

bool ortho = true

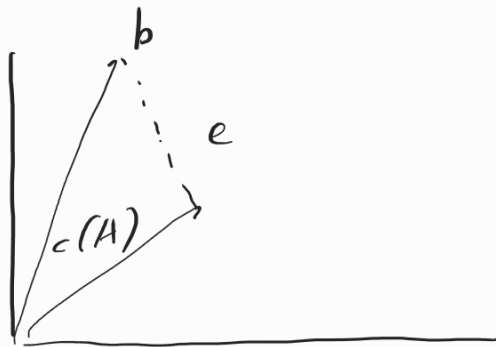
for v_i in V

for w_i in W

if $v_i \cdot w_i \neq 0$

return false

return true



$AX=b$ has no sol η

$A\hat{x}=b$ is our best solution

$$e = b - p$$

$$p = A\hat{x}$$

$$e = b - A\hat{x}$$

$$A^T(b - A\hat{x}) = 0$$

$$A^T b = A^T A \hat{x}$$

$$(A^T A)^{-1} A^T b = \cancel{(A^T A)^{-1}} A^T A \hat{x}$$

$$\hat{x} = (A^T A)^{-1} A^T b$$

* Qui2

$$\vec{p} = A\hat{x}$$

$$\vec{p} = A(A^T A)^{-1} A^T b$$

$$P_{\text{matrix}} = A(A^T A)^{-1} A^T \rightarrow \text{if } A^T A \text{ is singular, then invalid}$$

$$P_{\text{matrix}} = A(A^{-1} A^{T-1}) A^T = \underbrace{A A^{-1}}_I \underbrace{A^{T-1} A^T}_I$$

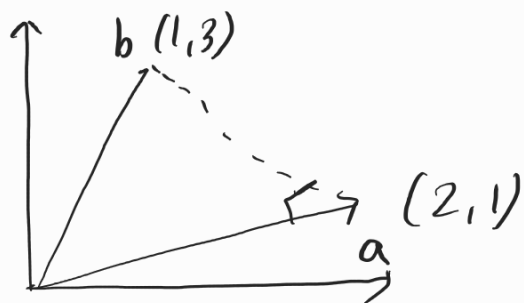
b onto A

$$A = \begin{bmatrix} 3 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \quad A^T = 3 \times 3$$

3×2

A is on $n \times n$, A^{-1} might exist
Not is singular

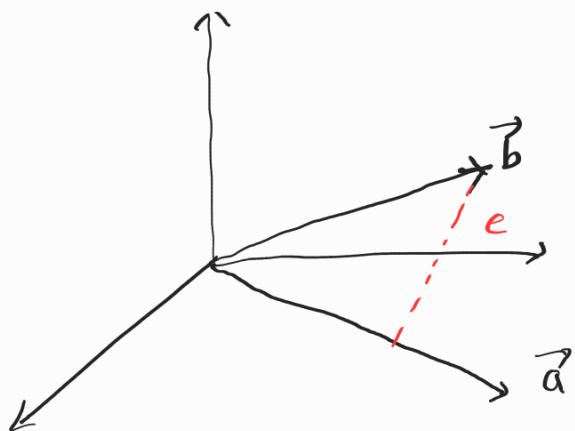
project \vec{b} onto \vec{a}



$$\vec{p} = a \frac{a^T b}{a^T a}$$

① $\vec{a} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

Project \vec{b} onto \vec{a}



$$\vec{p} = \frac{a^T b}{a^T a} \vec{a}$$

$$= \frac{\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}}{\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \frac{4}{2} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

② $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

Project \vec{b} onto \vec{A}

$$\vec{p} = A(A^T A)^{-1} A^T b$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \right) \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

3×2 2×3 3×2 2×3 3×1



