$$A_1 = L\,U$$

$$A = \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}$$

$$E_{21}A = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

$$EA = U$$

$$E^{-1}EA = E^{-1}U \longrightarrow A = E^{-1}U = LU$$

$$E^{-1} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \longrightarrow A = LDU$$

$$A = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix}$$

A ① $E_{21}A \longrightarrow$ modifies A

3x3

② $E_{31}(E_{21}A)$

③ $E_{32}(E_{31}E_{21}A) = U$

$E_{32}(E_{31}E_{21}A) = U$

How to get A?

$$\cancel{E_{32}^{-1}E_{32}}(E_{31}E_{21}A) = E_{32}^{-1}U$$

$$\cancel{E_{31}^{-1}E_{31}}(E_{21}A) = E_{31}^{-1}E_{32}^{-1}U$$

$$\cancel{E_{21}^{-1}E_{21}}A = \underline{E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}}U$$

$$L = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}$$

ex)

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}$$

* $$E_{32}E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 10 & -5 & 1 \end{bmatrix}$$

$$E_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

* $$E_{21}^{-1}E_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\cdots_{21} \cdots_{32} = \begin{bmatrix} \cdots \\ 2 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \quad E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix}$$

$$L = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -a_{21} & 1 & 0 \\ -b_{31} & -c_{32} & 1 \end{bmatrix}$$