$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$$

$$\begin{bmatrix} a & & & \\ & f & g & h \\ & j & k & l \\ & n & o & p \end{bmatrix} + \begin{bmatrix} & b & & \\ e & & g & h \\ i & & k & l \\ m & & o & p \end{bmatrix} + \begin{bmatrix} & & c & \\ e & f & & h \\ i & j & & l \\ m & n & & p \end{bmatrix}$$

$$+ \begin{bmatrix} & & & d \\ e & f & g & \\ i & j & k & \\ m & n & o & \end{bmatrix}$$

Recursive

$$A = \begin{bmatrix} -2 & -3 & 0 \\ -2 & -4 & 1 \\ 1 & 4 & -3 \end{bmatrix}$$

split along Row 1

$$ad - bc = 8$$

$$A_{11} = \begin{bmatrix} -4 & 1 \\ 4 & -3 \end{bmatrix} \qquad C_{11} = (-1)^{1+1} |A_{11}|$$

$$= 1 \cdot 8 = 8$$

$$A_{12} = \begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix} \quad C_{12} = (-1)^{1+2} |A_{12}|$$

$$= -1 \cdot 5 = -5$$

$$A_{13} = \begin{bmatrix} -2 & -4 \\ 1 & 4 \end{bmatrix} \quad C_{13} = (-1)^{1+3} |A_{13}|$$

$$= -4$$

$$|A| = \underset{(1,1)}{(-1)^2(-2)} \begin{vmatrix} -4 & 1 \\ 4 & -3 \end{vmatrix} + (-1)^3(-3) \begin{vmatrix} -2 & 1 \\ 1 & -3 \end{vmatrix} +$$

$$(-1)^4(0) \begin{vmatrix} -2 & -4 \\ 1 & 4 \end{vmatrix}$$

$$= -1$$

$x^T y$ = $\cdot$ prod = scalar

$x y^T$ = outterproduct = matrix

work any 2 vectors of length n

$\boxed{\text{3D Vectors}}$

Cross product

$$\underset{3\times 1}{U} \times \underset{3\times 1}{V} = \underset{3\times 1}{}$$

$$\begin{vmatrix} i & j & k \\ U_1 & U_2 & U_3 \\ V_1 & V_2 & V_3 \end{vmatrix} = (U_2V_3 - U_3V_2)\,\hat{i} + (U_3V_1 - U_1V_3)\,\hat{j} + (U_1V_2 - U_2V_1)\,\hat{k}$$

$$\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 1\cdot 1 - 2\cdot 1 \\ 2\cdot 0 - 4\cdot 1 \\ 4\cdot 1 - 1\cdot 0 \end{bmatrix} \end{array} \begin{array}{c} i \\ j \\ k \end{array} = \begin{bmatrix} -1 \\ -4 \\ 4 \end{bmatrix}$$

$ad-bc$ 2×2

$$\underset{2\times 2}{A^{-1}} = \frac{1}{ad-bc}\begin{bmatrix} -d & c \\ b & -a \end{bmatrix}$$

$Ax = b$

$Ax =$ some multiple of x.

$$\boxed{Ax = \lambda x}$$

$\lambda \to$ eigenvalue

$x \to$ eigenvector

$Ax = 0$

eigvecs live in $N(A)$

$\lambda = 0$

if A is singular

$\lambda = 1$ for I

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \lambda x$$

$$\boxed{\underbrace{|A - \lambda I|}_{\det = 0} = 0} \to \text{characteristic equation}$$

new $A - \lambda I$ is singular.

$\boxed{n \text{ eigenvalues / eigvec}}$

Not guaranteed to be unique

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 1 & 2 \\ \end{bmatrix} - \begin{bmatrix} \lambda & \cancel{2\lambda}\ 0 \\ \end{bmatrix} \right| = 0$$

$$\begin{bmatrix} 2 & 1 \end{bmatrix} \quad \begin{bmatrix} \cancel{2 \cdot \lambda} & \lambda \end{bmatrix} \Big| \qquad 0$$

$$\left| \begin{bmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix} \right| = 0$$

$$(1-\lambda)^2 - 4 = 0$$

$$\boxed{\lambda^2 - 2\lambda - 3 = 0} \qquad \begin{aligned} \lambda_1 &= -1 \\ \lambda_2 &= 3 \end{aligned}$$

Quadratic formula

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{trace} = \sum a_{ii} = 1 + 1 = 2$$

$$|A| = -3$$

λ's should add to trace(A)

should multiply to det(A)

* Triple check

$A - \lambda I$ is singular

$\lambda = -1$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$