PHYS101
New Topic:
Fluid Mechanics

SJW^2 Ch 13 and 14

Fluid Statics, Hydraulics and Archimede’s principle

SJW^2: 13.1-13.5

What is a fluid?

- The 3 states of matter can be described as:
  - **Gas:** a substance that flows to take up the shape of its container AND totally fills the volume.
  - **Liquid:** a substance that flows to take up the shape of its container BUT does NOT fill the volume
  - **Solid:** a substance than has its own shape and volume.
- Fluids are substances that can flow, i.e. fluids can be either liquids or gases. Fluids can be compressible (gases), or incompressible (liquids).
- Fluids exert a force on their container but only NORMAL to the surface of the container and fluids CANNOT support a shear stress.

Density

- Definition of density

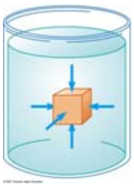
$$\boxed{\rho = \frac{m}{V} \left(= \frac{dm}{dV} \right)}$$

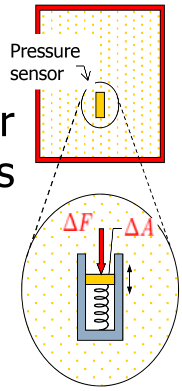
Unit is kg/m^3

Material	Density (kg/m^3)
Interstellar space	10^{-20}
Best lab vacuum	10^{-17}
Air (STP)	1.21
Water (20˚C)	1.00×10^3
Mercury (metal)	13.6×10^3
Gold	19.3×10^3
Earth (average)	5.5×10^3
Nucleus	$\sim 10^{17}$
Neutron star (core)	10^{18}

Pressure

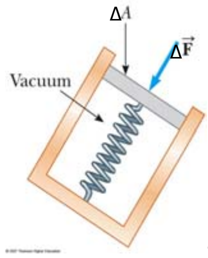
- Consider a small pressure sensor the fluid exerts a force ΔF that is normal to the area ΔA.





Define the pressure as

$$p = \frac{\Delta F}{\Delta A} \left(= \frac{dF}{dA} \right)$$



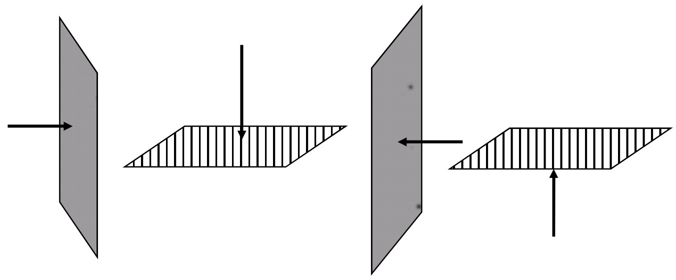
- Unit of pressure is N/m^2 = Pascal (Pa)

Problem

What happens when you walk over soft ground? Why will a woman's high heels sink into the soft ground, but the larger shoes of a bigger man will not?

Check your understanding

Consider each of the following surfaces at the same depth in the ocean. On which side of the surfaces would the pressure be greatest?



Check Understanding

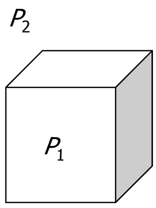
You are walking out on a frozen lake and you begin to hear the ice cracking beneath you. What is your best strategy for getting off the ice safely?

1) stand absolutely still and don't move a muscle
2) jump up and down to lessen your contact time with the ice
3) try to leap in one bound to the bank of the lake
4) shuffle your feet (without lifting them) to move toward shore
5) lie down flat on the ice and crawl toward shore

Note that it is only the magnitude of the force which appears in the $p=F/A$ relationship. Pressure is not a vector quantity. The direction of the force associated with the pressure of a fluid is always perpendicular to the surface on which the pressure acts.

If a box of fluid with pressure P_1 is immersed in fluid with pressure P_2

1. What force direction is felt by the surfaces of the box if P_1 is greater than P_2?
2. What force direction is felt by the surfaces of the box if P_2 is greater than P_1?

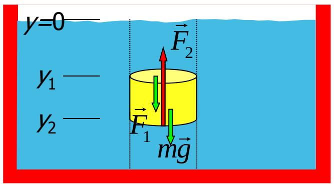


Removing the air from a container has very real consequences.

MYTHBUSTERS : Compressed diver

Hydrostatics

- Consider a small cylinder of fluid in *static equilibrium* (stationary and forces in balance).
- The fluid **above** the cylinder is exerting a force on area A equal to:



$$F_1 = m_1 g = \rho_F V_1 g = \rho_F A y_1 g \equiv P_1 A$$

- At depth y_2 there is a reaction force F_2 that is equal to the weight of ALL the fluid above this depth:

$$F_2 = m_2 g = \rho_F V_2 g = \rho_F A y_2 g \equiv P_2 A$$

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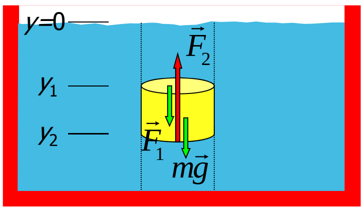
Hydrostatics

- Since the test cylinder is in static equilibrium, the forces are balanced.

$$F_2 = F_1 + m_{cyl} g$$

$$p_2 = p_1 + \rho g (y_2 - y_1)$$



- Thus, with the surface as reference, the absolute pressure at depth h in a fluid is given by:

$$\boxed{p = p_0 + \rho_f g h}$$

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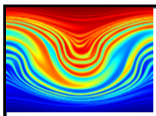
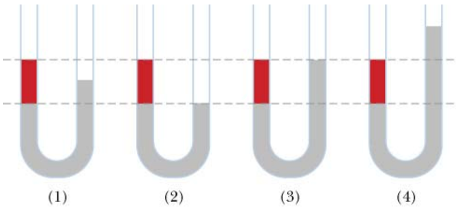


Check your understanding

- The figure shows four situations in which a red liquid and a grey liquid are in a U-tube open at both ends. In one situation the liquids cannot be in static equilibrium.

a) Which situation is that?

b) For the other three situations is the density of the red liquid greater than, less than, or equal to the density of the grey liquid.



Check Understanding

- How deep can a diver descend in ocean water without damaging his watch, which will withstand an absolute pressure of 5.5 bar?

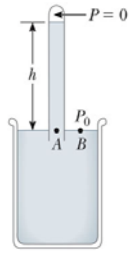
Mercury barometer

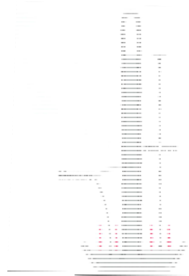
- The relationship

$$\boxed{p = p_0 + \rho_f g h}$$

can be used to measure pressure.

- In a barometer, a long closed tube is filled with mercury and inverted in a dish of mercury. The closed end is nearly a vacuum (P=0).
- Measures atmospheric pressure (P_o) which is directly linked to the height of the mercury column.
- Atmospheric pressure 760 mm (of Hg).





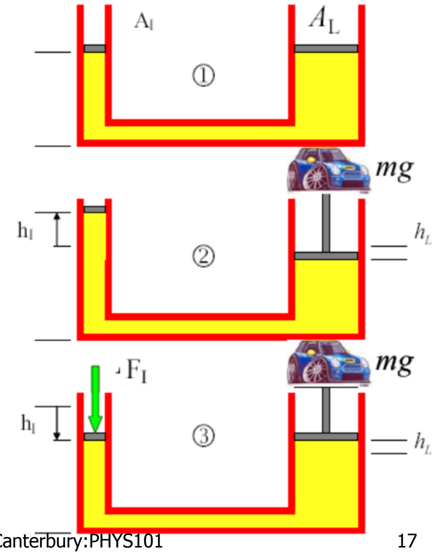
Pascal's Principle

- A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.
- Alternative (more precise) definition: A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

DEMO: Pascal's Vases

Hydraulic jack/lever

1. Unloaded the hydraulic lever has equal levels and thus pressures on each side.
2. With the weight *mg* on the RHS, the levels change.
3. Applying force F_I to the LHS to restore levels.



In the final situation:

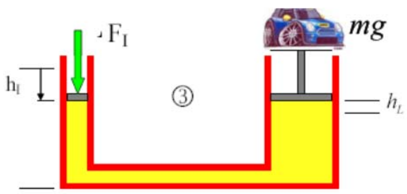
$$P_{\text{lhs}} = P_{\text{rhs}} \Rightarrow \frac{F_I}{A_I} = \frac{F_L}{A_L}$$

$$F_I = mg\frac{A_I}{A_L}$$

Hydraulic jack

- The force required to lift the car can be written as:

$$F_I = mg\frac{A_I}{A_L}$$



- By selecting $A_I < A_L$ the force required to lift something can be significantly reduced.
- Are we getting something for nothing?

Archimedes Principle

Archimedes Principle:

When a body is fully or partially submerged in a fluid, a buoyant force is directed upward that has a magnitude equal to the weight of the fluid that has been displaced by the body.

Alternative statement: the buoyant force on an object immersed in a fluid is equal to the weight of the fluid displaced by the object.

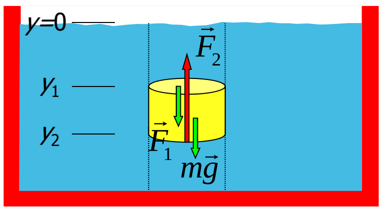
MYTHBUSTERS : Floating on Invisible water

Archimedes Principle

- For a cylinder of fluid we deduced that the difference between F_1 and F_2 must be equal to the weight of the fluid cylinder.

$$F_2 = F_1 + m_{cyl} g$$

$$p_2 = p_1 + \rho g (y_2 - y_1)$$



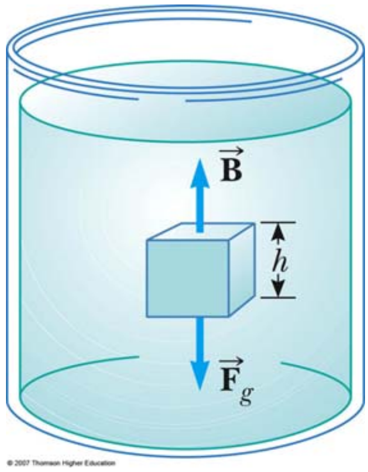
- F_1 and F_2 remain the same if the fluid cylinder is replaced with an object. F_2 - F_1 is the net force, from the fluid, on the object. It arises because of the difference in pressures at y_1 and y_2 and is called the **buoyancy force** for the object in the fluid and is the weight of the fluid displaced by the object.

Archimedes Principle

- The pressure at the top of the cube causes a downward force of $P_{top} A$.
- The pressure at the bottom of the cube causes an upward force of $P_{bot} A$.
- *Buoyancy force is given by:*

  $B = (P_{bot} - P_{top})\ A$

  $= \rho_{fluid}\ g\ V = Mg$



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Check Understanding

Imagine holding two identical bricks in place underwater. Brick 1 is just beneath the surface of the water, and brick 2 is held about 0.6m down. The force needed to hold brick 2 in place is:

1) greater
2) the same
3) smaller

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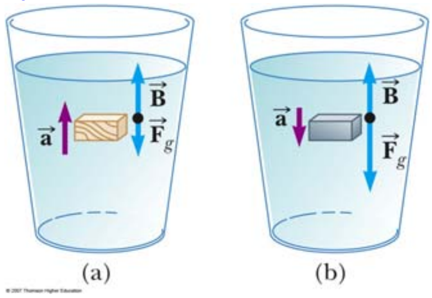
Submerged objects

- A submerged object that was held under and then let go experiences the following forces.

Buoyancy $F_B = \rho_f V g$,Weight $F_w = \rho_o V g$

Therefore, net upward force is

$$F_{net} = (\rho_f - \rho_o)Vg$$



- If $\rho_f > \rho_o$ object starts to rise
- If $\rho_f < \rho_o$ object starts to sink
- If $\rho_f = \rho_o$ object has neutral buoyancy – object does not move.

PLAY
ACTIVE FIGURE 13.9

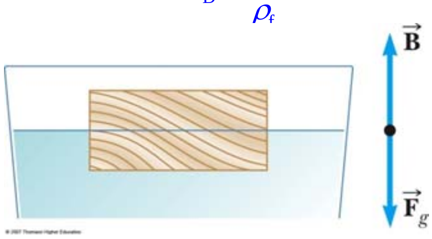
Floating objects

Buoyancy $F_B = \rho_f V_D g$, where V_D is the **displaced** volume

Weight $F_w = \rho_o V g$

The net force is zero (no motion) $\rho_f V_D g = \rho_o V g$

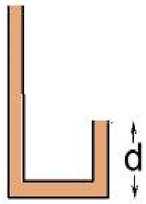
The displaced volume is $V_D = \dfrac{\rho_o}{\rho_f} V$



PLAY
ACTIVE FIGURE 13.10

Practice Example

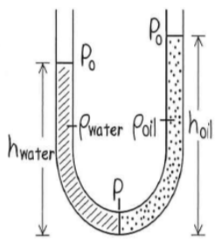
A drainage release valve opens when the force applied to it is 9.8N. Using the schematic of the system below what is the total height of the water needed in the long arm to just operate the release valve? Assume that the plastic tube in the figure has a cross-sectional area of 5.00 $cm^{2.}$ The short arm of the tube has a length d=0.800m and can be assumed to be full of water.



We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.

Practice Example

- A U tube is partially filled with water. Oil (which does not mix with the water) is poured into the left arm of the tube until the oil-water interface is at the midpoint of the tube as shown. Determine a relationship between h_{oil} and h_{water}.



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FLUID DYNAMICS AND BERNOULLI'S EQUATION

SJW²: 13.5 AND CHAPTER 14

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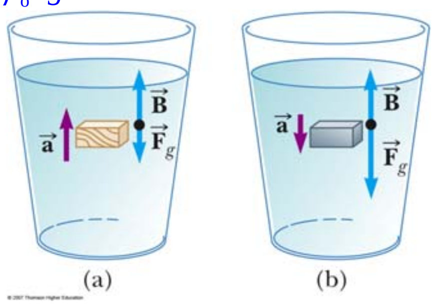

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ACTIVE FIGURE 13.9

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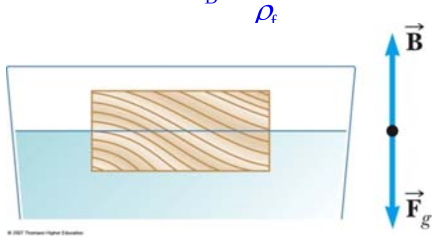


PLAY
ACTIVE FIGURE 13.10

Example

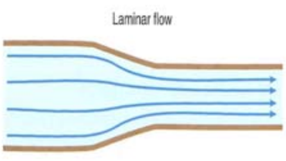
You notice that a block floats in an unknown liquid with 4.6 cm of the side of the block submerged. When the block is placed in water it also floats but with 6.0 cm submerged. Determine the density of the unknown liquid?

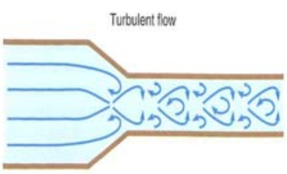
Check your understanding

- A boat carrying a car is floating on a lake. The car is then driven overboard and sinks. What happens to the water level in the lake (with respect to the shore)?
    1) rises
    2) drops
    3) remains the same
    4) depends on the size of the car

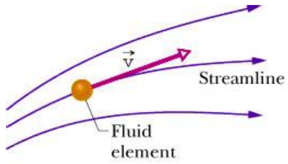
Properties of an Ideal Fluid

- **Steady flow:** Laminar, no turbulence.
- **Incompressible:** Density has constant and uniform value.
- **Non viscous flow:** No heat generated in the flow.
- **Irrotational flow:** No angular momentum present - no eddies, whirlpools etc.

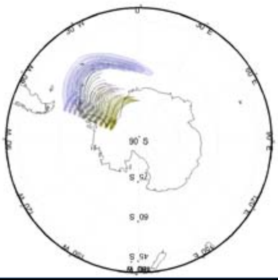


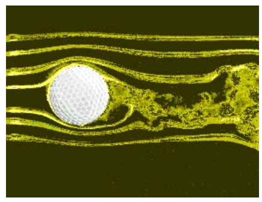


Streamlines

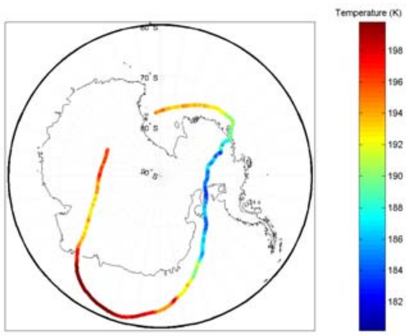


- In **laminar** flow the fluid particles follow **streamlines** which are imaginary paths following the velocity of a fluid particle.
- The velocity is always tangential to the streamline.
- Streamlines cannot cross - a crossing point represents **two** different velocities which is not possible.





VORCORE: super-pressure Balloons



A five day balloon trajectory during VORCORE showing changes in temperature 20km above Antarctica.

In September and October 2005, the VORCORE campaign flew 27 super-pressure balloons from McMurdo, Antarctica into the stratospheric polar vortex. Long-duration flights greater than two months were achieved. The balloons act as tracers of the atmospheric flow around Antarctica following smooth streamlines in their journey in the stratosphere (the region of the ozone layer roughly 20 km above the Earth's surface).

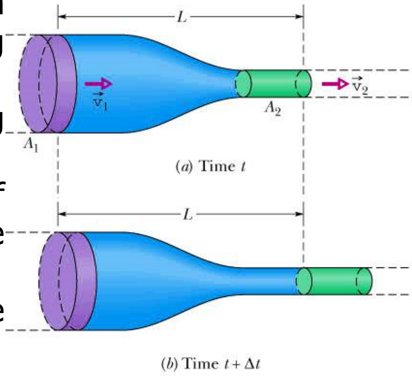
Equation of Continuity

- The equation of continuity is a result of an ideal fluid being incompressible.
- Consider a pipe with changing diameter.
- In time Δt there is a volume of fluid $A_1 \Delta x_1$ that moves in from the left end.
- This **same volume** must move out the right end.

$$A_1 \Delta x_1 = A_2 \Delta x_2$$

$$A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

The product of area and velocity is constant at all points along the flow.



$$\boxed{A_1 v_1 = A_2 v_2}$$

Equation of Continuity

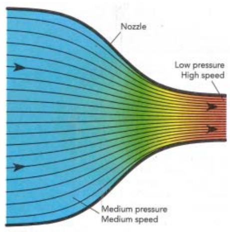
- The equation of continuity says that the volume of an incompressible fluid entering one part of a flow tube in a given time interval must be matched by an equal volume leaving downstream in the same time.

- NOTE: A flow tube can be considered to be a bundle of neighboring streamlines and could represent flow in a pipe or a stream.
- An important consequence of the equation of continuity is that flow is faster in narrower parts of the tube and slower in wider parts.
- The product, Av, is called the *volume flux* or the *flow rate.*

Equation of Continuity

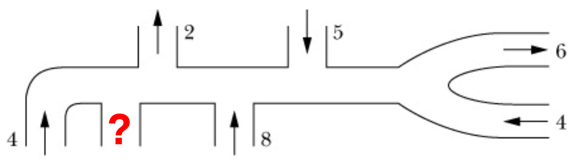
- When water flows through the nozzle at the end of a hose, it exchanges its pressure potential energy for kinetic energy and sprays out. The nozzle's narrowing channel initiates the energy transformation so that low speed, high pressure water entering the nozzle becomes fast moving, atmospheric pressure water leaving the nozzle.

- Water flowing through a nozzle speeds up and its pressure drops. The narrowing spacing between streamlines indicates that the flow speed is increasing, while the colour change indicates that the pressure is dropping.



Check your understanding

- The figure below shows a pipe and gives the volume flow rate (in cm^3/s) and the direction of flow for all but one section.
- What is the volume flow rate and the direction of flow for that section?

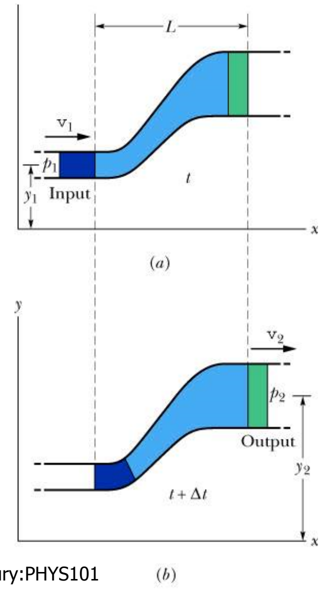


Bernoulli's Equation

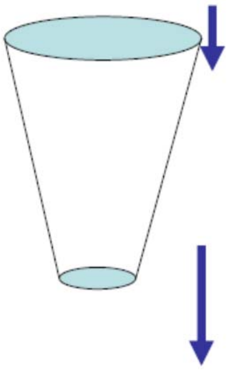
- In figure (a), a volume of fluid is about to enter the pipe from the left.
- After this volume has entered, an equivalent volume must have exited from the right - figure (b).
- By considering the mechanical conservation of energy, it can be shown that the following expression holds true:

$$\boxed{p + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant}}$$

Proof



Flow from a tap



A vertical flow of water from a tap tapers and this can be explained by Bernoulli's equation.

The water in the flow tube increases in speed as it descends because of acceleration due to gravity.

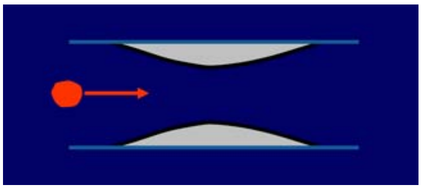
The increase in speed results from the increase in kinetic energy due to decrease in potential energy. For a distributed mass we need to use density:

$$\frac{1}{2}\rho v^2 + \rho g h = constant$$

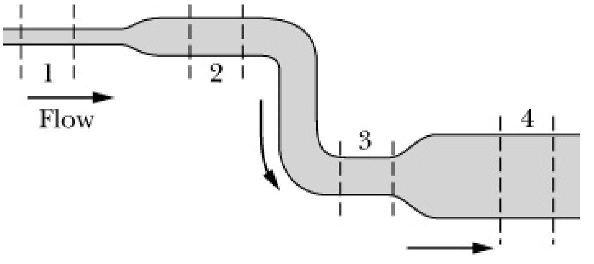
Check Understanding

A blood platelet drifts along with the flow of blood through an artery that is partially blocked. As the platelet moves from the wide region into the narrow region, the blood pressure:

1) increases
2) decreases
3) stays the same
4) drops to zero



Check your understanding

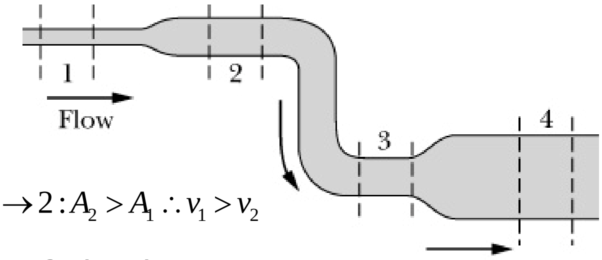


Water flows smoothly through the pipe shown in the figure. Rank the numbered sections of the pipe, greatest first, according to:

a) the volume flow through them.
b) the flow speed v through them
c) the water pressure within them

Explanation for b)

Rank the numbered sections of the pipe, greatest first, according to the flow speed within them.



$$\boxed{A_1 v_1 = A_2 v_2}$$

$$1 \to 2 : A_2 > A_1 \therefore v_1 > v_2$$

$$2 \to 3 : A_2 = A_3 \therefore v_2 = v_3$$

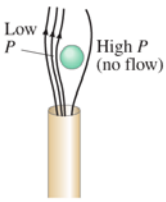
$$3 \to 4 : A_4 > A_3 \therefore v_3 > v_4$$

Rankings

$v_1 > v_2 = v_3 > v_4$

Bernouilli Demonstration

$$\boxed{p + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant}}$$



We can make a ping-pong ball float above a blowing jet of air; if the ball begins to leave the jet of air, the higher pressure in the still air outside the jet pushes the ball back in.

MYTHBUSTERS : Hurricane window

Practice Example

- What volume V of helium is needed to lift a load of 180 kg (including the weight of the empty balloon)? The density of air is 1.29 kg/m^3 and the density of helium is 0.179 kg/m^3.

Practice Example

- A man makes an automatic whiskey and ginger machine by connecting some old pipes together. What velocity must the flow of whiskey and ginger be to make a drink with the volume of whiskey being 15% that of the ginger, if the whiskey tube is 3 times the area of the ginger tube?

We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.

FLUID DYNAMICS AND PHYS101 REVIEW

SJW^2: CHAPTER 14

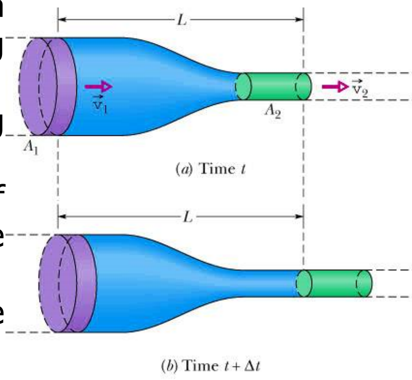
Equation of Continuity

- The equation of continuity is a result of an ideal fluid being incompressible.
- Consider a pipe with changing diameter.
- In time Δt there is a volume of fluid $A_1 \Delta x_1$ that moves in from the left end.
- This **same volume** must move out the right end.

$$A_1 \Delta x_1 = A_2 \Delta x_2$$

$$A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

The product of area and velocity is constant at all points along the flow.

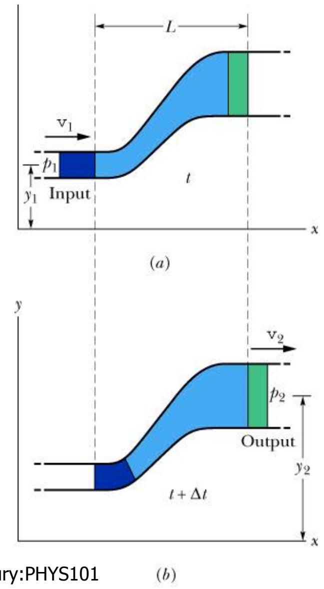


$$\boxed{A_1 v_1 = A_2 v_2}$$

Bernoulli’s Equation

- In figure (a), a volume of fluid is about to enter the pipe from the left.
- After this volume has entered, an equivalent volume must have exited from the right - figure (b).
- By considering the mechanical conservation of energy, it can be shown that the following expression holds true:

$$\boxed{p + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant}}$$



Proof

Practice Example

Small hydroelectric plants sometimes obtain their water supply from reservoirs. For a 100 cm diameter pipe, which connects the reservoir and the plant, where the pipe at the base of the dam is 50 m below the reservoir surface and the water drops 200 m through the tube before flowing into the turbine through a 50 cm diameter nozzle.

Determine the flow speed at the turbine?

Determine the flow speed at the top of the intake pipe?

We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.

Practice Example

- Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.5 m/s through a 2 cm diameter pipe in the cellar under a pressure of 3.0 atms, what will the flow speed and pressure be in a 1 cm diameter pipe in the second floor 5 m above?

We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.

Practice Example

- The width of the stream of water emerging from a tap reduces as it falls because the gravitational force increases the speed of the stream. Determine the speed of the flow from the tap if the cross sectional area of the stream changes from 1.2 cm^2 to 0.35 cm^2 in a vertical distance of 0.5 m.

We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.

REVIEW

- WE WILL USE THIS PERIOD TO REVIEW PHYS101 PRACTICE EXAMPLES. THIS RELIES ON STUDENTS IDENTIFYING PROBLEMS THEY HAD WITH PARCTICULAR PRACTICE EXAMPLES.