$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 4 & 10 \end{bmatrix}$$

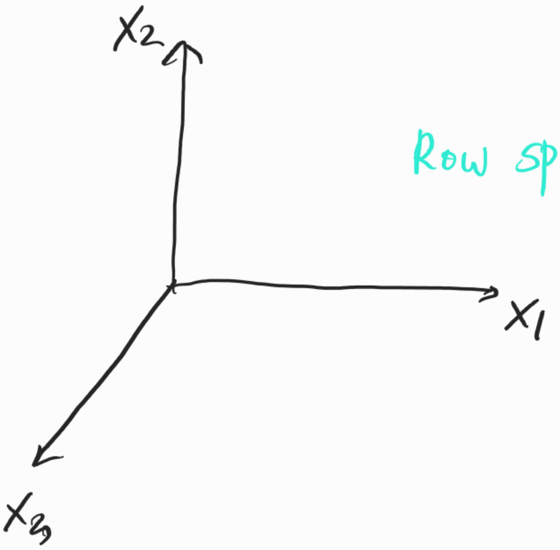
Rank 1 $\rightarrow$ # of basis vec for $C(A)$ & $C(A^T)$

$$1x_1 + 2\boxed{x_2} + 5\boxed{x_3} = 0$$

$\dim(N(A)) = 2$

$n = 3$ $\qquad$ $n - r$ = # basis for $N(A)$

r = # pivots in A.



Row space $\perp$ Nullspace

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \rightarrow C(A^T) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$B = \begin{bmatrix} 1 & 2 \end{bmatrix}$ $\qquad$ set $x_2 = 1$

$$\begin{bmatrix} \cdots \\ 0 & 0 \end{bmatrix}$$

$$x_1 + 2 = 0$$

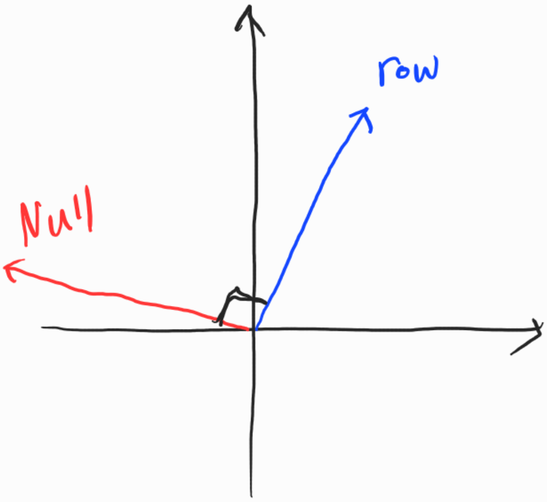
$$x_1 = -2$$

$$N(A) = \begin{bmatrix} -2 \\ 1 \end{bmatrix} x_2$$

$n = 2$

$\text{rank} = 1$

$$n - r = \dim(N(A)) = 2 - 1 = 1$$



$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = -2 + 2 = 0$$

$$1x_1 + 2x_2 + 5x_3 = 0$$

$x_2 = 1,\ x_3 = 0$ $\qquad$ $x_2 = 0\ \ x_3 = 1$

$x_1 = -2.$ $\qquad$ $x_1 = -5$

$$N(A) = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} x_3$$

$$C(A^T) = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \qquad N(A) = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} \right\}$$

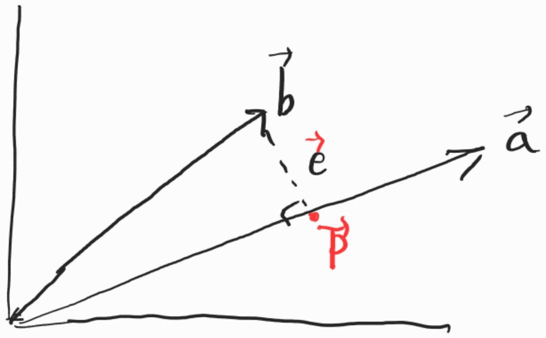
① $\begin{bmatrix} 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} -2 \\ \cdots \end{bmatrix}$

① $[1\ 2\ 3]\begin{bmatrix} \\ 1 \\ 0 \end{bmatrix} = 0$

② $[1\ 2\ 5]\begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} = 0$

$Ax = b$ has no solution $\rightarrow$ b is not in the $C(A)$

We are projecting b to the closest point in A's col space



p is the projection of b onto a.

$\vec{p} = x\vec{a}$

$\vec{a}^T\vec{e} = 0$

$\vec{a}^T(\vec{b} - \vec{p}) = 0$

$\vec{a}^T(\vec{b} - x\vec{a}) = 0$

$\vec{a}^T\vec{b} - \vec{a}^T x\vec{a} = 0$

$\vec{a}^T\vec{b} - x\vec{a}^T\vec{a} = 0$

$\underbrace{\vec{a}^T\vec{b}}_{\text{Scalar}} = x\underbrace{\vec{a}^T\vec{a}}_{\text{scalar}}$

$x = \dfrac{a^Tb}{a^Ta}$

proj $\boxed{p = xa = \dfrac{a^Tb}{a^Ta}\vec{a}}$

$p = Pb$

$P = \dfrac{aa^T}{\ }$ $\qquad xy^T =$ singular

$$\frac{\quad}{a^T a}$$

square

P is rank 1

P ends up being symmetric

↳ $P^T = P$

$\vec{p} = a_1 x_1 + a$

$p = A\hat{x}$

$e = b - p$

$e = b - A\hat{x}$

$a_1^T (b - A\hat{x}) = 0$ $\qquad$ $a_2^T (b - A\hat{x}) = 0$

$$\begin{bmatrix} a_1^T \\ a_2^T \end{bmatrix} (b - A\hat{x}) = 0$$

$$A^T (b - A\hat{x}) = 0$$

$$A^T b - A^T A \hat{x} = 0$$

$$A^T b = (A^T A)\hat{x}$$

$$(A^T A)^{-1} A^T b = \hat{x}$$

$$p = A\hat{x}$$

$$\boxed{\vec{p} = A(A^T A)^{-1} A^T b}$$

$$P = A(A^T A)^{-1} A^T$$

matrix