Eigenvectors

solving the NULL space of $A - A\lambda$

$$A - A_1 I = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \xrightarrow{R} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$x_1 + 1 = 0$

$\lambda_2 = 3$

$$A - A_2 I = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \xrightarrow{R} \begin{bmatrix} +1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\lambda_1 = -1$ $\quad$ $\lambda_2 = +3$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Basis eigenvectors

$Ax = \lambda x$

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check λ_1

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

Find λ's and eigenvectors

$$AAx = \lambda\lambda x$$

$$A^2x = \lambda^2 x$$

Raise A^k, λ^k, $\boxed{x \text{ stays the same}}$

eigvals $(AB) \neq$ eigvals of A or B

Eigenvectors

Diagonalizing A

X is matrix of n independent eigenvecs

$$X = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ x_1 & \vdots & \vdots & x_n \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}_{n \times n}$$

$$\boxed{X^{-1} A X = \Lambda}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

A by itself.

$$X X^{-1} A X = X \Lambda$$

$$A X X^{-1} = X \Lambda X^{-1}$$

$$A = X \Lambda X^{-1}$$

verify

$$X = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \quad A = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \quad X^{-1} = \begin{bmatrix} -.5 & .5 \\ .5 & .5 \end{bmatrix}$$

✓ $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -.5 & .5 \\ .5 & .5 \end{bmatrix}$

$X = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

order doesn't matter as long as λ's correspond to eigvecs

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} .5 & -.5 \\ .5 & .5 \end{bmatrix}$$

$A = X\Lambda X^{-1}$

$$A^2 = X\Lambda \underbrace{X^{-1} \quad X}_{I} \Lambda X^{-1}$$

$$= X\Lambda\Lambda X^{-1}$$

$$= X\Lambda^2 X^{-1}$$

$$\boxed{A^K = XA^K X^{-1}}$$

$A^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

$X = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} (-1)^2 & 0 \\ 0 & 3^2 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix}$$

$$(2-\lambda)^2$$

$$\lambda_1 = , \lambda_2 = 2 \qquad \boxed{\text{Multiplicity of } 2}$$

$$N(A - 2I)$$

$$\begin{bmatrix} 2-2 & 1 \\ 0 & 2-2 \end{bmatrix} \qquad X = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \qquad X = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad (ad - bc = 0)$$

* X^{-1} does not exist, so we can't set $A = X \Lambda X^{-1}$!