$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

- 2 independent vectors
- they live in R^2

$$\begin{bmatrix} 1 & 0 & b \\ 0 & 1 & a \\ 0 & c & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A

$m \times n$

rank r

# of independent cols

- $C(A)$ has number of independent cols in it.
- Basis vectors form the column space

$$\begin{bmatrix} 1 & 2 & 0 \\ 3 & 6 & 1 \\ 5 & 10 & 1 \end{bmatrix}$$

$cv_1 + dv_2$

$C(A)$'s basis would be c_1 & c_3.

If rank(A) is full,

then $Ax = 0$ will be $x = 0$

Then $Ax = 0$ only has $x = 0$

$C(A)$ has n basis vectors

$m \times n$

2×3

$(m < n)$

$$\begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 6 \\ 2 & 3 & 4 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2 elems per column

→ The vector set is dependent.

- $C(A)$ lives in $\boxed{\mathbb{R}^m}$
- $N(A)$ lives in $\boxed{\mathbb{R}^n}$

it is a set of vectors in $\mathbb{R}^n$.

A

$$\begin{array}{cc} r_1 & \\ r_2 & \end{array} \begin{bmatrix} 1 & 1 & 6 \\ 2 & 3 & 4 \end{bmatrix}$$

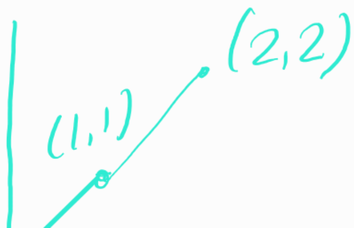
$$\begin{array}{ccc} c_1 & c_2 & c_3 \end{array}$$

The row space lives in $\mathbb{R}^n$.

$$A^T = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 6 & 4 \end{bmatrix}$$

row space is $C(A^T)$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$



$$C(A) \neq \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \boxed{C(A) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$$C(A) \neq C(R)$$

$$C(A^T) \overset{r_1}{=} \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$\xrightarrow{r_1} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\underline{r_1 = [1 \quad 1]}$$