Voronoi Fortune Algorithm Introduction

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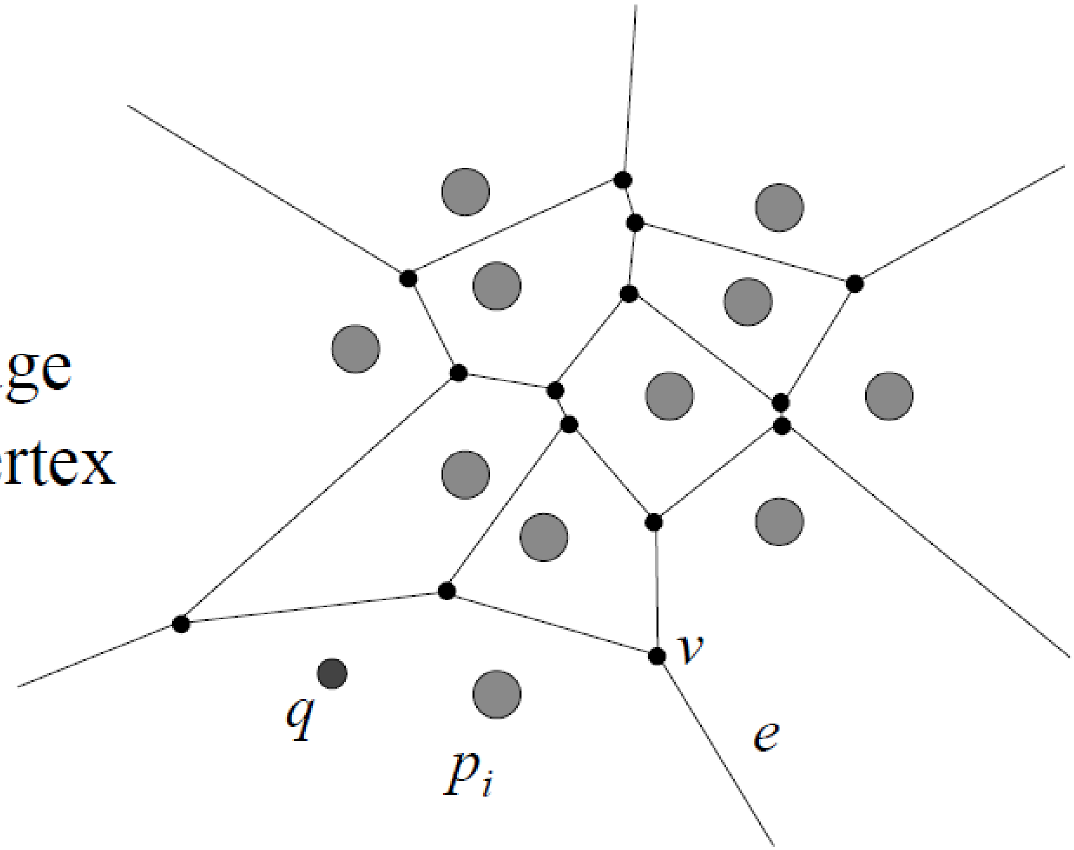
Post Office: What is the area of service?

p_i : site points

q : free point

e : Voronoi edge

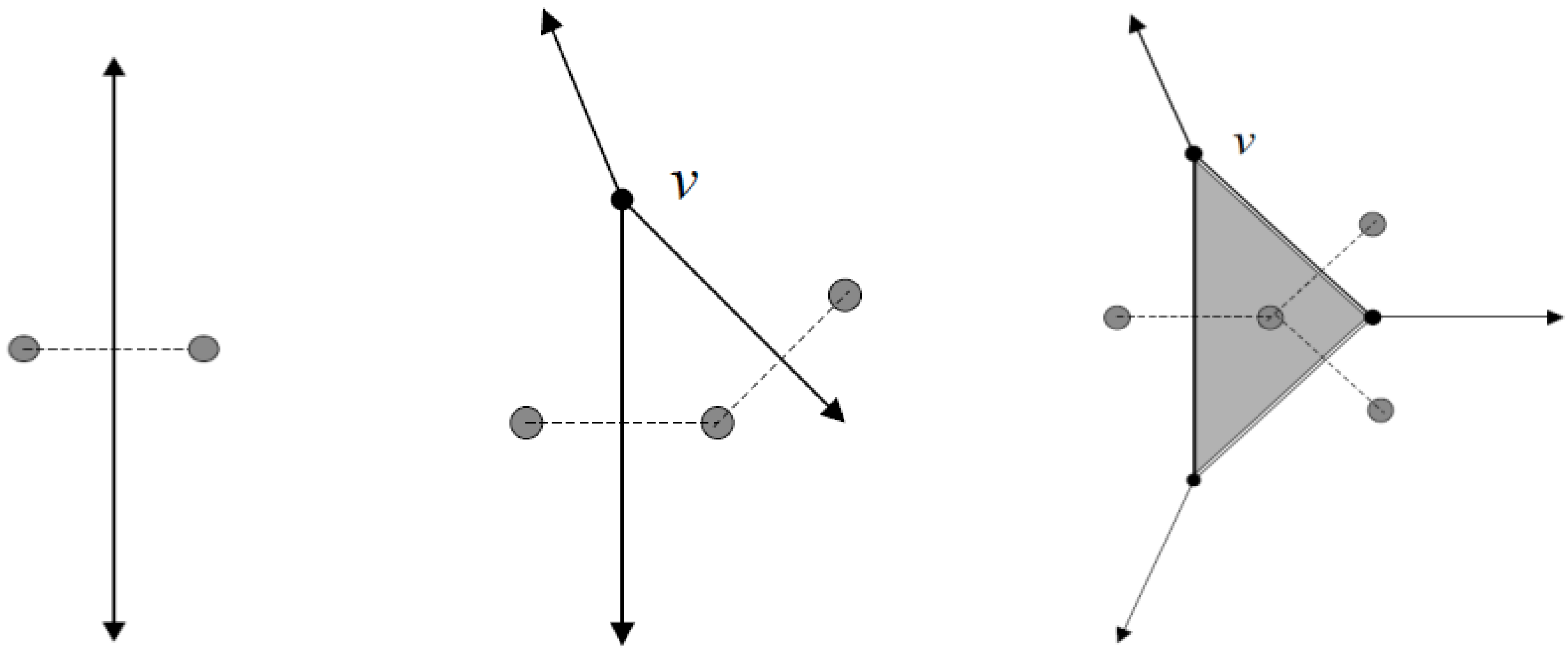
v : Voronoi vertex



Definition of Voronoi Diagram

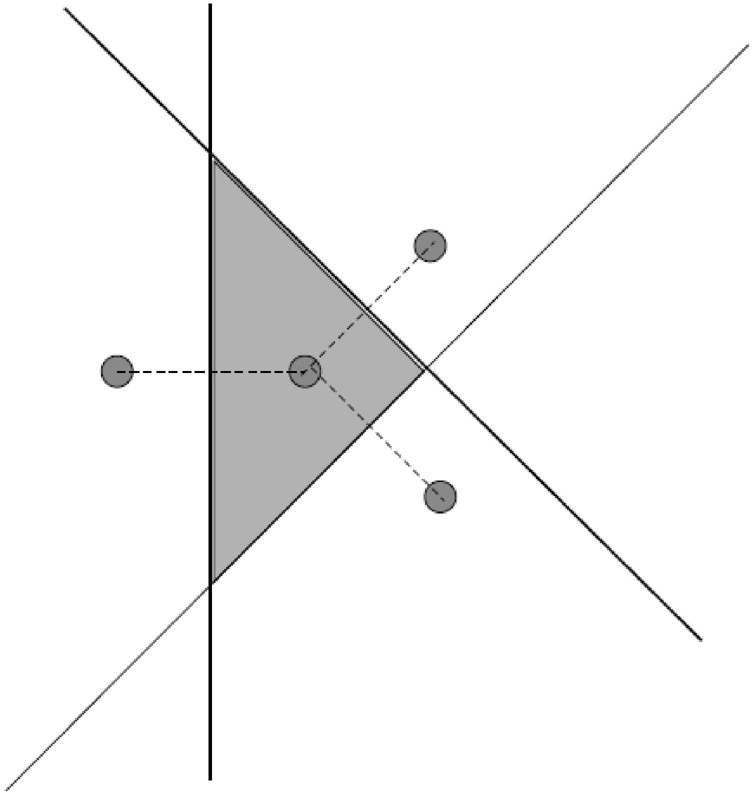
- Let P be a set of n distinct points (sites) in the plane.
- The Voronoi diagram of P is the subdivision of the plane into n cells, one for each site.
- A point q lies in the cell corresponding to a site $p_i \in P$ *iff*
  Euclidean_Distance(q, p_i) < Euclidean_distance(q, p_j),
  for each $p_i \in P, j \neq i$.

Typical Voronoi Diagram



Half Plane Intersection Algorithm

Repeat for each site



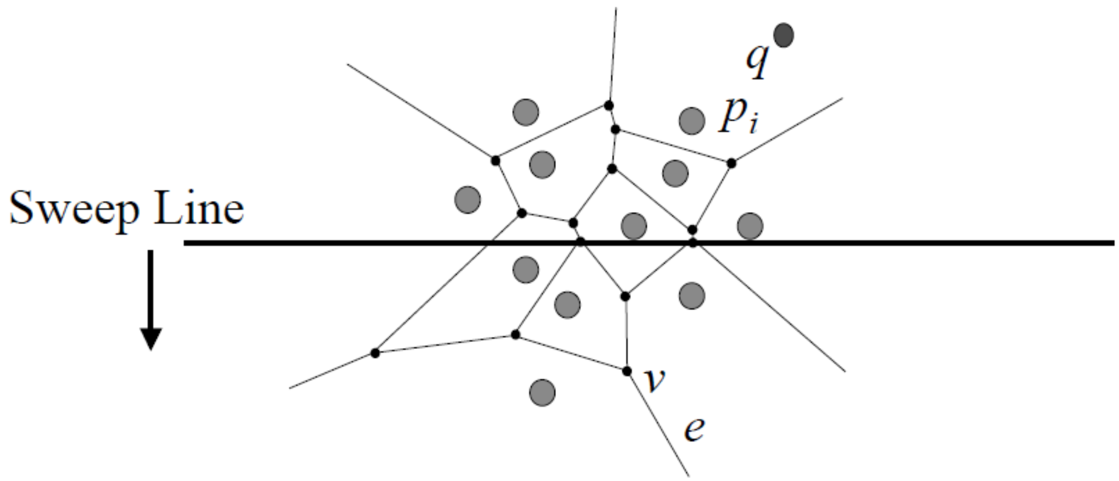
Running Time:
O($n^2 \log n$)

Fortune Algorithm (Sweep Line Algorithm)

- It is an incremental construction
- A horizontal line is swept among the sites from top to bottom
- It maintains portion of Voronoi diagram which does not change due to the appearance of new sites below sweep line;
- It keeps track of incremental changes of the Voronoi diagram that is caused for the appearance of each site on the sweep line.

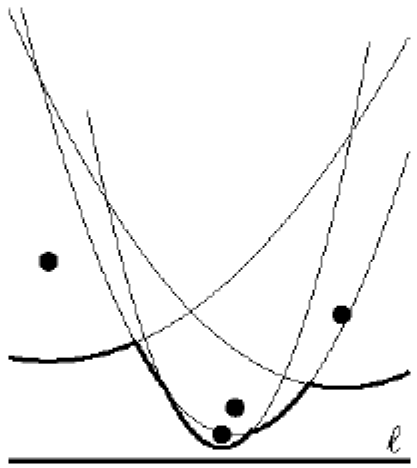
Fortune Algorithm (Sweep Line Algorithm)

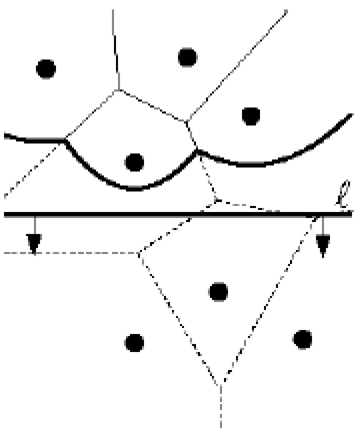
What is the invariant we are looking for?



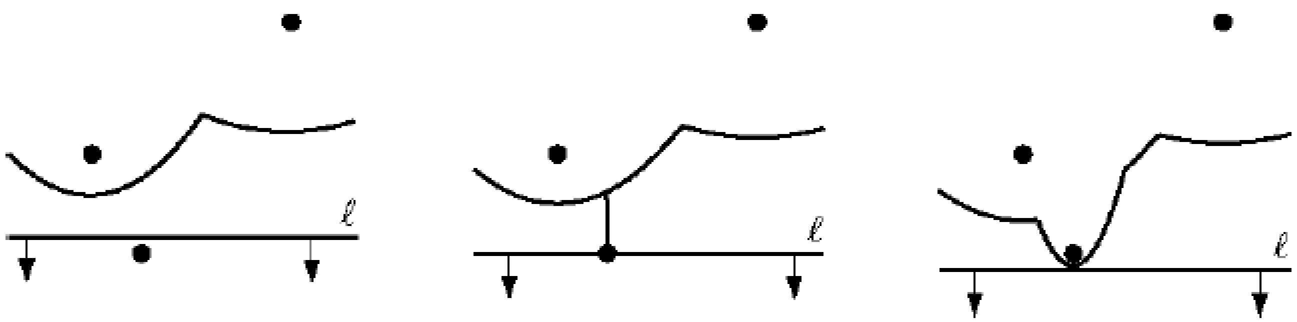
Maintain a representation of the locus of points q that are closer to some site p_i *above* the sweep line than to the line itself (and thus to any site below the line).

Fortune Algorithm (Sweep Line Algorithm)

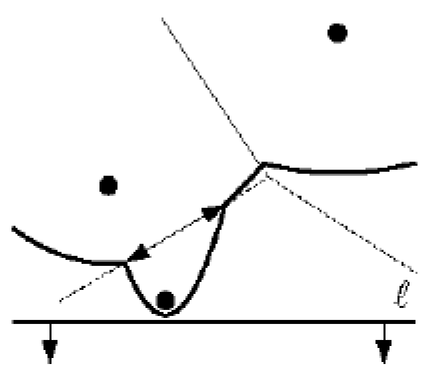




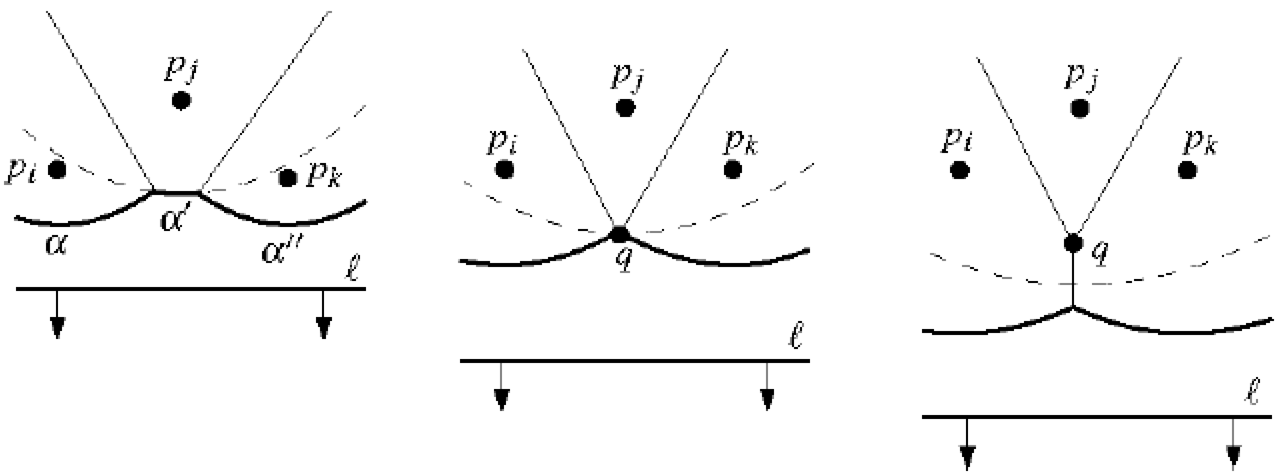
Fortune Algorithm (Sweep Line Algorithm)



site event



Fortune Algorithm (Sweep Line Algorithm)



circle event

Fortune Algorithm (Sweep Line Algorithm)

Beach Line properties

- Voronoi edges are traced by the break points as the sweep line moves down.
  - Emergence of a new break point(s) (from formation of a new arc or a fusion of two existing break points) identifies a new edge
- Voronoi vertices are identified when two break points meet (fuse).
  - Decimation of an old arc identifies new vertex

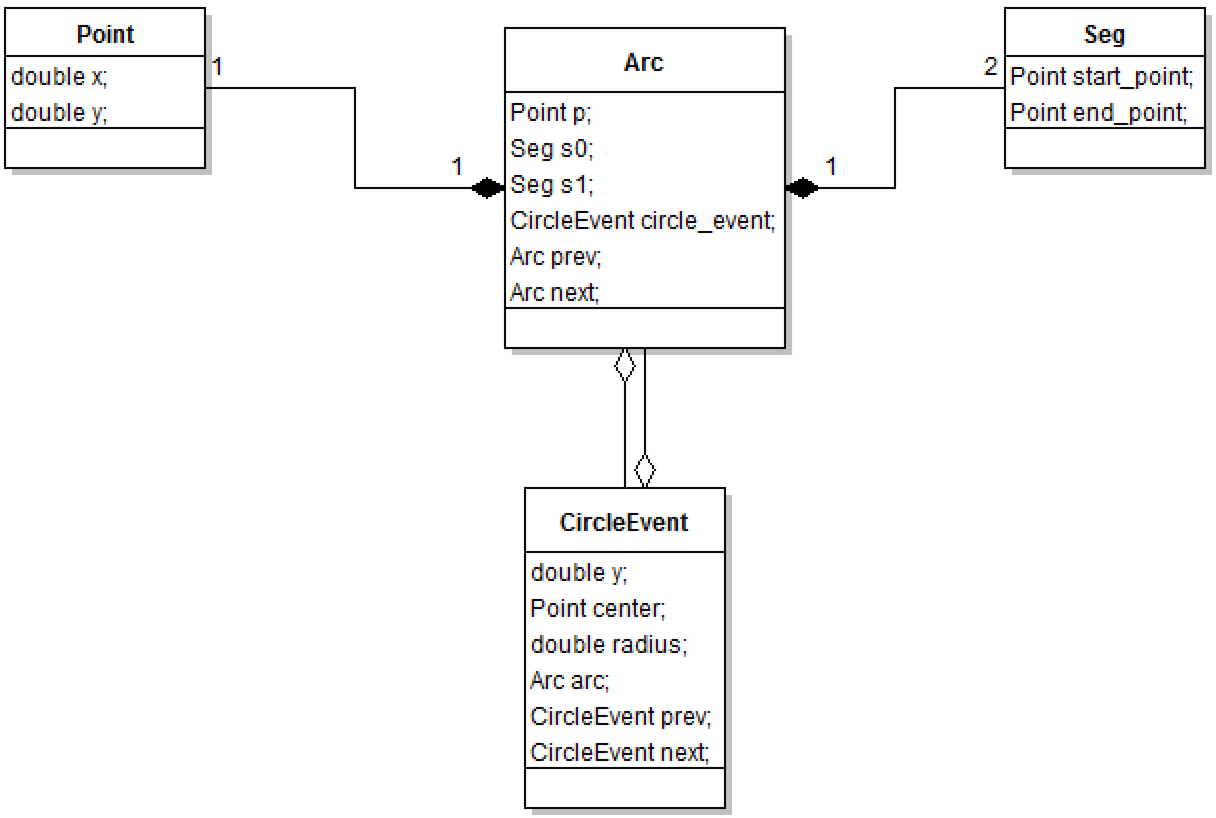
Fortune Algorithm (Sweep Line Algorithm)

Total Running Time

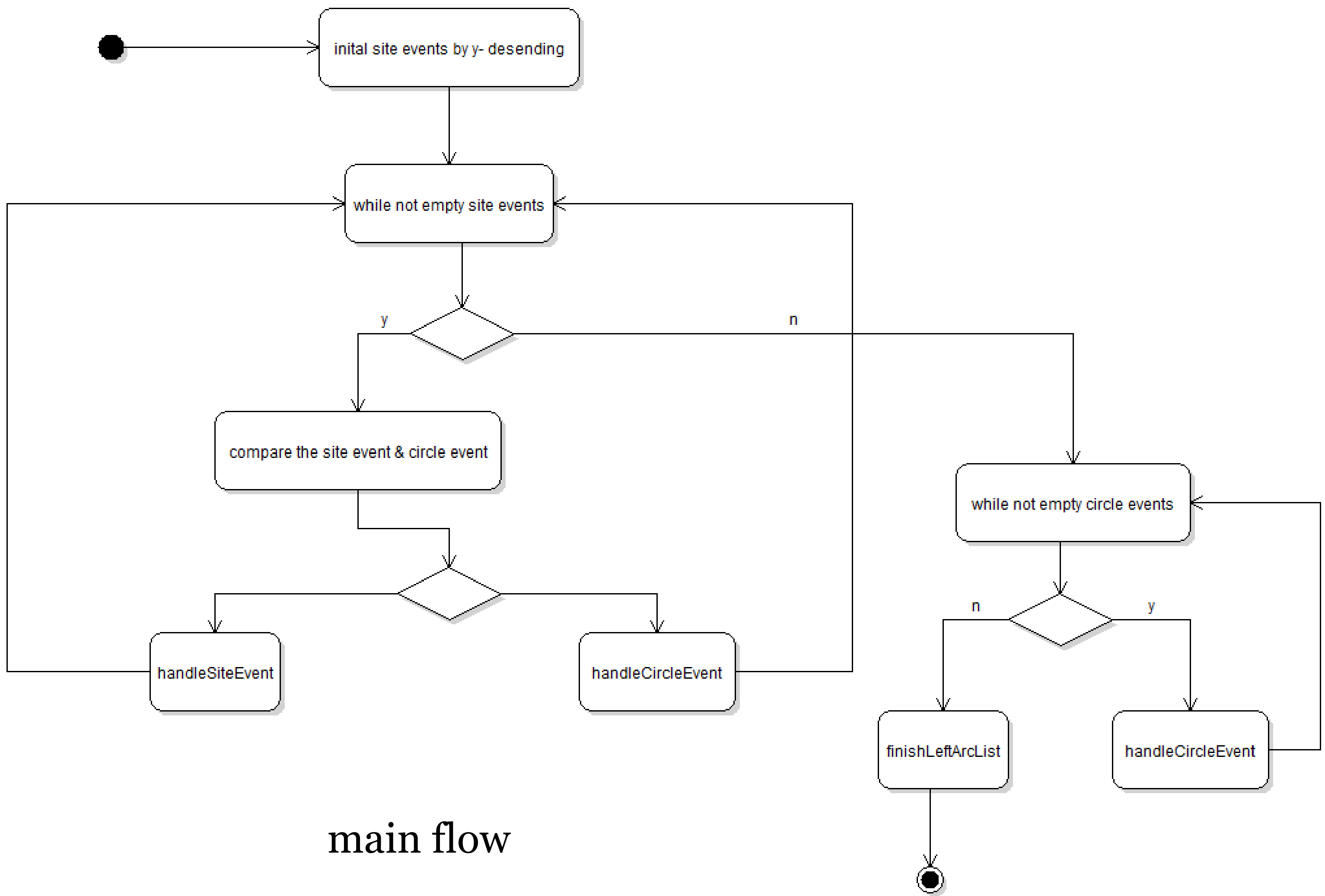
- Each new site can generate at most two new arcs
  - → beach line can have at most $2n - 1$ arcs
  - → at most $O(n)$ site and circle events in the queue
- Site/Circle Event Handler $O(\log n)$

→ $O(n \log n)$ total running time

Fortune Algorithm (Sweep Line Algorithm)

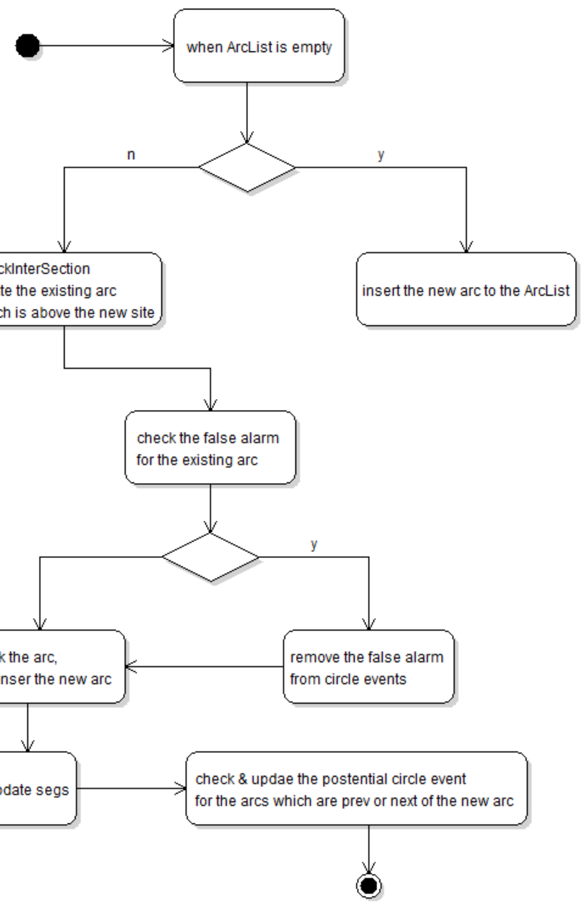


Fortune Algorithm (Sweep Line Algorithm)



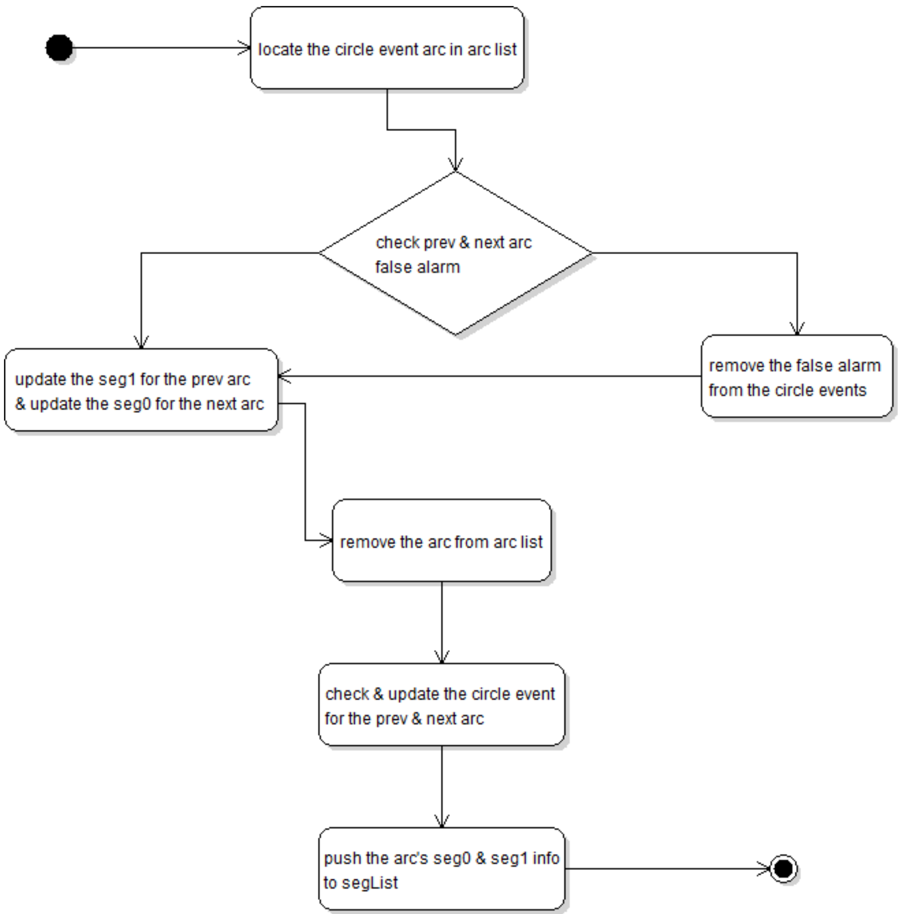
main flow

Fortune Algorithm (Sweep Line Algorithm)



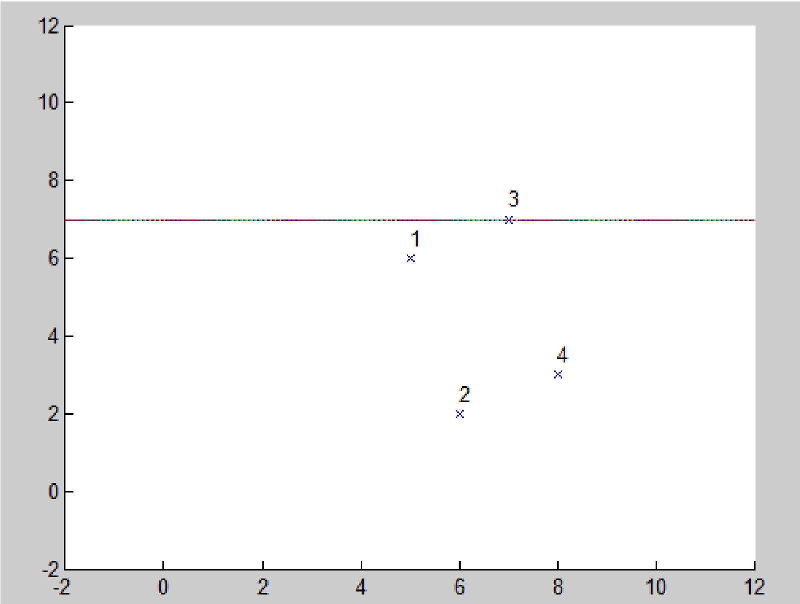
site event

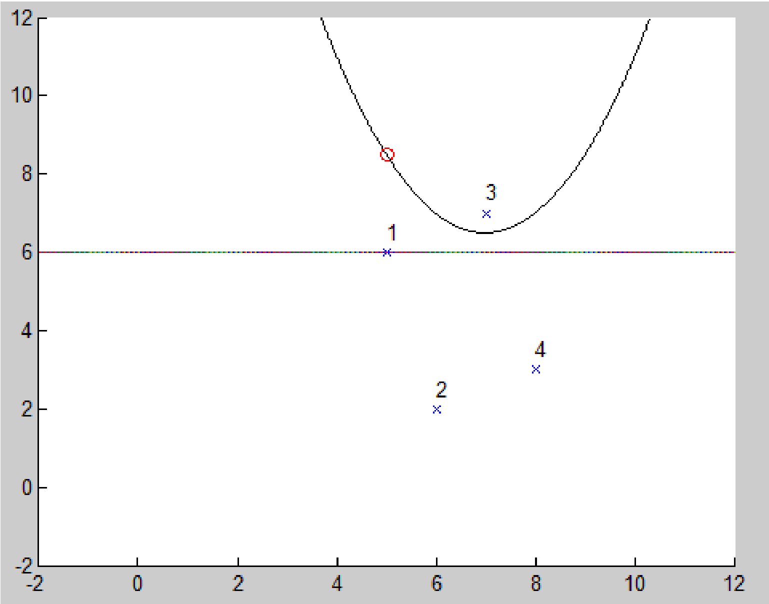
Fortune Algorithm (Sweep Line Algorithm)



circle event

Fortune Algorithm (Sweep Line Algorithm)

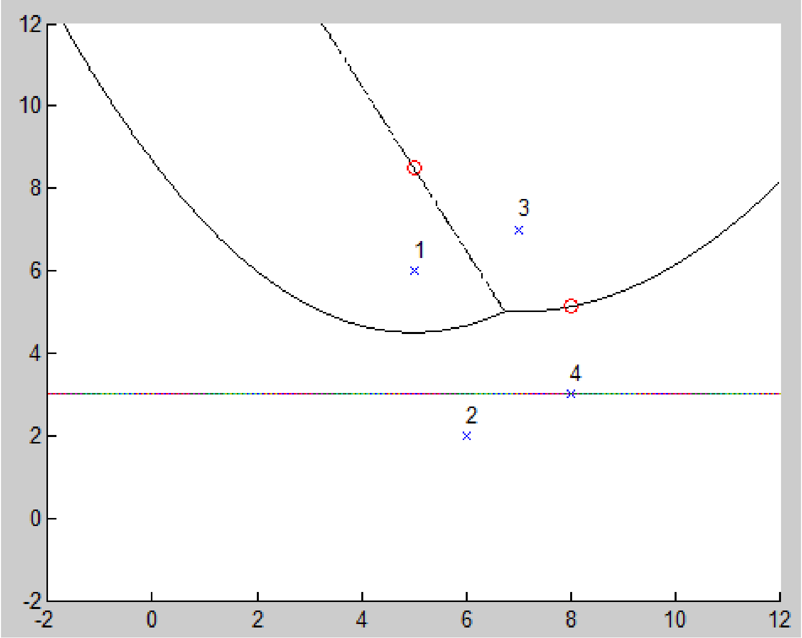


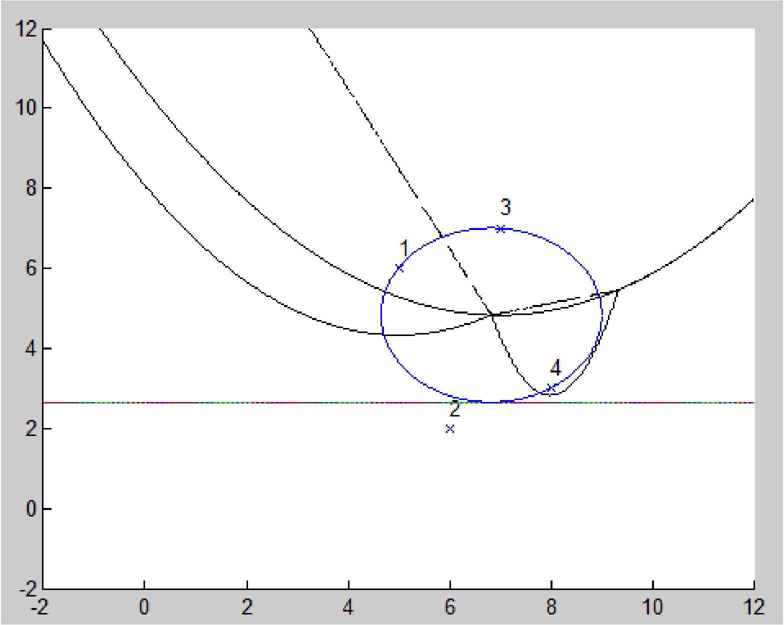


Arc(p3) -> ArcList

Arc(p3)<->Arc(P1)<->Arc(p3)

Fortune Algorithm (Sweep Line Algorithm)





Arc(p3)<->Arc(P1)<->Arc(p3)<->Arc(p4)<->Arc(p3)

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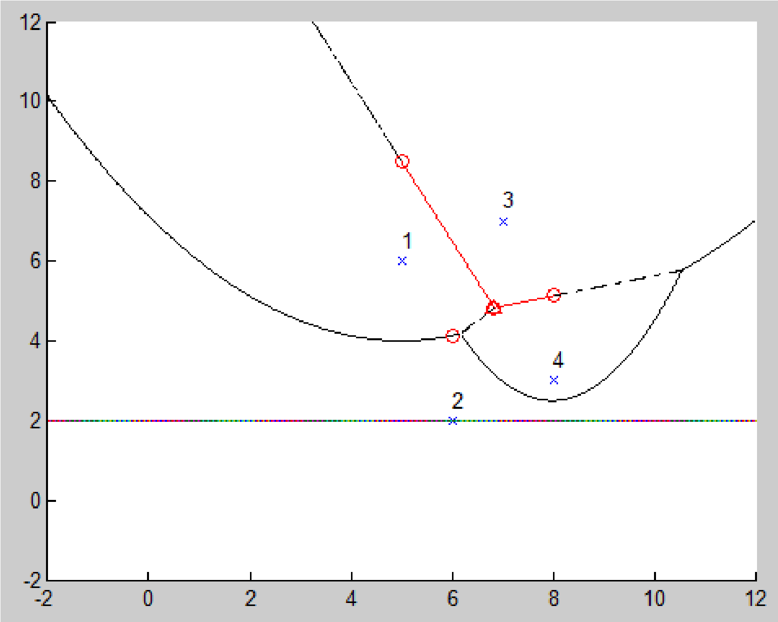
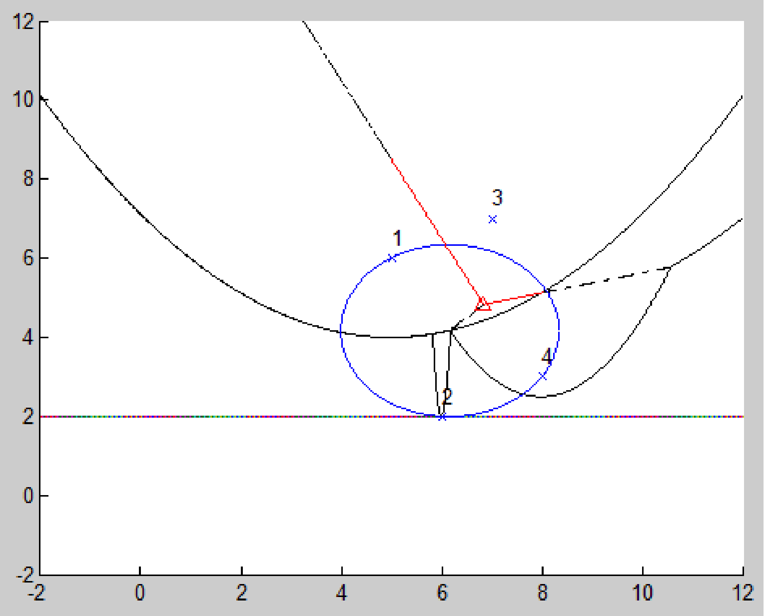
Fortune Algorithm (Sweep Line Algorithm)





Arc(p3)<->Arc(P1)<->Arc(p2)<->Arc(p1)<->Arc(p4)<->Arc(p3)

Arc(p3)<->Arc(P1)<->Arc(p2)<->Arc(p1)<->Arc(p4)<->Arc(p3)

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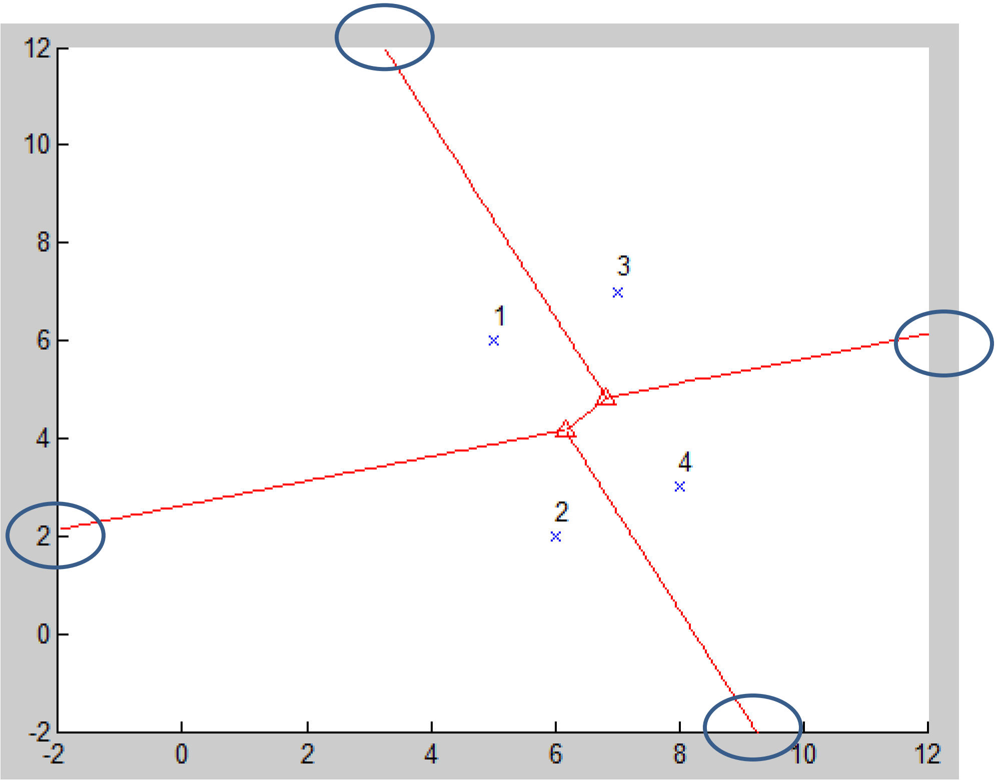

Arc(p3)<->Arc(P1)<->Arc(p2)<->Arc(p4)<->Arc(p3)

Question?