an odd function

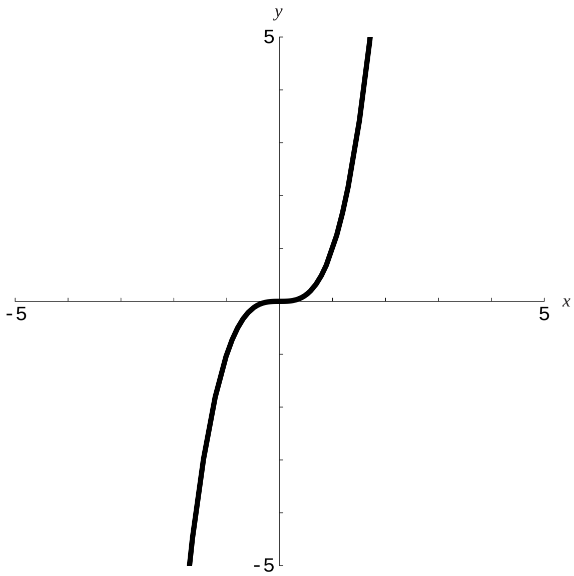
12. $f(x) = 0$ is

(a) an even function only

(b) an odd function only

(c) both an even and an odd function

(d) neither an even nor an odd function



13.
The function graphed above is

(a) an even function only
(b) an odd function only
(c) both an even and an odd function
(d) neither an even nor an odd function

14. Answer true or false. $f(x) = |x| + \cos x$ is an even function.

15. $f(x) = 3|x| + 2\cos x$ is symmetric about

(a) the x-axis
(b) the y-axis
(c) the origin
(d) nothing

16. $f(x) = 5x^3 - 2x$ is symmetric about

(a) the x-axis
(b) the y-axis
(c) the origin
(d) nothing

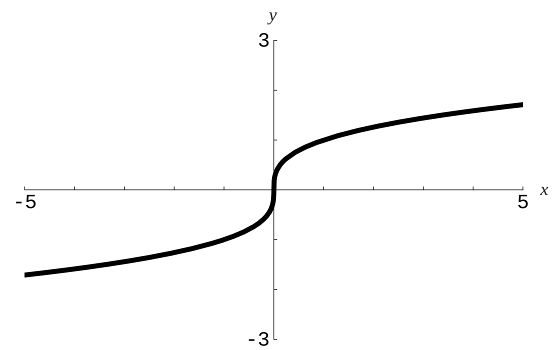
Section 0.3

1. What do all members of the family of lines of the form $y = 5x + b$ have in common?

(a) Their slope is 5.
(b) Their slope is -5.
(c) They go through the origin.
(d) They cross the x-axis at the point $(5,0)$.

2. What points do all graphs of the form $y = x^n$, n is odd, have in common?

(a) $(0,0)$ only
(b) $(0,0)$ and $(1,1)$
(c) $(-1,-1)$, $(0,0)$ and $(1,1)$
(d) none



3.
The equation whose graph is given is

(a) $y = \sqrt{x}$
(b) $y = \sqrt[3]{x}$
(c) $y = \dfrac{1}{x^2}$
(d) $y = \dfrac{1}{x^3}$

4. Answer true or false. The graph of $y = -3(x-4)^3$ can be obtained by making transformations to the graph $y = x^3$.

5. Answer true or false. The graph of $y = x^2 + 6x + 9$ can be obtained by transforming the graph of $y = x^2$ to the left three units.

6. Answer true or false. There is no difference in the graphs of $y = \sqrt{|x|}$ and $y = |\sqrt{x}|$.

7. Determine the vertical asymptote(s) of $y = \dfrac{x+3}{x^2+2x-8}$.

(a) $x = -4$, $x = 2$
(b) $x = 4$
(c) $x = -2$, $x - 4$
(d) $x = 8$

8. Find the vertical asymptote(s) of $y = \dfrac{x^6}{3x^6 - 3}$.

(a) $x = 0$
(b) $x = -1$, $x = 1$
(c) $x = 3$
(d) $x = \dfrac{1}{3}$

9. For which of the given angles, if any, is all of the trigonometric functions negative?
   (a) $\dfrac{\pi}{3}$
   (b) $\dfrac{2\pi}{3}$
   (c) $\dfrac{4\pi}{3}$
   (d) No such angle exists
10. Use the trigonometric function of a calculating utility set to the radian mode to evaluate $\sin\left(\dfrac{\pi}{7}\right)$.
   (a) 0.0078
   (b) 0.4339
   (c) 0.1424
   (d) 0.1433
11. Answer true or false. The amplitude of $5\cos\left(3x-\dfrac{\pi}{3}\right)$ is 3.
12. Answer true or false. The phase shift of $2\sin\left(x-\dfrac{\pi}{3}\right)$ is $\dfrac{\pi}{3}$.
13. Use a graphing utility to graph $y_1 = \sin\left(x-\dfrac{\pi}{3}\right)$ and $y_2 = \sin\left(2\left(x-\dfrac{\pi}{3}\right)\right)$.
   (a) y_1 has the greatest phase shift.
   (b) y_2 has the greatest phase shift.
   (c) y_1 and y_2 have the same phase shift.
   (d) Neither y_1 nor y_2 have a phase shift.
14. Answer true or false. The period of $y=\cos\left(5x-\dfrac{\pi}{3}\right)$ is 10π.
15. Answer true or false. A force acting on an object, $F=\dfrac{k}{x^2}$, that is inversely proportional to the square of the distance from the object to the source of the force is found to be 25 N when $x=1$ m. The force will be 100 N if x becomes 2 m.

Section 0.4

1. Answer true or false. The functions $f(x)=\sqrt[3]{x+3}$ and $g(x)=x^3-3$ are inverses of each other.
2. Answer true or false. The functions $f(x)=\sqrt[6]{x}$ and $g(x)=x^6$ are inverses of each other.
3. Answer true or false. The trigonometric function $y=\sec x$ is a one-to-one function.
4. Find $f^{-1}(x)$ if $f(x)=x^7$.
   (a) $\sqrt[7]{x}$
   (b) $\dfrac{1}{x^7}$
   (c) $-\sqrt[7]{x}$
   (d) $-\dfrac{1}{x^7}$
5. Find $f^{-1}(x)$ if $f(x)=3x-4$.
   (a) $\dfrac{1}{3x-4}$
   (b) $\dfrac{x+4}{3}$
   (c) $\dfrac{x}{3}+4$
   (d) $\dfrac{1}{3x}-4$
6. Find $f^{-1}(x)$ if $f(x)=\sqrt[9]{x+3}$.
   (a) x^9-3
   (b) $(x+3)^9$
   (c) x^9+3
   (d) $\dfrac{1}{\sqrt[9]{x+3}}$
7. For the function defined by

$$f(x)=\begin{cases} -x^2, & x<0 \\ x^2, & x\geq 0 \end{cases}$$

   which of the following is the inverse function $f^{-1}(x)$ (if it exists)?
   (a) $f^{-1}(x)=\begin{cases} -\sqrt{-x}, & x<0 \\ \sqrt{x}, & x\geq 0 \end{cases}$
   (b) $f^{-1}(x)=\begin{cases} -1/x, & x<0 \\ 1/x, & x\geq 0 \end{cases}$
   (c) $f^{-1}(x)=\sqrt{|x|}$
   (d) The inverse function does not exist.
8. Answer true or false. If f has a domain of $0\leq x\leq 10$, then f^{-1} has a domain of $0\leq x\leq 10$.
9. The graphs of f and f^{-1} are reflections of each other about the
   (a) x-axis
   (b) y-axis
   (c) line $y=x$
   (d) origin
10. Answer true or false. A 200 foot fence is used as a perimeter about a rectangular plot. The formula for the length of the fence is the inverse of the formula for the width of the fence.
11. Find the domain of $f^{-1}(x)$ if $f(x)=(x+3)^2$ for $x\geq -3$.
   (a) $x\geq -3$
   (b) $x\geq 3$

(c) $x \geq 0$

(d) $x \leq 0$

12. Find the domain of $f^{-1}(x)$ if $f(x) = -\sqrt{x-5}$.

(a) $x \leq 0$

(b) $x \geq 0$

(c) $x \leq 5$

(d) $x \geq 5$

13. Let $f(x) = x^2 + 6x + 9$. Find the smallest value of k such that $f(x)$ is a one-to-one function on the interval $[k, \infty)$.

(a) 0

(b) -3

(c) 3

(d) -9

14. Answer true or false. The function $f(x) = -x$ is its own inverse.

15. Answer true or false. To have an inverse, a trigonometric function must have its domain restricted to $[0, 2\pi]$.

16. Find the exact value of $\sin^{-1}(1)$.

(a) 0

(b) $\pi/2$

(c) π

(d) $3\pi/2$

17. Find the exact value of $\cos^{-1}(\cos(3\pi/4))$.

(a) $3\pi/4$

(b) $\pi/4$

(c) $-\pi/4$

(d) $5\pi/4$

18. Use a calculating utility to approximate x if $\tan x = 3.1$ and $-\pi/2 < x < \pi/2$.

(a) 1.2582

(b) 1.2588

(c) 1.2593

(d) 1.2595

19. Use a calculating utility to approximate x if $\sin x = 0.15$ and $\pi/2 < x < 3\pi/2$.

(a) 0.1506

(b) 2.9910

(c) 3.2932

(d) no solution

20. Answer true or false. $\cos^{-1} x = \dfrac{1}{\cos x}$ for all x.

21. Approximate $\sin(\sin^{-1} 0.3)$.

(a) 3.0000

(b) 0.3096

(c) 0.3000

(d) 0

22. A ball is thrown at 5 m/s and travels 2.45 m horizontally before coming back to its original height. Given that the acceleration due to gravity is 9.8 m/s^2, and air resistance is negligible, the range formula is $R = \dfrac{v^2}{9.8} \sin 2\theta$, where θ is the angle above the horizontal at which the ball is thrown. Find all possible positive angles in radians above the horizontal at which the ball can be thrown.

(a) 1.5708

(b) 1.5708 and 3.1416

(c) 0.7854 and 1.5708

(d) 0.7854

23. Answer true or false. $\sin^{-1} x$ is an odd function.

24. Answer true or false. $\tan^{-1}(1) + \tan^{-1}(2) = \tan^{-1}(-3)$.

Section 0.5

1. $2^{-5} =$

(a) $\dfrac{1}{10}$

(b) $-\dfrac{1}{10}$

(c) $\dfrac{1}{32}$

(d) $-\dfrac{1}{32}$

2. Use a calculating utility to approximate $\sqrt[6]{29}$. Round to four decimal places.

(a) 1.7528

(b) 5.3852

(c) 1.7530

(d) 5.3854

3. Use a calculating utility to approximate $\log 28.4$. Round to four decimal places.

(a) 3.3464

(b) 3.3462

(c) 1.4533

(d) 1.4535

4. Find the exact value of $\log_3 81$.
   (a) 12
   (b) $\frac{3}{4}$
   (c) $\frac{1}{4}$
   (d) 4

5. Use a calculating utility to approximate $\ln 39.1$ to four decimal places.
   (a) 1.5920
   (b) 3.6661
   (c) 1.5922
   (d) 3.6663

6. Answer true or false. $\ln \frac{a^3 b}{c^2} = 3\ln a + \ln b - 2\ln c$.

7. Answer true or false. $\log(5x\sqrt{x-2}) = (\log 5)(\log x)(\log^{1/2}(x-2))$.

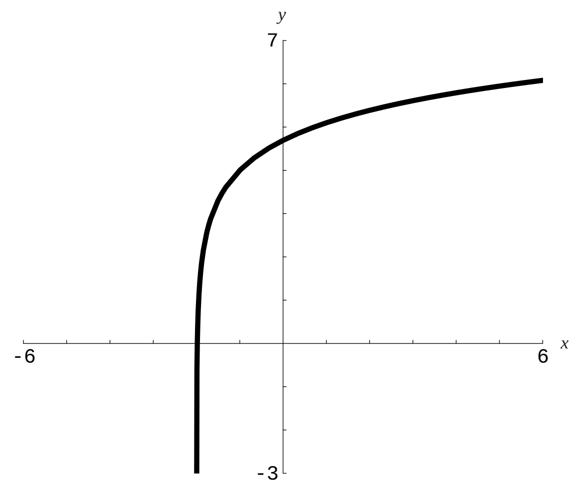
8. Rewrite $5\log 2 - \log 12 + \log 24$ as a single logarithm.
   (a) $\log 64$
   (b) $\log 22$
   (c) $\frac{\log 24}{5}$
   (d) $\log 44$

9. Solve $\log_{10}(x+5) = 1$ for x.
   (a) 5
   (b) -5
   (c) 0
   (d) no solution

10. If $\log_{10} x^{5/2} - \log_{10} x^{3/2} = 4$ then find x.
   (a) 4
   (b) 40
   (c) $1,000$
   (d) $10,000$

11. Solve $4^{-2x} = 6$ for x to four decimal places.
   (a) 0.6462
   (b) -0.6462
   (c) 1.2925
   (d) -1.2925

12. Solve for x if $3e^x - xe^x = 0$.
   (a) 3
   (b) -3
   (c) $\frac{1}{3}$
   (d) $-\frac{1}{3}$

13. 

This is a graph of
   (a) $4 - \ln(2+x)$
   (b) $4 + \ln(2+x)$
   (c) $4 - \ln(x-2)$
   (d) $4 + \ln(x-2)$

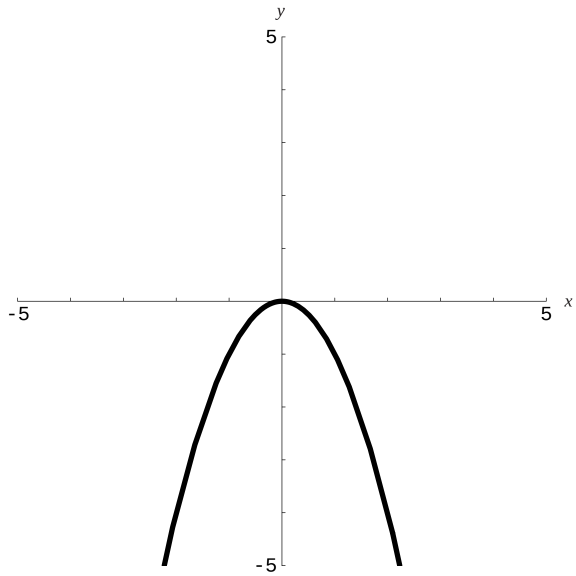
14. Use a calculating utility and the change of base formula to find $\log_3 4$.
   (a) 0.2007
   (b) 0.7925
   (c) 1.2619
   (d) 0.4621

15. The equation $Q = 6e^{-0.052t}$ gives the mass Q in grams of a certain radioactive substance remaining after t hours. How much remains after 6 hours?
   (a) 4.3919 g
   (b) 4.3920 g
   (c) 4.3921 g
   (d) 2.3922 g

Chapter 0 Test

1. Answer true or false. For the equation $y = x^2 + 10x + 21$, the values of x that cause y to be zero are 3 and 7.
2. Answer true or false. The graph of $y = x^2 - 4x + 9$ has a minimum value.
3. A company has a profit/loss given by $P(x) = 0.1x^2 - 2x - 10,000$, where x is the time in years, good for the first 20 years. After how many years will the graph of the profit/loss equation first begin to rise?
   (a) 5 years
   (b) 10 years
   (c) 15 years
   (d) 0 years
4. Find the natural domain of $g(x) = \dfrac{1}{|x-2|}$.
   (a) all real numbers
   (b) all real numbers except 2
   (c) all real numbers except -2
   (d) all real numbers except -2 and 2
5. Answer true or false. If $f(x) = \dfrac{2x}{x^2+1}$, then $f(1) = 1$.
6. Find the hole(s) in the graph of $f(x) = \dfrac{x-4}{x^2-5x+4}$.
   (a) $x = 4$
   (b) $x = 1$
   (c) $x = 1, 4$
   (d) $x = -4$
7. The cumulative number of electrons passing through an experiment over time, given in seconds, is given by $n(t) = t^2 + 4t + 6$. How many electrons pass through the experiment in the first 5 seconds?
   (a) 13
   (b) 39
   (c) 15
   (d) 51
8. What is the entire domain of $f(x) = \sqrt{49 - x^2}$?
   (a) all real numbers
   (b) $0 \le x \le 7$
   (c) $-7 \le x \le 7$
   (d) $0 \le x \le 49$
9. Answer true or false. The graph of $f(x) = |x+8|$ touches the x-axis.
10. Answer true or false. The graph of $y = \dfrac{x^2-8}{x-9}$ would produce a false line segment on a graphing utility on the domain of $-10 \le x \le 10$.
11. The graph of $y = (x-2)^3$ is obtained from the graph of $y = x^3$ by
    (a) translating vertically 2 units upward
    (b) translating vertically 2 units downward
    (c) translating horizontally 2 units to the left
    (d) translating horizontally 2 units to the right
12. If $f(x) = x^3 + 2$ and $g(x) = x^2$, then $g \circ f(x) =$
    (a) $x^3 + 2$
    (b) $(x^3+2)^2$
    (c) $x^3 + x^2 + 2$
    (d) $x^6 + 2$
13. Answer true or false. $f(x) = |x| + \sin x$ is an odd function.
14. A particle initially at $(1,3)$ moves along a line of slope $m = 4$ to a new position at (x, y). Find y if $x = 4$.
    (a) 12
    (b) 15
    (c) 16
    (d) 19
15. The slope-intercept form of a line having a slope of 5 and a y-intercept of 3 is
    (a) $x = 5y + 3$
    (b) $x = 5y - 3$
    (c) $y = 5x + 3$
    (d) $y = 5x - 3$
16. A spring is initially 3 m long. When 2 kg are suspended from the spring it stretches 4 cm. How long will the spring be if 10 kg are suspended from it?
    (a) 3.20 m
    (b) 3.02 m
    (c) 3.40 m
    (d) 3.04 m

17. The equation whose graph is given is



(a) $y = x^2$

(b) $y = (-x)^2$

(c) $y = -x^2$

(d) $y = x^{-2}$

18. Answer true or false. The only asymptote of $y = \dfrac{x+5}{x^2+6x+5}$ is $y = -1$.

19. Answer true or false. The functions $f(x) = \sqrt[3]{x-3}$ and $g(x) = x^3 + 3$ are inverses of each other.

20. If $f(x) = \dfrac{1}{x^3+2}$ then find $f^{-1}(x)$.

(a) $\sqrt[3]{x-2}$

(b) $\sqrt[3]{\dfrac{1-2x}{x}}$

(c) $\sqrt[3]{1+2x}$

(d) $\sqrt[3]{\dfrac{1+2x}{x}}$

21. Find the domain of $f^{-1}(x)$ if $f(x) = \sqrt{x-6}$.

(a) $x \geq 0$

(b) $x \leq 0$

(c) $x \geq 6$

(d) $x \geq -6$

22. Use a calculating utility to approximate $\log(41.3)$ to three decimal places.

(a) 1.610

(b) 1.613

(c) 1.616

(d) 1.618

23. Answer true or false. $\log \dfrac{ab^3}{\sqrt{c}} = \log a + 3\log b - \dfrac{1}{2}\log c$

24. If $5^{2x} = 8$ then solve for x.

(a) 1.292

(b) 0.646

(c) 0.204

(d) 0.102

25. Use a calculating utility to approximate x if $\sin x = 0.42$ and $3\pi/2 < x < 5\pi/2$.

(a) 6.715

(b) 6.717

(c) 6.719

(d) 6.723

Chapter 0: Answers to Sample Tests

Section 0.1

1. false	2. false	3. c	4. c	5. a	6. true	7. b	8. true
9. c	10. c	11. false	12. a	13. c	14. c	15. b	

Section 0.2

1. b	2. b	3. b	4. d	5. false	6. false	7. c	8. d
9. b	10. c	11. d	12. c	13. b	14. true	15. b	16. c

Section 0.3

1. a	2. c	3. b	4. true	5. true	6. false	7. a	8. b
9. d	10. b	11. false	12. true	13. c	14. false	15. false	

Section 0.4

1. true	2. false	3. false	4. a	5. b	6. a	7. a	8. false
9. c	10. true	11. c	12. a	13. b	14. true	15. false	16. b
17. a	18. b	19. b	20. false	21. c	22. d	23. true	24. false

Section 0.5

1. c	2. a	3. c	4. d	5. b	6. true	7. false	8. a
9. a	10. d	11. b	12. a	13. b	14. c	15. a	

Chapter 0 Test

1. false	2. true	3. b	4. b	5. true	6. a	7. d	8. c
9. true	10. true	11. d	12. b	13. false	14. b	15. c	16. a
17. c	18. false	19. true	20. b	21. a	22. c	23. true	24. b
25. b							