Product: $(AB)^T = B^T A^T$ (reverse order)

Inverse: $(A^{-1})^T = (A^T)^{-1}$

$(Ax)^T = x^T A^T$

$3\times 3 \quad 3\times 1 \qquad 1\times 3 \quad 3\times 3$

$= 3\times 1 \qquad\qquad = 1\times 3$

$$\boxed{(A^T)_{ij} = A_{ji}}$$

* if A^{-1} exists does $(A^T)^{-1}$?

$$Ax = x_1 \begin{bmatrix} c_1 \end{bmatrix} + x_2 \begin{bmatrix} c_2 \end{bmatrix} + \dots + x_n \begin{bmatrix} c_n \end{bmatrix}$$

Ax combines cols of A.

$x^T A^T \rightarrow [x_1 \; x_2] \begin{bmatrix} a & b \\ c & d \end{bmatrix} =$ combines rows of A

$$A = (LDU)^T = U^T D^T L^T$$

Dot : $v \cdot w$

$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ $\underset{2\times 1}{v} * \underset{2\times 1}{w}$ (can't calculate)

Inner Product

$$v^T w = [v_1 \ v_2] \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = v_1 w_1 + v_2 w_2$$

scalar

$$\underset{2\times 1}{v} \ \underset{1\times 2}{w^T} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} [w_1 \ w_2] = \begin{bmatrix} v_1 w_1 & v_1 w_2 \\ v_2 w_1 & v_2 w_2 \end{bmatrix}$$

$v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $w = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ Outer Product

$$v w^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix} [2 \ 1] = \boxed{\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}}$$

vw^T is singular product

$$(\underset{3\times 3}{A}\underset{3\times 1}{x})^T y = \underset{1\times 3}{x^T}(\underset{3\times 3}{A^T} y)$$

$Ax = 3\times 1$

$(Ax)^T = 1\times 3$

$(1\times 3)(3\times 1) = 1\times 1$

float f;
32 bits

1 0 1 1 0 1 1 0

$1.0 \ \varepsilon \ 1.00063$

$$\begin{bmatrix} 0 & 1 \\ \end{bmatrix} \begin{bmatrix} .1 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ \end{bmatrix}$$

PA = LU

$LU(A)$

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$*$ P^{-1} exist?

$$P^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$*$ If A is Symmetric, $P^{-1} = P^T$

$$I = P^{-1}P = PP^{-1} = P^TP = PP^T$$

$$\boxed{A = LDU = LDL^T}$$ (if A is symmetric)

$$R = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

2×3

$$R^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

3×2

$$\underset{3\times 2}{R^T}\ \underset{2\times 3}{R} = \begin{bmatrix} 17 & 22 & 27 \\ 22 & 29 & 36 \\ 27 & 36 & 45 \end{bmatrix}$$

Symmetric!

$$R R^T = \begin{bmatrix} 14 & 32 \\ 32 & 77 \end{bmatrix}$$

Symmetric!

$$(R^T R)^T$$

$$R^T (R^T)^T$$

$$= R^\top (R^\top)^\top$$

$$= R^\top R .$$