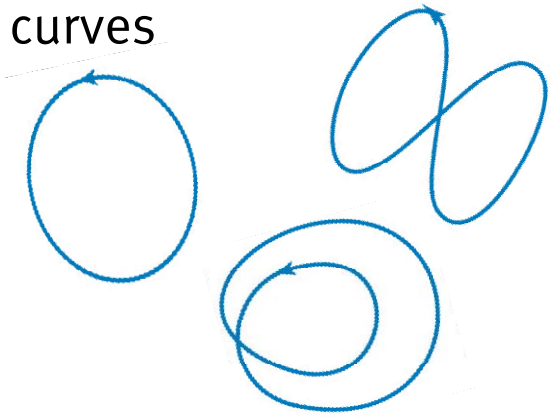
Discrete Curves

Warmup: Smooth Setting

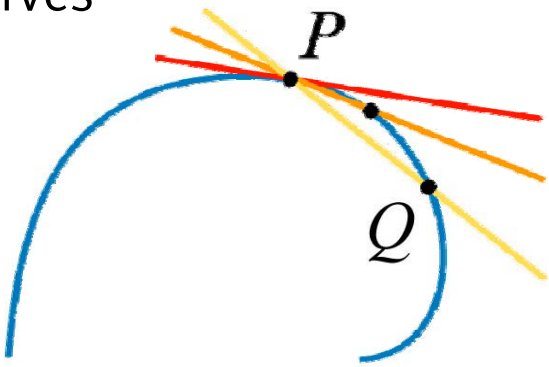
Univariate curves



Warmup: Smooth Setting

Univariate curves

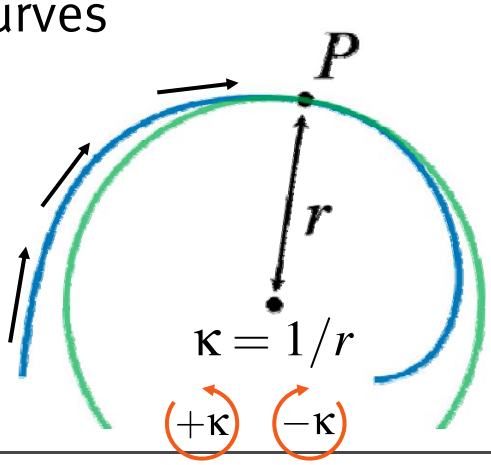
- secant
- tangent



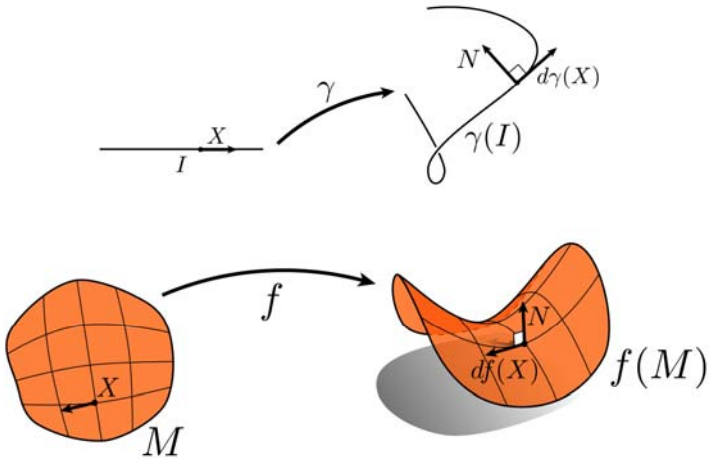
Warmup: Smooth Setting

Univariate curves

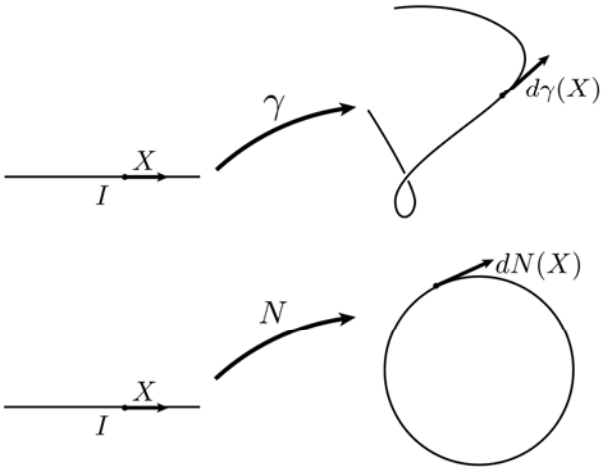
- secant
- tangent
- circle
- curvature
  - signed



Differential Setup



The Normal



Gauß Map: $N(s)$

arc length param

Map to unit circle (or sphere)

- its differential

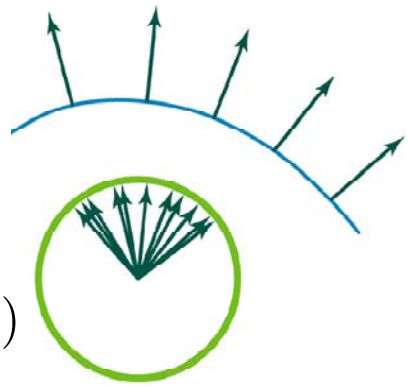
$$dN(X) = df(SX)$$

tangent vector

Shape Operator

- more generally

$$II(X,Y) = g(SX,Y)$$



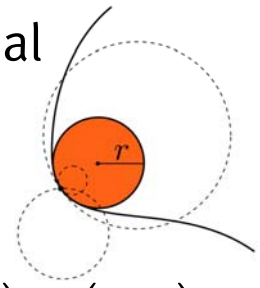
Framed Curve

Tangent, normal, bi-normal

- Frenet frame

$$d\gamma = T \quad dT = \kappa N$$

$$B = T \times N$$



$$\begin{pmatrix} dT \\ dN \\ dB \end{pmatrix} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix} \begin{pmatrix} T \\ N \\ B \end{pmatrix}$$