



Lecture 4: Power, Kinetic and Potential Energy and Conservation of Energy

SJW² 6.3-6.5, 7.1-7.3

Some of the material in this Lecture is review material and has previously been covered in NCEA (Standard 90521) and/or equivalents.

University of Canterbury:PHYS101

1



Introduction to Energy

- The concept of energy is one of the most important topics in science and engineering.
- Every physical process that occurs in the Universe involves energy and energy transfers or transformations.
- Energy is not easily defined.
- The energy approach to describing motion is particularly useful when Newton's Laws are difficult or impossible to use.

University of Canterbury:PHYS101

2



Power

- Power is the time rate of energy transfer.
- The average power is given by:

$$\overline{P} = \frac{W}{\Delta t}$$

when the method of energy transfer is work.

- The instantaneous power is the limiting value of the average power as Δt approaches zero.

$$\overline{P} = \lim_{\Delta t \rightarrow 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

- The power is valid for any means of energy transfer.

University of Canterbury:PHYS101

3



Kinetic Energy

- Kinetic Energy is the energy of a particle due to its motion.
 - $K = \frac{1}{2} m v^2$
 - K is the kinetic energy
 - m is the mass of the particle
 - v is the speed of the particle
- When work is done on a system and the only change in the system is in the speed, the net work done on the system equals the change in kinetic energy of the system.

University of Canterbury:PHYS101

4



Kinetic Energy

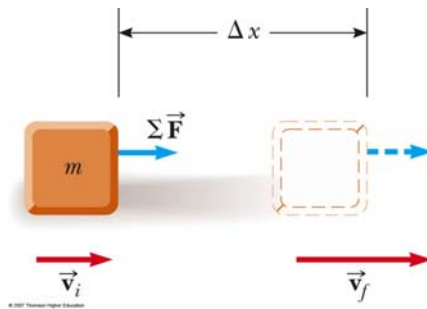
- Calculating the work:

$$W = \int_{x_i}^{x_f} \sum F dx = \int_{x_i}^{x_f} ma dx$$

$$W = \int_{v_i}^{v_f} mv dv$$

$$\sum W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{net} = K_f - K_i = \Delta K$$



Proof

University of Canterbury:PHYS101

5



Example

- A two man bob-sled has a mass of 390 kg. Starting from rest, the two racers push the sled for 50 m with a net force of 270 N. Neglecting friction, what is the speed of the sled at the end of the 50 m?

University of Canterbury:PHYS101

6



Potential Energy

- Potential energy is energy related to the configuration of a system in which the components of the system interact by forces.
- Alternatively, it can be considered to be the energy stored in the forces between or within an object with the potential to do work.
- Examples:
 - Gravitational potential energy
 - Chemical potential energy
 - Elastic potential energy
 - Nuclear potential energy
 - Electrostatic potential energy

University of Canterbury:PHYS101

7



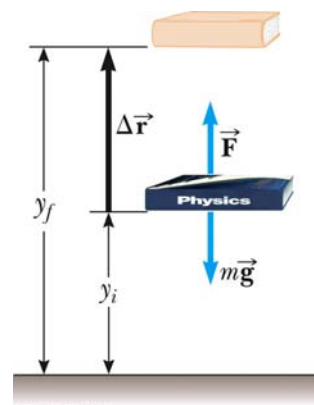
Gravitational Potential Energy

- Work is done on the book by lifting vertically against the force of gravity.
- This produces an increase in the energy of the system.
- **Gravitational potential energy is therefore the energy associated with an object at a given location above the surface of the Earth.**

$$W = (\vec{F}_{\text{app}}) \cdot \Delta \vec{r}$$

$$W = (mg\hat{j}) \cdot [(y_f - y_i)\hat{j}]$$

$$W = mgy_f - mgy_i$$



PLAY
ACTIVE FIGURE 6.16

University of Canterbury:PHYS101

8



Elastic Potential Energy

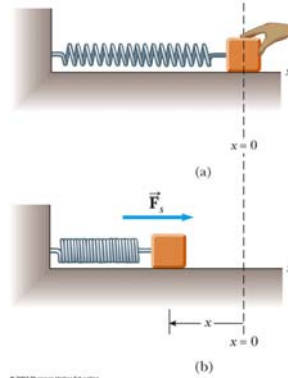
- The force the spring exerts is given by Hooke's law to be:

$$F_s = -kx$$

- The work done by an external applied force on a spring-block system is:

$$W = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2$$

- The work is equal to the difference between the initial and final values of an expression related to the configuration of the system and is called the **Elastic potential energy**.
- Elastic potential energy is stored in the spring only when it is stretched or compressed and is always positive.
- The elastic potential energy is a maximum when the spring has reached its maximum extension or compression.



9



Check understanding

A ball is connected to a light spring suspended vertically as shown. When pulled downward from its equilibrium position and released the ball oscillates up and down. **In the system of the ball, spring and the Earth, what forms of energy are there during the motion?**

- (a) Kinetic and elastic potential energy
- (b) Kinetic and gravitational potential energy
- (c) Kinetic, elastic potential and gravitational potential energy
- (d) Elastic and gravitational potential energy



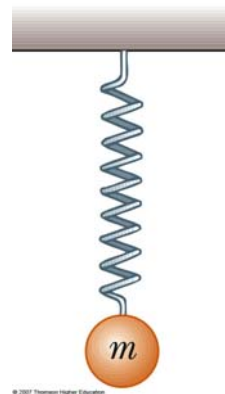
University of Canterbury: PHYS101

10



Check understanding 2

A ball is connected to a light spring suspended vertically as shown. When pulled downward from its equilibrium position and released the ball oscillates up and down. **In the system of the ball and the spring, what forms of energy are there during the motion?**



- (a) Kinetic and elastic potential energy
- (b) Kinetic and gravitational potential energy
- (c) Kinetic, elastic potential and gravitational potential energy
- (d) Elastic and gravitational potential energy

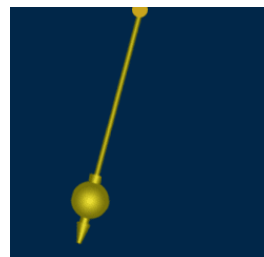
University of Canterbury:PHYS101

11



Isolated and Non-isolated systems

- Non-isolated systems
 - Energy can cross the system boundary in a variety of ways.
 - Total energy of the system changes.
- Isolated systems
 - Energy does not cross the boundary of the system.
 - Total energy of the system is constant.



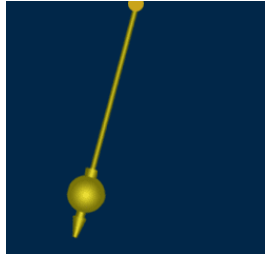
Is this an isolated system?

University of Canterbury:PHYS101

12



Isolated Systems?



The pendulum ignoring any friction is an Isolated System

$$\Delta E_{\text{tot}} = 0$$



The child and swing is not isolated – the woman is applying an external force which results in WORK.

$$\Delta E_{\text{tot}} = W$$

University of Canterbury:PHYS101

13



Energy Conservation

- The total energy of a system is always conserved – but all forms of energy must be included in the sum.
 - mechanical energy (kinetic and potential), thermal energy and internal energy.

$$\Delta E_{\text{tot}} = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}$$

- A conservative force is one in which the work done does not depend on the path.
- Forces which only transfer energy between kinetic and potential energy are called conservative.
- Reversing the path returns the system to the same combination of kinetic and potential energy.

University of Canterbury:PHYS101

14



Conservative forces

- A conservative force is one in which the work done does not depend on the path.
- Forces which only transfer energy between kinetic and potential energy are called conservative.
- Reversing the path returns the system to the same combination of kinetic and potential energy.



MYTHBUSTERS : Swinging for Science

DEMO: Newton's Cradle

University of Canterbury:PHYS101

15



Check Understanding

Which of the following is a conservative force? (All refer to a car on a slope.)

- a) The force you exert on the car pushing it uphill.
- b) The force exerted by rain drops falling on the car.
- c) The frictional force of the road on the car.
- d) The gravitational force acting on the car.
- e) The force you exert on the car (pushing it uphill) after it starts to slide downhill.

University of Canterbury:PHYS101

16



Conservation of mechanical energy

▪ When a **conservative force** does work W on an object within the system, it transfers energy between kinetic energy K of the object and potential energy U of the system.

▪ The change ΔK in kinetic energy is $\Delta K = W$.

▪ The change in potential energy is $\Delta U = -W$.

▪ Hence $\Delta K = -\Delta U$, or $\Delta K + \Delta U = 0$. This results in

$$K_2 - K_1 = -(U_2 - U_1)$$

or

$$K_1 + U_1 = K_2 + U_2 \quad \text{conservation of mechanical energy}$$

for the change in $K + U$ between points 1 and 2.

University of Canterbury:PHYS101

17



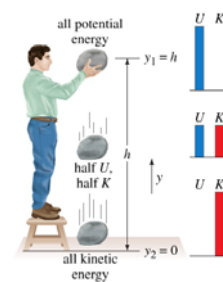
Conservation of mechanical energy

▪ If only conservative forces are doing work, the total mechanical energy of a system neither increases nor decreases in any process. It stays constant – it is conserved.

▪ In an **isolated** system with only conservative forces acting the kinetic energy and potential energy can change, but their sum cannot change.

▪ This result is called the principle of conservation of mechanical energy. It can be written as:

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0$$



The rock's potential energy changes to kinetic energy as it falls. The bar graphs represent the potential energy, U , and kinetic energy, K , for the three different positions.

University of Canterbury:PHYS101

18

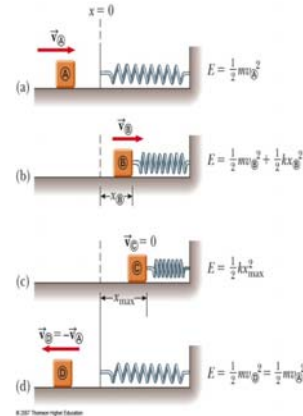


Energy Conservation

- This expression is the elastic potential energy:

$$U_s = \frac{1}{2} kx^2$$
- The elastic potential energy can be thought of as the energy stored in the deformed spring.
- The stored potential energy can be converted into kinetic energy.
- Observe the effects of different amounts of compression of the spring.

DEMO: Coupled Oscillations



PLAY
ACTIVE FIGURE 7.11

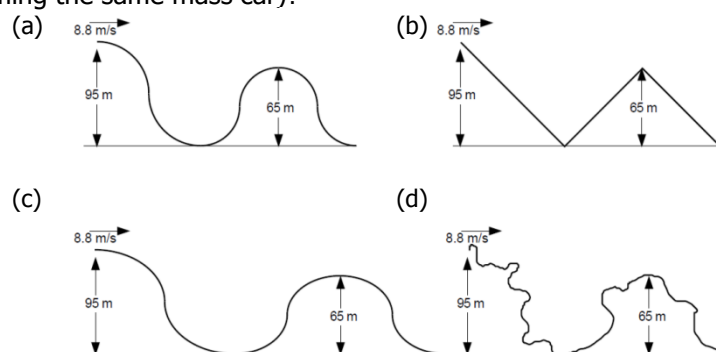
University of Canterbury:PHYS101

19



Check Understanding

The figures below show four roller-coaster designs. Rank the speed of the roller coaster on the second bump in ascending order (ignoring friction and assuming the same mass car)?



MYTHBUSTERS : Waterslide

University of Canterbury:PHYS101

20



Practice Example

A 10 kg block on a horizontal frictionless surface is attached to a light spring ($k = 0.75 \text{ kN/m}$). The block is initially at rest at its equilibrium position. A constant force of 70 N acting at 45° to the horizontal is applied. Assuming conservation of mechanical energy, determine the speed of the block when it is 12 cm from its equilibrium position.

We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.

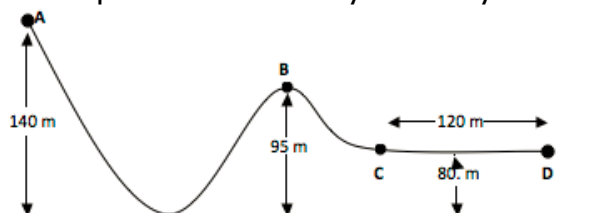
21



Practice Example

You are in charge of checking that a rollercoaster meets a set of specifications. For a 500 kg rollercoaster car released from rest at Point A, will the car be travelling at less than 25 m/s at Point B? Will the braking system that applies the brakes at Point C stop the car by Point D if the braking system applies a retarding force of 200N? You may assume there is no friction or air resistance between points A and C in your analysis.

We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.

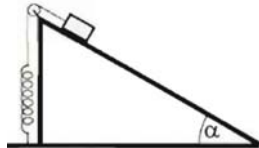


University of Canterbury:PHYS101

22



Practice Example



- A block of mass $m=5$ kg is held on an inclined plane (with angle $\alpha = 30^\circ$) and connected to an unstretched spring (with spring constant $k=80\text{N/m}$) as shown in the diagram. Assuming no friction, what is the lowest point reached on the block once it is released.

We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.

University of Canterbury:PHYS101

23



Lecture 5: Linear Momentum

SJW² 8.1-8.5

The majority of material in this Lecture is review material and has previously been covered in NCEA (Standard 90521) and/or equivalents.

University of Canterbury:PHYS101

24



Linear Momentum

The linear momentum of a particle is a vector \vec{p} , defined as

$$\vec{p} = m\vec{v}$$

in which m is the mass of the particle and \vec{v} is its velocity.

From Newton's second law:

$$\vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

Hence the force on a body is the rate of change of its momentum.



Change in momentum due to a force

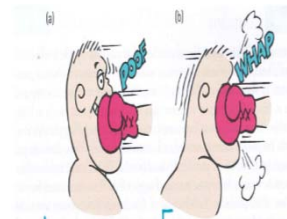
$$\vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

Conversely we note that the change in momentum is given by the time integral of the net force acting:

$$\Delta\vec{p} = \int_{t_i}^{t_f} \sum \vec{F} dt$$

The quantity on the left-hand side is called the impulse of the net force:

$$\vec{I} = \int_{t_i}^{t_f} \sum \vec{F} dt$$



In both cases, the impulse provided by the boxer's jaw reduces the momentum of the punch. (a) When the boxer moves away (rides the punch), he extends the time and diminishes the force. (b) If the boxer moves into the gloved fist, the time is reduced and must withstand greater force.



Conservation of Linear Momentum

- If no net external force acts on a system of particles, the total linear momentum of the system cannot change.
- This is because

$$\vec{F} = 0$$

implies $d\vec{P}/dt = 0$ or in other words:

$$\vec{P} = \text{a constant,}$$

where \vec{P} is the sum of the momenta of the individual particles, or

$$\vec{P} = \Sigma \vec{p}_i$$

- This result is called the law of conservation of linear momentum. It can also be written as $\vec{P}_{\text{initial}} = \vec{P}_{\text{final}}$.

University of Canterbury:PHYS101

27



Example

A box with mass $m = 6.0$ kg slides with speed $v = 4.0$ m/s across a frictionless floor in the positive direction of an x axis. It suddenly explodes into two pieces. One piece, with mass $m_1 = 2.0$ kg, moves in the positive direction of the x -axis with speed $v_1 = 8.0$ m/s.

What is the velocity of the second piece, with mass m_2 ?

University of Canterbury:PHYS101

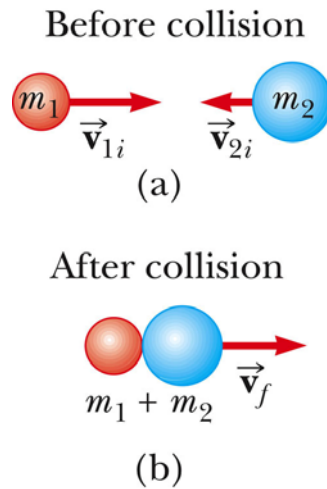
28



Perfectly Inelastic Collisions

- An inelastic collision is one in which the momentum is conserved. However, in an inelastic collision the total kinetic energy of the system is not conserved.
- In an inelastic collision, some kinetic energy is lost, but the objects do not stick together.
- A perfectly inelastic collision is one in which the two objects stick together after the collision.

PLAY
ACTIVE FIGURE 8.7



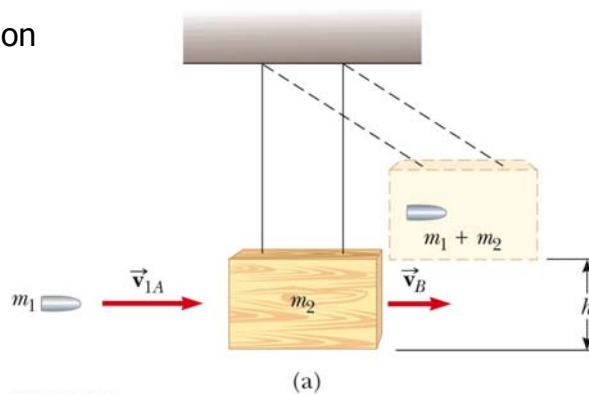
University of Canterbury:PHYS101

29



Ballistic pendulum

- Momentum conservation in action.



Ballistic pendulum by Nicholas Johnson

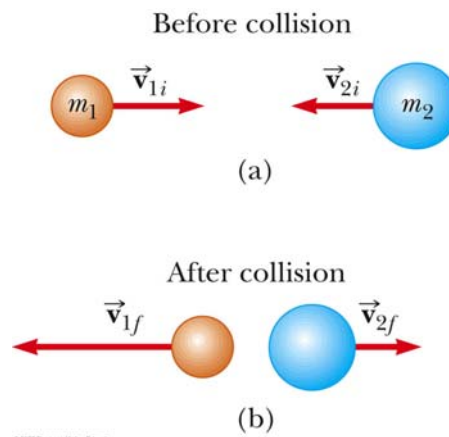
University of Canterbury:PHYS101

30



Elastic Collisions

- An elastic collision is one in which the momentum and the total kinetic energy of the system is conserved.
- Elastic and perfectly inelastic collisions are limiting cases, most actual collisions fall in between these two types.



MYTHBUSTERS : Ballistics Barrel

University of Canterbury:PHYS101

31



Check Understanding

- You are lying in bed (blinking teenagers) and you want to shut your bedroom door. You have a bouncy ball and a blob of clay (both with the same mass) sitting next to your bed. Which one would be more effective to throw at your door to close it?
 - 1) the bouncy ball
 - 2) the blob of clay
 - 3) it doesn't matter—they will be equally effective
 - 4) my mum will do it eventually

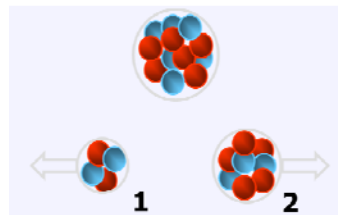
University of Canterbury:PHYS101

32



Check Understanding

- A uranium nucleus (at rest) undergoes fission and splits into two fragments, one heavy and the other light. Which fragment has the greater momentum?
 - 1) the heavy one
 - 2) the light one
 - 3) both have the same momentum
 - 4) impossible to say



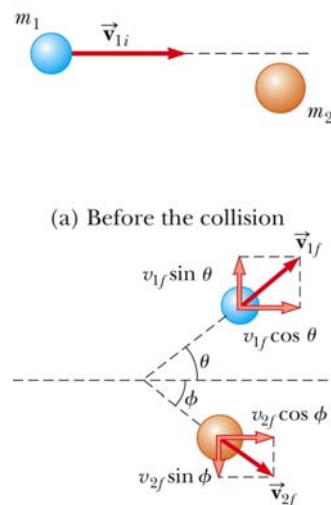
University of Canterbury:PHYS101

33



Collisions in two dimensions

- For problems in two dimensions, the momentum is conserved in all directions. Thus, velocity components in specific directions will be used.
- If the collision is elastic, the use of conservation of kinetic energy can also be used.



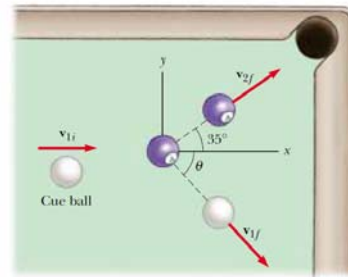
University of Canterbury:PI

(b) After the collision



Practice Example

A billiard player wants to sink the target ball in the corner pocket (see Figure). If the angle to the corner pocket is 35° , at what angle is the cue ball deflected? Assume both balls have the same mass and the collision is elastic.



We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.

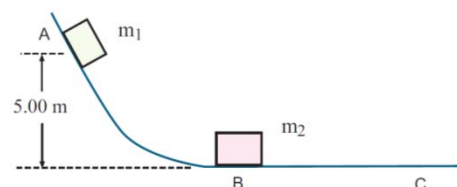
University of Canterbury:PHYS101

35



Practice Example

- Consider a frictionless track in a factory as shown in the Figure below. A block of mass $m_1 = 5.0 \text{ kg}$ is released from A (5m above ground level). It makes a head-on elastic collision with a block of mass $m_2 = 5.0 \text{ kg}$ at B, initially at rest. Will the maximum height to which m_1 rises after the collision be less than a 1.5m safety limit.



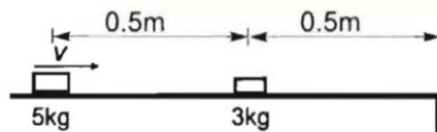
We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.

36



Practice Example

- Two blocks of mass $m_1 = 5 \text{ kg}$ and $m_2 = 3 \text{ kg}$ are initially at rest on a table at a distance 0.5 m from each other. Block m_2 is also 0.5 m from the edge of the table (see diagram). Determine the velocity that should be given to block m_1 to ensure that block m_1 just reaches the edge of the table after an elastic collision between the two blocks.



We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.

37



Lecture 6: Circular and Rotational Motion

SJW² 3.5, 4.8, 9.1-9.3

The majority of material in this Lecture is review material and has previously been covered in NCEA (Standard 90521) and/or equivalents.

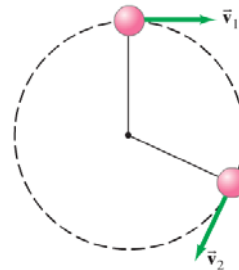
University of Canterbury:PHYS101

38



Uniform Circular Motion

- **Uniform circular motion** occurs when an object moves in a circular path with a constant speed.
- The associated analysis motion is a *particle in uniform circular motion*.
- An acceleration exists since the *direction* of the motion is changing.
 - This change in velocity is related to an acceleration.
- The velocity vector is always tangent to the path of the object.



A small ball moves in a circle, showing how the velocity changes. At each point, the instantaneous velocity is in a direction tangent to the circular path.

University of Canterbury:PHYS101

39



Centripetal Acceleration

- The acceleration is always perpendicular to the path of the motion.
- The acceleration always points toward the center of the circle of motion.
- This acceleration is called the **centripetal acceleration**.
- The magnitude of the centripetal acceleration vector is given by

$$a_c = \frac{v^2}{r}$$
- The direction of the centripetal acceleration vector is always changing, to stay directed toward the center of the circle of motion.

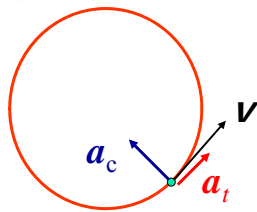
DEMO: Tennis ball on string

University of Canterbury:PHYS101

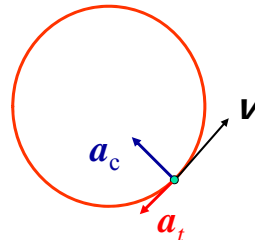
40



Centripetal acceleration vector always points toward centre of circle



moving anti-clockwise;
speeding up



moving anti-clockwise;
slowing down

“Centripetal” means “center-seeking.” The magnitude of a_c depends on both v and r . However, regardless of speed or tangential acceleration, a_c always points toward the center. That is, a_c is always radial (along the radius).

University of Canterbury:PHYS101

41



Forces

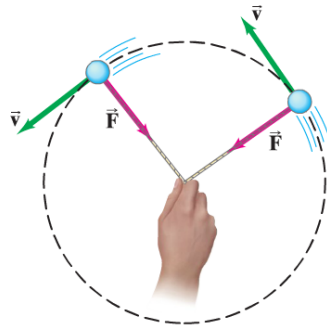
- The centripetal acceleration required to keep an object in uniform circular motion means there must be a force acting on the object.
- The type of force depends on the situation:
 - For planets orbiting the sun it is the gravitational force
 - For a ball being whirled at a constant speed in a horizontal circular path it is a tension exerted by the string
 - For a ball in a channel it is a normal force provided by the side of the channel.

University of Canterbury:PHYS101

42



Curving a bullet



A force is required to keep an object moving in a circle. If the speed is constant, the force is directed toward the circle's centre.

MYTHBUSTERS : Curving a bullet

University of Canterbury:PHYS101

43



Check Understanding

Which description is most accurate when describing what happens when we release the string when swinging a ball in a horizontal circle. Select one:

- the ball flies out in the radial direction defined by the string at the instant you release the ball, because the centrifugal force is no longer counteracted by the string.
- the ball flies out in the radial direction defined by the string at the instant you release the ball, because balls naturally fly straight out.
- The ball flies out along a tangent line perpendicular to the string, and that it then drops straight down to the ground, because there is no centrifugal force.
- the ball flies out along a tangent line perpendicular to the string, and that it then drops straight down to the ground because balls fall straight down when released.
- Neither, because although there is no centrifugal force, and the ball's velocity is tangent to the circle at the instant of release, the ball then follows a parabolic trajectory.

University of Canterbury:PHYS101

44



Check Understanding

You swing a ball at the end of a string in a vertical circle. Because the ball is in circular motion there has to be a centripetal force. At the top of the ball's path, what is F_c equal to?

- 1) $F_c = T - mg$
- 2) $F_c = T + N - mg$
- 3) $F_c = T + mg$
- 4) $F_c = T$
- 5) $F_c = mg$

University of Canterbury:PHYS101

45



Period

- The ***period***, T , is the time required for one complete revolution.
- The speed of the particle would be the circumference of the circle of motion divided by the period. Therefore, the period is defined as:

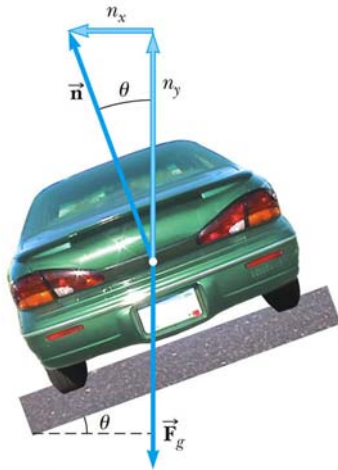
$$T \equiv \frac{2\pi r}{v}$$

University of Canterbury:PHYS101

46



Circular motion on an banked track



© 2007 Thomson Higher Education

An engineer wishes to design a curved road in such a way that a car will not have to rely on friction to round a curve. If the speed limit for the curve is 60 km/h and the radius of the curve is 50 m. What angle should the roadway be banked at?

University of Canterbury:PHYS101

47



Rotation of Rigid Objects

- Extended objects, such as a wheel or a CD, rotating around their axis can not be analyzed by modeling the object as a particle since the different parts of the object have different velocities and accelerations.
- When analyzing the motion of an extended object analysis is simplified by considering the object to be rigid.
- A rigid object is one that is non-deformable.
 - A *rigid body* is a body that can rotate with all its parts locked together and without any change in its shape.
 - The relative locations of all particles making up the object remain constant.

University of Canterbury:PHYS101

48



Rotation of Rigid Objects

- Extended objects, such as a wheel or a CD, rotating around their axis can not be analyzed by modeling the object as a particle since the different parts of the object have different velocities and accelerations.
- When analyzing the motion of an extended object analysis is simplified by considering the object to be rigid.
- A rigid object is one that is non-deformable.
 - A *rigid body* is a body that can rotate with all its parts locked together and without any change in its shape.
 - The relative locations of all particles making up the object remain constant.

University of Canterbury:PHYS101

49

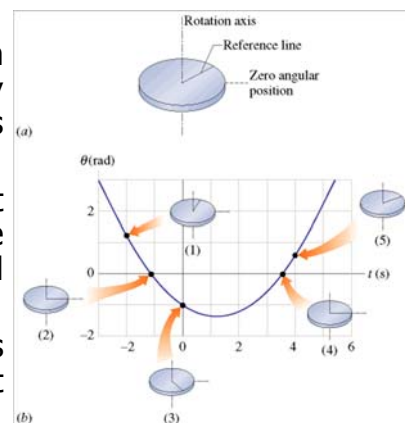


Angular Position and displacement

- We can associate the angle θ with the entire rigid object (every particle on the object rotates through the same angle).
- The angular position of the object is the angle θ between the reference line on the object and the fixed reference line.
- The *angular displacement* is defined as the angle the object rotates through:

$$\Delta\theta = \theta_f - \theta_i$$

University of Canterbury:PHYS101



50



Average and Instantaneous Angular Speed

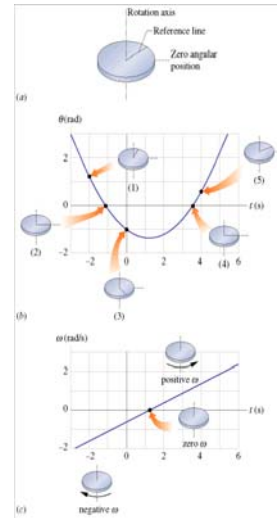
- The average angular speed, ω_{avg} , of a rotating rigid object is the ratio of the angular displacement to the time interval.

$$\omega_{avg} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

- The instantaneous angular speed is defined as the limit of the average speed as the time interval approaches zero.

$$\omega \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- The SI unit for ω is radians/second (rad/s), but sometimes questions will use revolutions per second or revolutions per minute (rpm). ω is negative for clockwise motion.



University of Canterbury:PHYS101

51



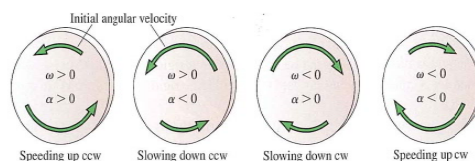
Average and Instantaneous Angular Acceleration

- The average angular acceleration, α , of an object is defined as the ratio of the change in the angular speed to the time it takes for the object to undergo the change:

$$\alpha_{avg} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

- The instantaneous angular acceleration is defined as the limit of the average angular acceleration as the time goes to 0.

$$\alpha \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

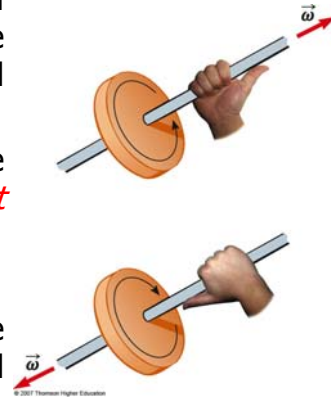


52



Directions

- Strictly speaking, the angular speed and acceleration (ω , α) are the magnitudes of the angular velocity and acceleration vectors.
- The direction of the vector specifies the **direction of the rotation axis, and *not* the direction of any motion.**
- The right hand rule:**
when the fingers of the right hand are curled around the rotation axis and point in the direction of rotation, then the thumb points in the direction of the angular velocity vector.



University of Canterbury:PHYS101

53



Rotation with constant angular acceleration

If a rotating body is subject to **constant angular acceleration**, the equations describing its motion are equivalent to the linear equations of motion:

Rotational Motion About a Fixed Axis

$$\begin{aligned}\omega_f &= \omega_i + \alpha t \\ \theta_f &= \theta_i + \omega_i t + \frac{1}{2} \alpha t^2 \\ \omega_f^2 &= \omega_i^2 + 2\alpha(\theta_f - \theta_i) \\ \theta_f &= \theta_i + \frac{1}{2}(\omega_i + \omega_f)t\end{aligned}$$

Translational Motion

$$\begin{aligned}v_f &= v_i + at \\ x_f &= x_i + v_i t + \frac{1}{2} at^2 \\ v_f^2 &= v_i^2 + 2a(x_f - x_i) \\ x_f &= x_i + \frac{1}{2}(v_i + v_f)t\end{aligned}$$

© 2007 Thomson Higher Education

University of Canterbury:PHYS101

54



Relationship Between Angular and Linear Quantities

- Displacements

$$s = \theta r$$

- Speeds

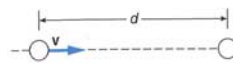
$$v = \omega r$$

- Accelerations

$$a = \alpha r$$

- Every point on the rotating object has the same angular motion.
- Every point on the rotating object does **not** have the same linear motion.

Linear motion

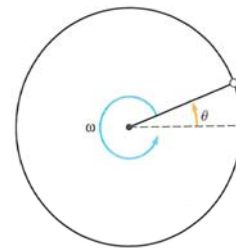


Displacement = d

Velocity = $\frac{d}{t} = v$

Acceleration = $\frac{\Delta v}{t} = a$

Rotational motion



Displacement = θ

Velocity = $\frac{\theta}{t} = \omega$

Acceleration = $\frac{\Delta \omega}{t} = \alpha$



Relating linear and angular variables

For a particle describing uniform motion in a circle:

- Position: $s = r\theta$ (θ in radians)

- Speed, velocity: $\frac{ds}{dt} = r \frac{d\theta}{dt}$
 $v = r\omega$ (ω in radians/s)

- Acceleration: $\frac{dv}{dt} = r \frac{d\omega}{dt}$
 $a_t = r\alpha$ (α in rad/s²)

Here a_t is the tangential component of the acceleration. Also

$$a_r = v^2/r = \omega^2 r$$

is the radial acceleration component, directed towards the centre of the circle.

56

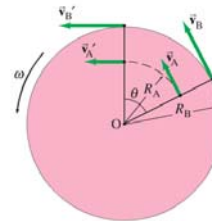


Speed Comparison

- The linear velocity is always tangent to the circular path and is called the tangential velocity.
- The magnitude is defined by the tangential speed:

$$v = \frac{ds}{dt} = r \frac{d\theta}{dt} = r\omega$$

- All points on the rigid object will have the same angular speed, but not the same tangential speed.



A wheel rotating anticlockwise. Two points on the wheel, at distances R_A and R_B from the center, have the same angular velocity because they travel through the same angle in the same time interval. But, the two points have different linear velocities because they travel different distances in the same time interval.

University of Canterbury:PHYS101

57



Acceleration Comparison

- The tangential acceleration is the derivative of the tangential velocity.

$$a_t = \frac{dv}{dt} = r \frac{d\omega}{dt} = r\alpha$$

- All points on the rigid object will have the same angular acceleration, but not the same tangential acceleration.
- The tangential quantities depend on r , and r is not the same for all points on the object.



On a rotating wheel whose angular velocity is increasing, a Point P has both tangential and radial (centripetal) components of linear acceleration.

University of Canterbury:PHYS101

58



Practice Example

- An amusement park ride consists of a vertical cylinder that spins about a vertical axis. When the cylinder spins sufficiently rapidly, any person inside is held against the wall. Suppose that the coefficient of static friction is 0.25. For a man of mass 60 kg and the radius of the cylinder to be 7m.
- Determine the safety critical angular velocity of the cylinder above which the man will not slide down the wall.
- Research shows that if the number of revolutions per second is greater than 20 at this critical velocity the rider will be ill? Is this ride going to get messy?

We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.

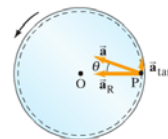
59



Practice Example

- A merry-go-round is initially at rest. At $t=0$, it is given a constant angular acceleration $\alpha=0.06 \text{ rad/s}^2$, which increase its angular velocity for 8 seconds.
- a) Determine the angular velocity of the merry-go-round?
- b) Determine the linear velocity of a child (at P) located 2.5 m from the center (O) of the merry-go-round?
- c) Determine the tangential acceleration of the child?
- d) Determine the centripetal acceleration of the child?

We will complete as many Practice examples as time allows. Students are encouraged to complete these examples in their own time if not completed in class. Model answers will be provided on the LEARN website.



University of Canterbury:PHYS101

60