$$A = \begin{bmatrix} 2 & 8 & 4 \\ 2 & 5 & 1 \\ 4 & 10 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$$

using Mathlab

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 11 \\ -4 \\ 3 \end{bmatrix}$$

4×5 $AX = 0 =$

$$\begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 3 & 9 & 0 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

pivots (x_1 x_2 x_4)

$$\begin{bmatrix} \underline{1} & 0 & -2 & 0 & 1 \\ 0 & \underline{1} & 3 & 0 & -4 \\ 0 & 0 & 0 & \underline{1} & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

No pivots on column 3?

x_3, x_5 are free variables.

① Set $x_3 = 1$, $x_5 = 0$

$$\begin{cases} x_1 - 2 = 0 \rightarrow x_1 = 2 \\ x_2 + 3 = 0 \rightarrow x_2 = -3 \\ x_4 = 0 \rightarrow x_4 = 0 \end{cases}$$

$$\begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3$$

② Set $x_3 = 0$, $x_5 = 1$

$$\begin{cases} x_1 + 1 = 0 \rightarrow x_1 = -1 \\ x_2 - 4 = 0 \rightarrow x_2 = 4 \\ x_4 - 1 = 0 \rightarrow x_4 = 1 \end{cases}$$

$$\begin{bmatrix} -1 \\ 4 \\ 0 \\ 1 \\ 1 \end{bmatrix} x_5$$

$N(A) = [v_1]x_3 + [v_2]x_5$ form

$Ax = b$ $\quad x = x_{particular} + x_{null\ space}$ $\quad b = \begin{bmatrix} -1 \\ 6 \\ 7 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 1 & 3 & 1 & 6 \end{bmatrix} \xrightarrow{rref} \left[\begin{array}{cccc|c} 1 & 3 & 0 & 2 & 1 \\ 0 & 0 & 1 & 4 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Consistent

$x = x_p + x_n$

set free variables $= 0$

$x_2, x_4 = 0$

$x_1 + 3(0) + 0x_3 + 2(0) = 1$

$\therefore x_1 = 1 \quad , \quad x_3 = 6$

$$x_p = \begin{bmatrix} 1 \\ 0 \\ 6 \\ 0 \end{bmatrix}$$

$$\boxed{Ax_p = b}$$

$N(A)$

① Set $x_2 = 1, \; x_4 = 0$

$$1x_1 + 3(1) + 0x_3 + 2(0) = 0$$

$$\boxed{Ax_n = 0}$$

$\therefore x_1 = -3$

$x_3 = 0$

$$\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_2$$

② set $x_2 = 0, \; x_4 = 1$

$\therefore x_1 = -2$

$x_3 = -4$

$$\begin{bmatrix} -2 \\ 0 \\ -4 \\ 1 \end{bmatrix} x_4$$

one solution

infinetely many solutions

$$x = \begin{bmatrix} 1 \\ 0 \\ 6 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -2 \\ 0 \\ -4 \\ 1 \end{bmatrix} x_4$$

$Ax_p = b$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 2 & 1 \\ 0 & 0 & 1 & 4 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$Ax_n = 0$