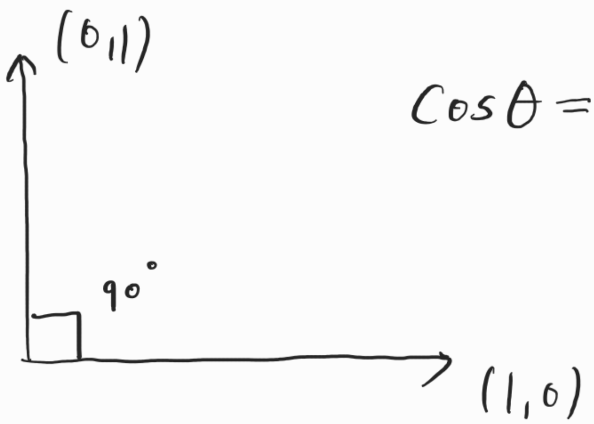
Orthogonal ($\perp$)

$$\begin{bmatrix}1\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\end{bmatrix}$$



$$\cos\theta = \frac{v \cdot w}{\|\vec{v}\|\|\vec{w}\|}$$

if $x^T y = 0$

$\rightarrow x \perp y$

$\boxed{x^T x = x \cdot x}$ $x = \begin{bmatrix}1\\2\end{bmatrix}$ (transform computation is faster)

$$\begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix} = 1 \cdot 1 + 2 \cdot 2 = 5$$

$$\|x\|^2 + \|y\|^2 = \|x+y\|^2$$

$$x^T x + y^T y = (x+y)^T (x+y)$$

$$= x^T x + \underline{x^T y + y^T x} + y^T y$$

$$= x^T x + 0 + y^T y$$

$$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\|x\|^2 = 1 \qquad \|y\|^2 = 1$$

$$x + y = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\|x+y\|^2 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2$$

* Any vector $v \perp 0$. ex) $\begin{bmatrix} 1. & 1.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$

If subspace $S \perp$ subspace T

then every vector in $S(s_1 \ldots s_n) \perp$ every vector in $T(t_1 \ldots t_n)$

$$Ax = 0 = \begin{bmatrix} \text{row } 1 \\ \vdots \\ \text{row } n \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

if $(\text{row}_1 \text{ of } A) \cdot x = 0$

then they are orthogonal.

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis

$$\text{Row space} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$N(A) = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_2$$

Basis

$$\text{row}_1 \cdot \text{Null}_1 = \begin{bmatrix} 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} = 0$$

$$\text{Row}_1 \cdot \text{Null}_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$c_1 (\text{row}_1)^T x + c_2 (\text{row}_2)^T x + \dots c_n (\text{row}_n)^T x = 0$$

- row $\perp$ NULL

  They are orthogonal complements.

S & T are $\perp$ complements

$T^{\perp}$ for S

$S^{\perp}$ for T.