$$A = \begin{array}{c} \begin{bmatrix} 1 & 3 & 4.5 \\ 2 & 6 & 9 \\ 3 & 9 & 5 \end{bmatrix} \\ \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \end{array} \rightarrow \begin{bmatrix} 1 & 3 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C(A) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$V = \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \end{array}$$

v_1 & v_2 are independent.

V's column space spans a plane in R^4.

shape

Line(1)

Plane(2)

Volumn(3)

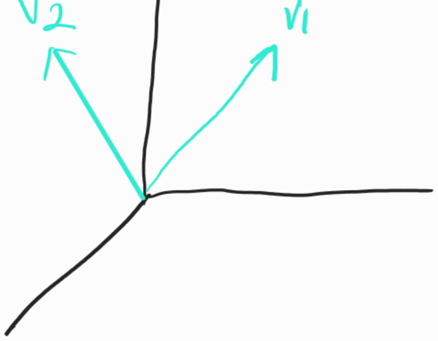
n-dimensional subspace (4+)

$$V = \begin{bmatrix} \vdots & \vdots \\ v_1 & v_2 \\ \vdots & \vdots \end{bmatrix} \quad \begin{array}{c} \begin{bmatrix} 1 & -2 \\ 3 & 6 \\ 0 & 0 \end{bmatrix} \\ \begin{array}{cc} v_1 & v_2 \end{array} \end{array}$$

2×1

is this still in R^3? Yes.

$V = R^3$



$cv_1 + dv_2 + ev_3.$

$Ax=b$ is solvable if $b \in C(A)$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$c_1 \quad c_2$

$$\begin{bmatrix} 5 \\ 6 \end{bmatrix} = x_1 c_1 + x_2 c_2$$

$$= x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= cv + dw$$

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

is $b = cv + dw + eu$ $\qquad 0 \neq 1$

$b = x_1 C_1 + x_2 C_2 + x_3 C_3$

$\boxed{\text{No. } \quad b \notin C(A)}$

Solving $AX=0$

for solutions that are not trivial

... not trivial.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

pivot free variable

$x_1 + 2x_2 = 0$

$0x_1 + 0x_2 = 0$

→ infinitely many sol

$x_1 + 2x_2 = 0$

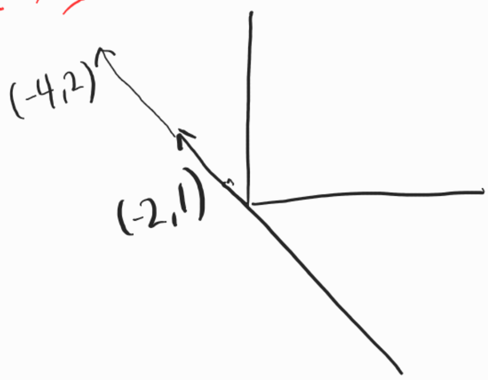
$0x_1 + 0x_2 \neq 0$

① $x_2 = 1$

② $x_1 + 2 = 0$

$x_1 = -2.$

$$\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$



$$N(A) = x_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

[vec] * free var

$3x_1 + 7x_2 - 5x_3 = 0$

$$\begin{bmatrix} 3 & 7 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

* row-reduced Echelon Form

$$\begin{bmatrix} 1 & \frac{7}{3} & -\frac{5}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

pivot x_1 free x_2 free x_3

2 Solutions

1. $x_3 = 0$, $x_2 = 1$

$1 \times 1 + \frac{7}{3} \cdot 1 + 0 = 0$

$x_1 = -\frac{7}{3}$

$\begin{bmatrix} -7/3 \\ 1 \\ 0 \end{bmatrix} x_2$

2. $x_2 = 0$, $x_3 = 1$

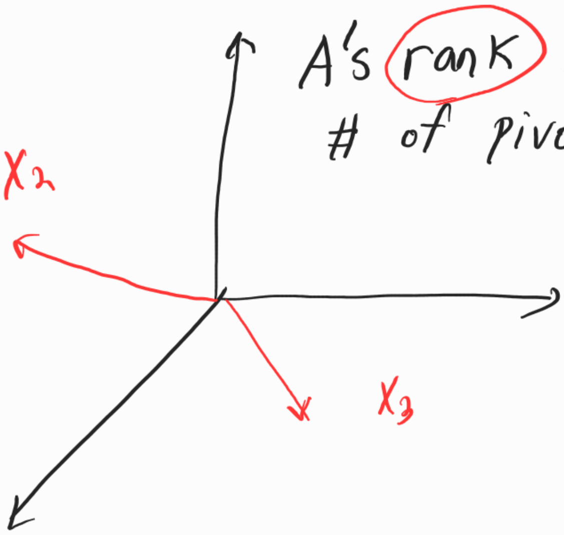
$1x_1 + 0 - 5/3 = 0$

$x_1 = 5/3$

$\begin{bmatrix} \frac{5}{3} \\ 0 \\ 1 \end{bmatrix} x_3$

A singular matrix is not full rank.

A's rank is # of pivots



$N(A) = \begin{bmatrix} -7/3 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 5/3 \\ 0 \\ 1 \end{bmatrix} x_3$

The nullspace has however many free vars as independent vectors.

free vars is $n - r$ = Nums of Nullspace.