Bijective Mappings

Jian-Ping Su

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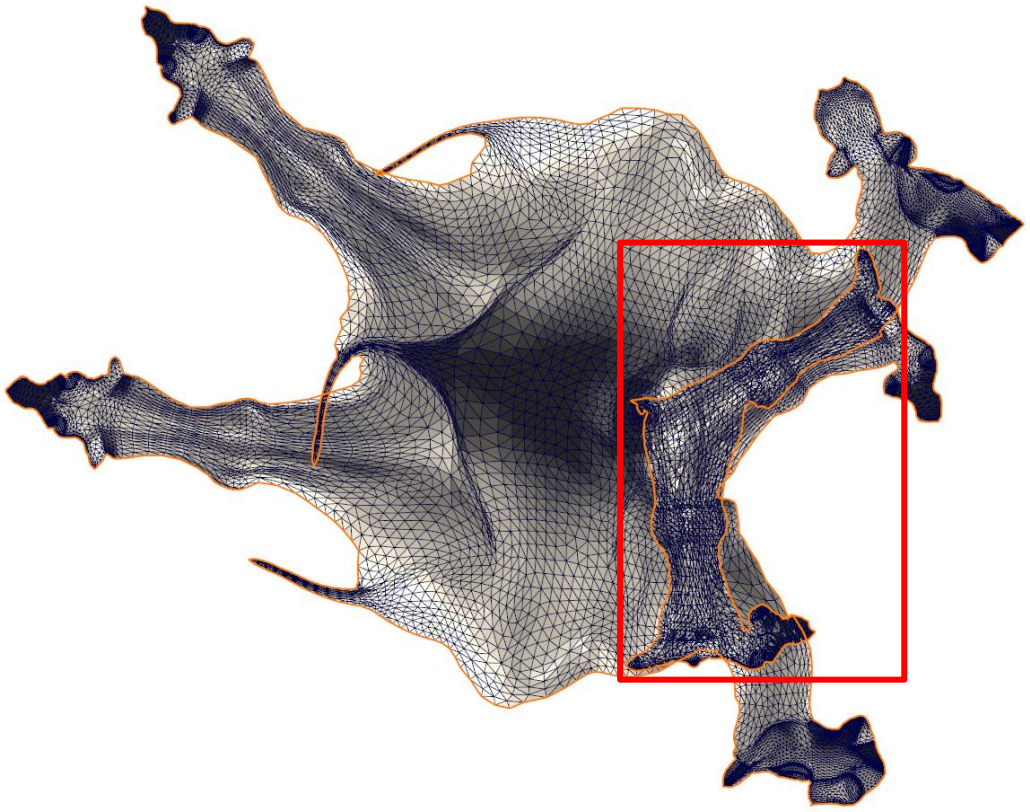
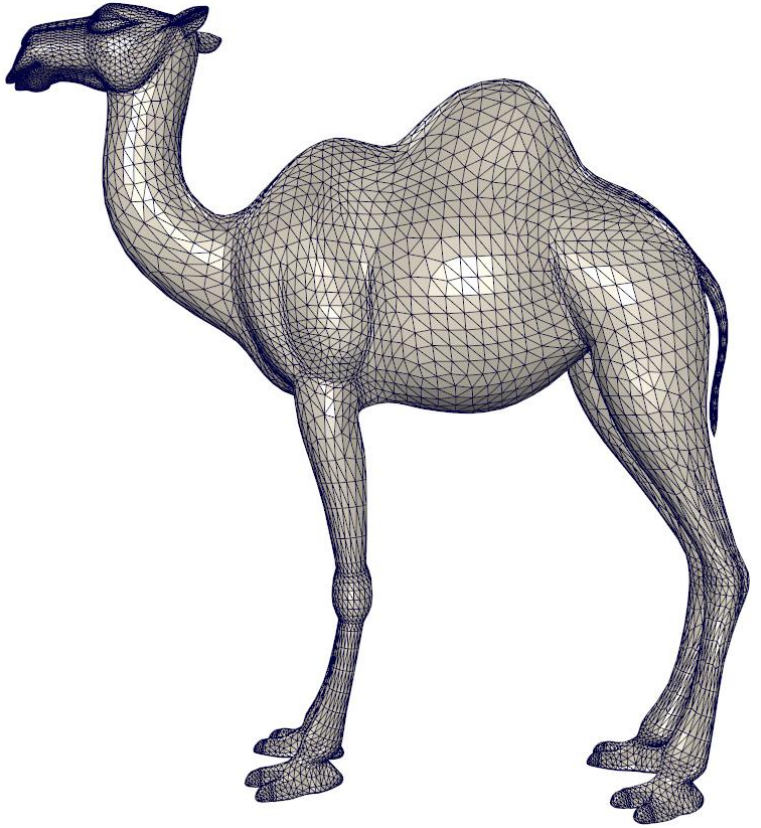
USTC 2020 Spring Digital Geometry Processing

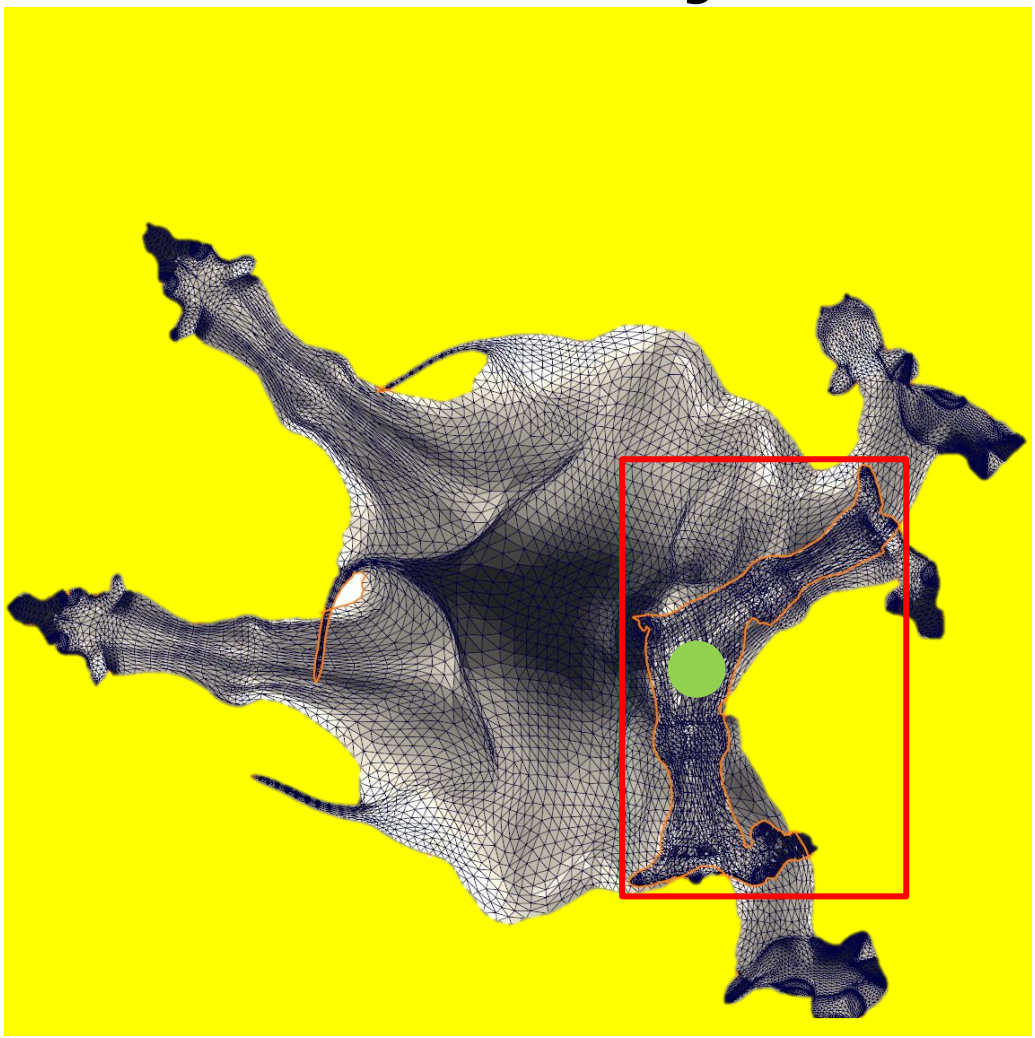
Bijective Mappings



Injective

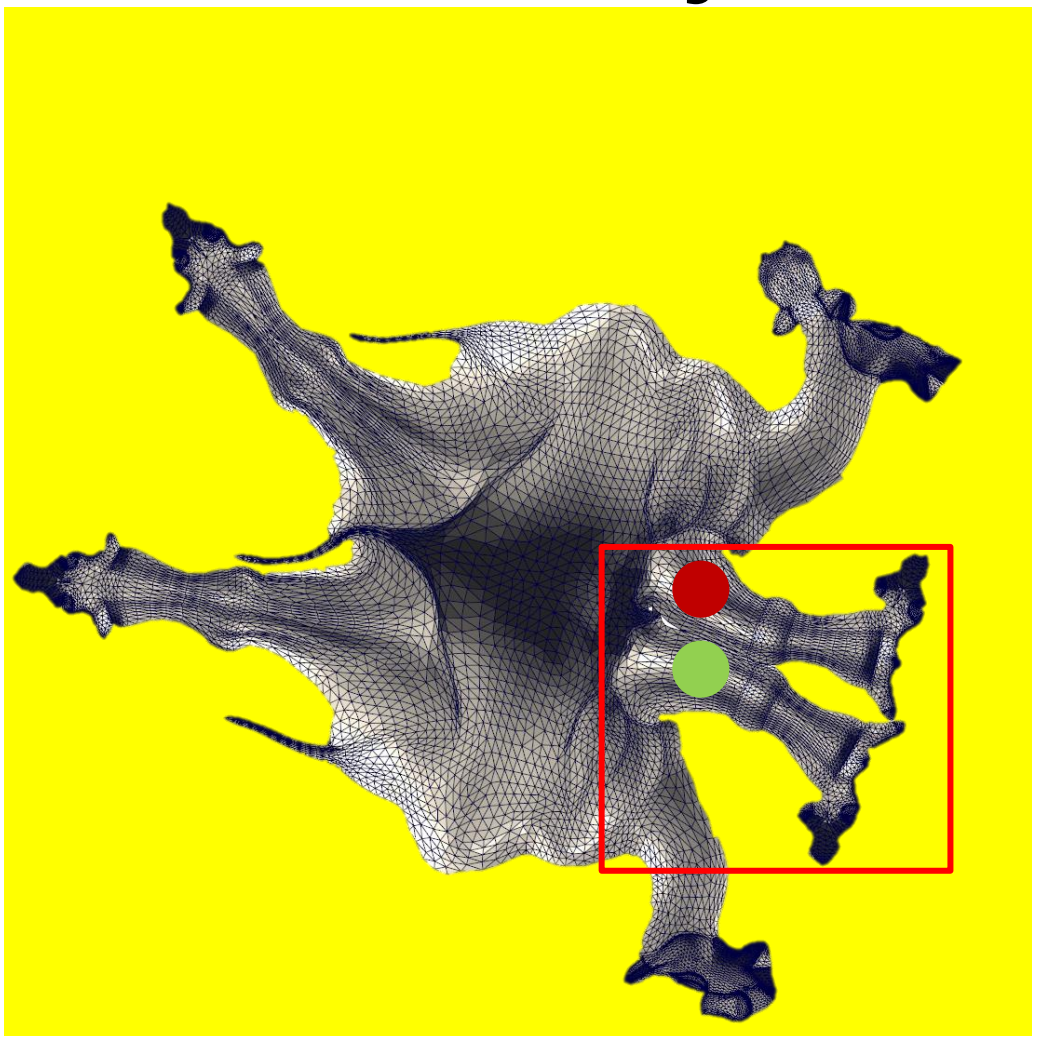


Bijective Mappings



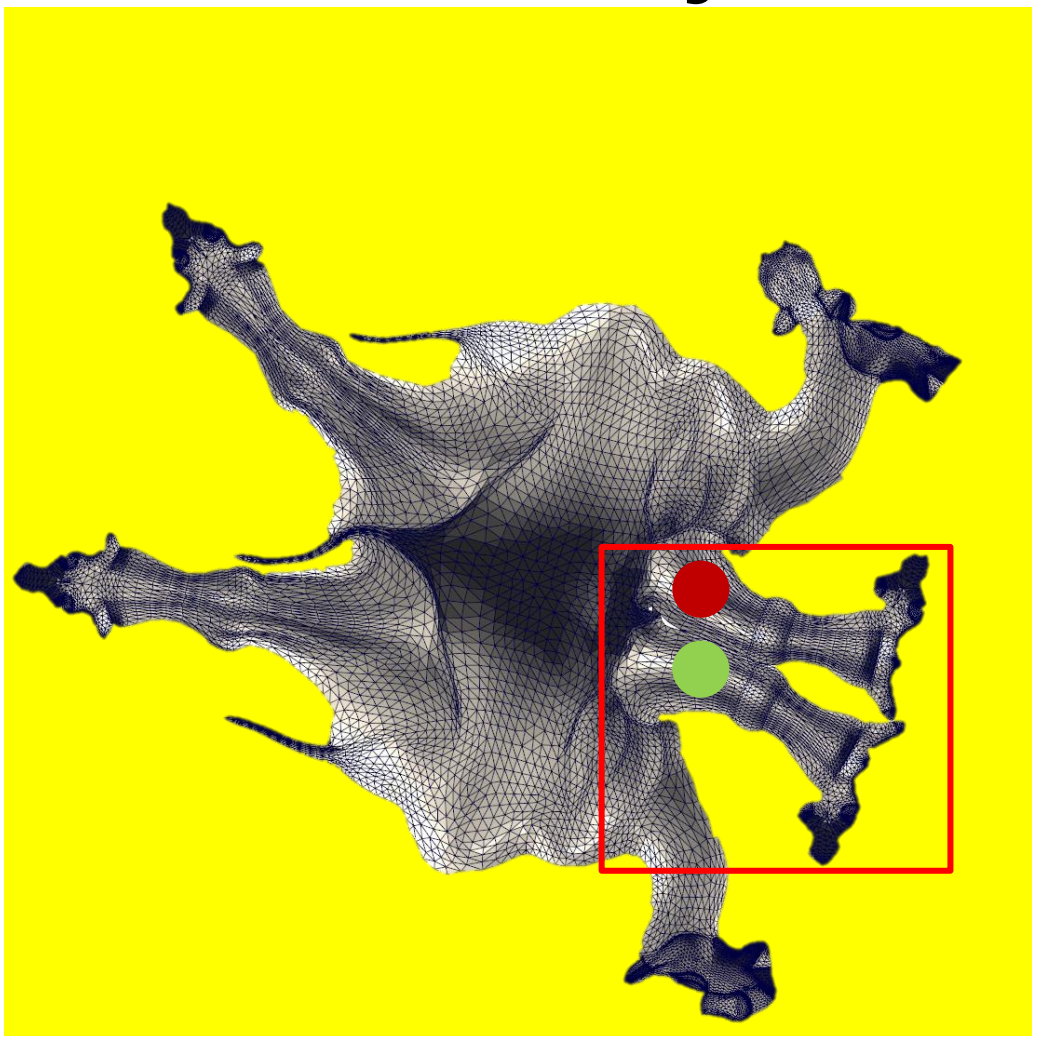
Injective

Bijective Mappings



Bijective

Bijective Mappings



Bijective

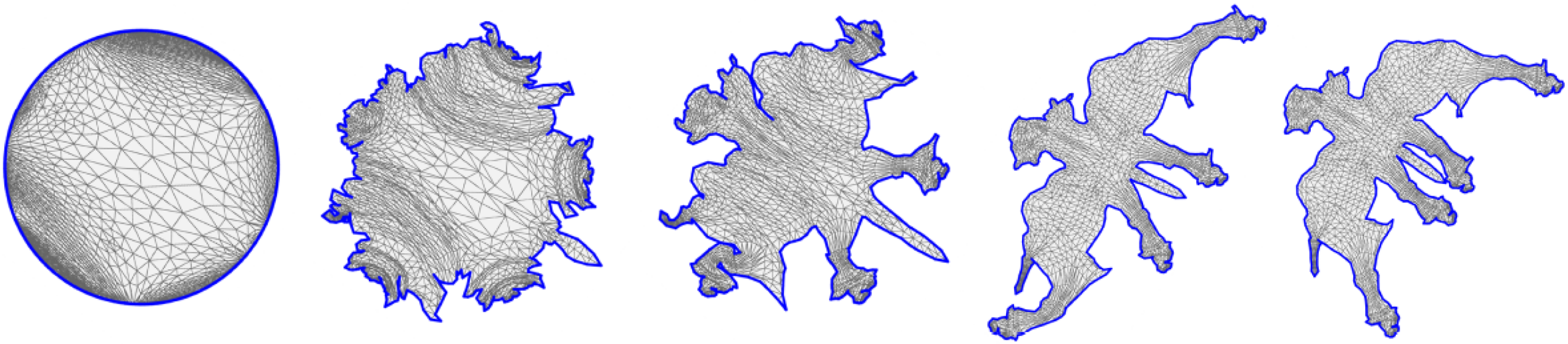
Existing work

Paper List

- Bijective Parameterization with Free Boundaries.
- Simplicial Complex Augmentation Framework for Bijective Maps.
- Efficient Bijective Parameterizations

Bijective Parameterization with Free Boundaries

Jason Smith, Scott Schaefer

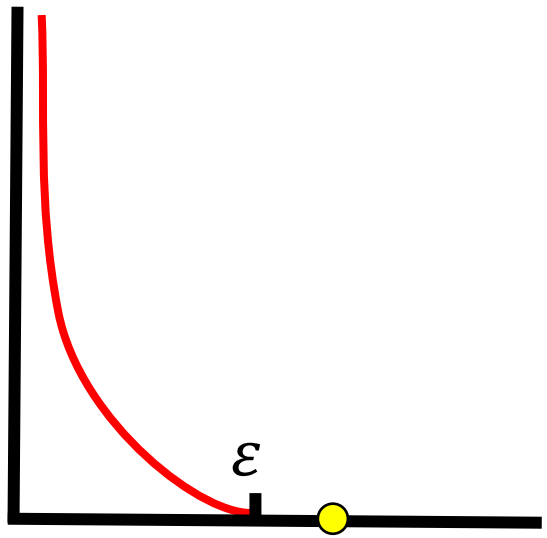


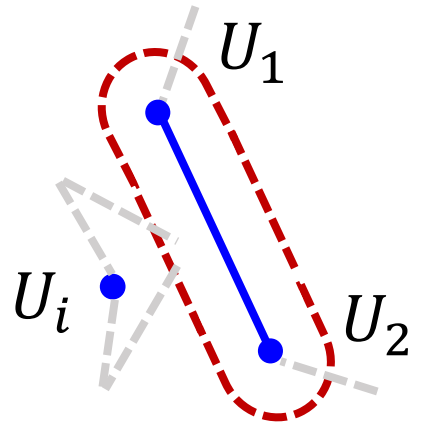
Barrier function

distortion barrier

$$\min \boxed{E_D} + \boxed{E_B}$$

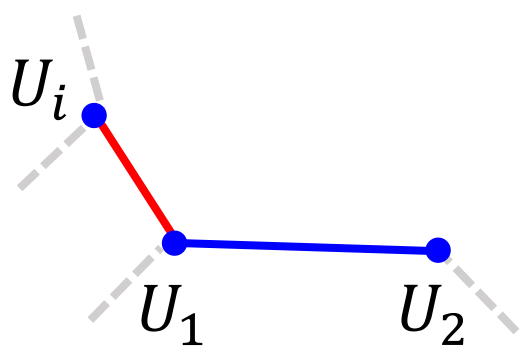
$$E_B = \max\left(0, \frac{\varepsilon}{dist(U_1, U_2, U_i)} - 1\right)^2$$

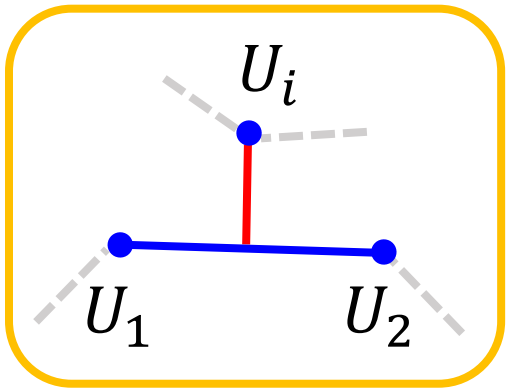


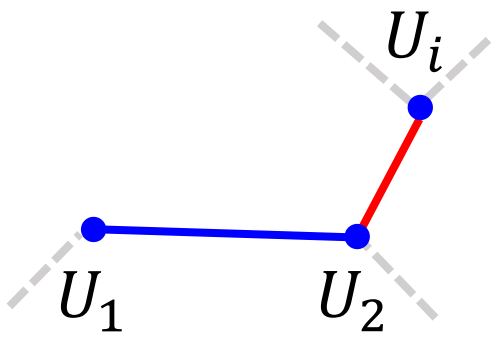


Definition of distance

- Distance is not C^2.
- Hessian of barrier function is difficult to compute.
- Convex-concave decomposition is not easy.

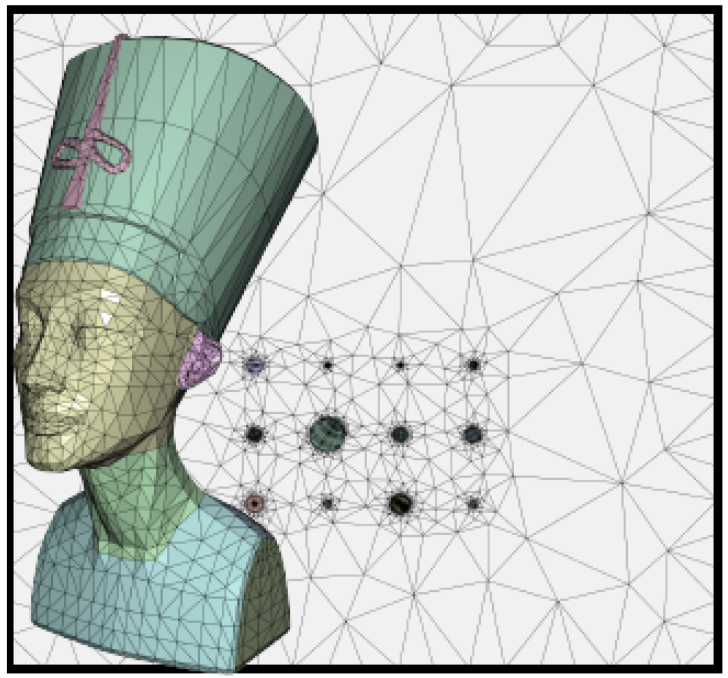
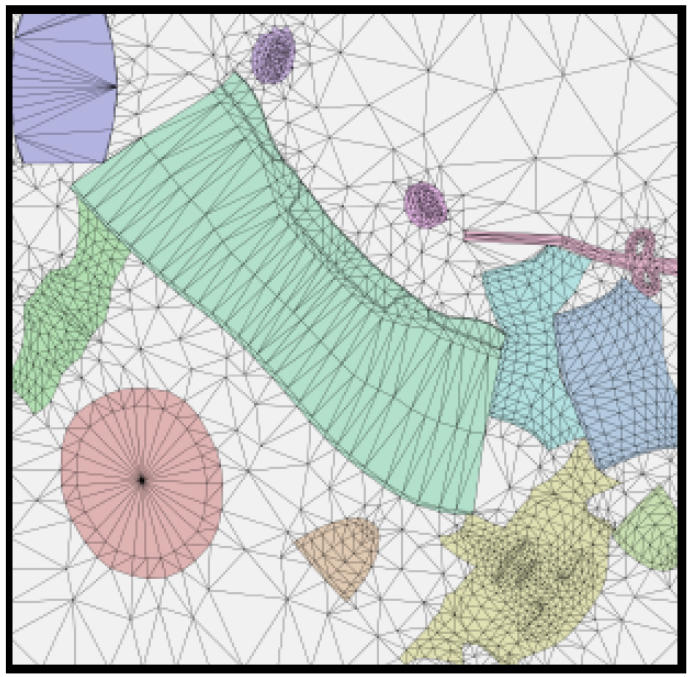
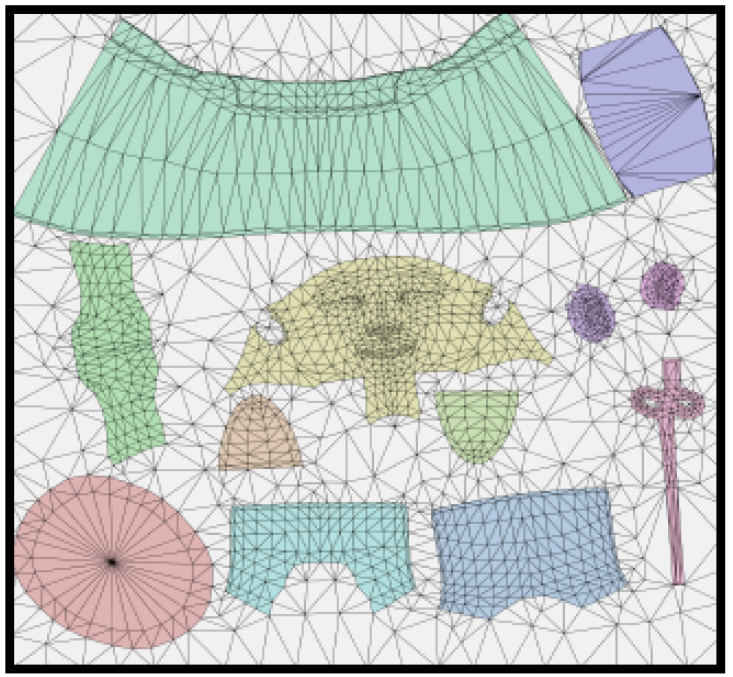
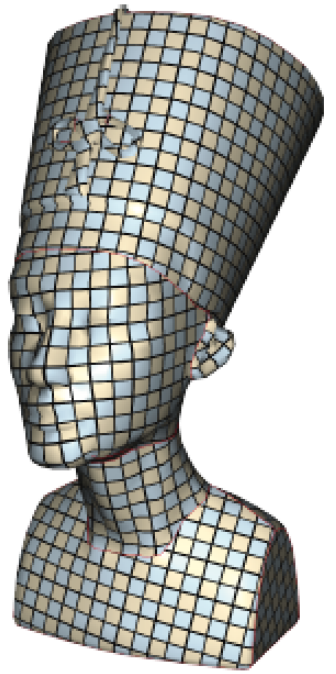






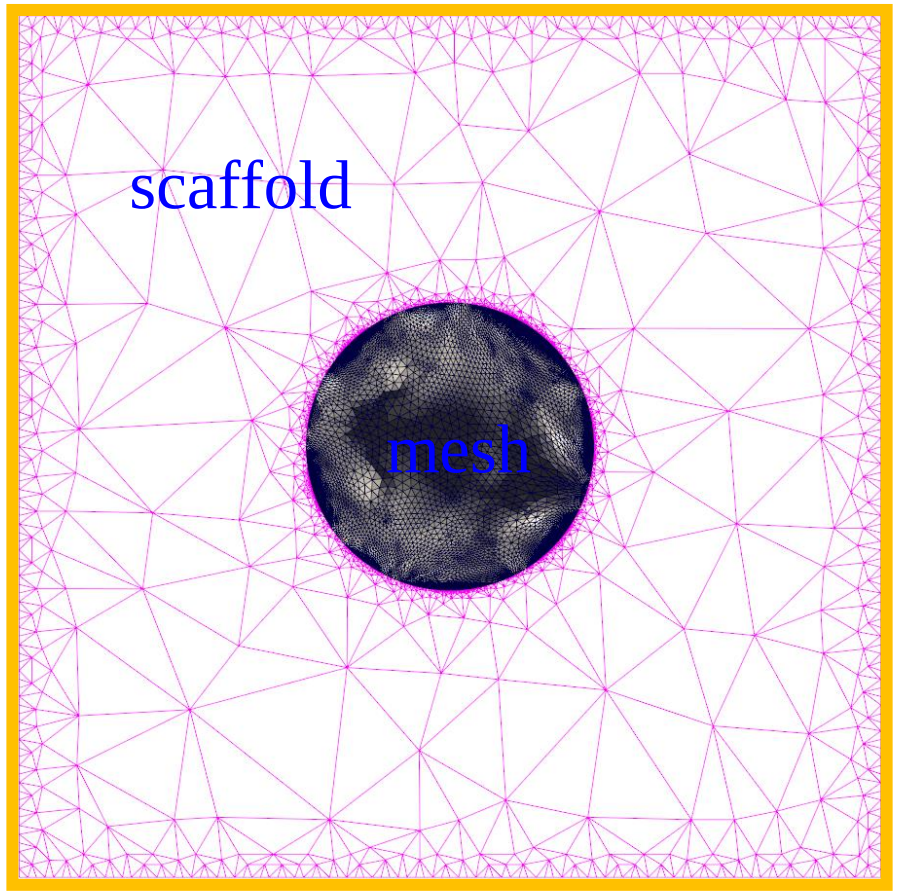
Simplicial Complex Augmentation Framework for Bijective Maps

Zhongshi Jiang, Scott Schaefer, Daniele Panozzo

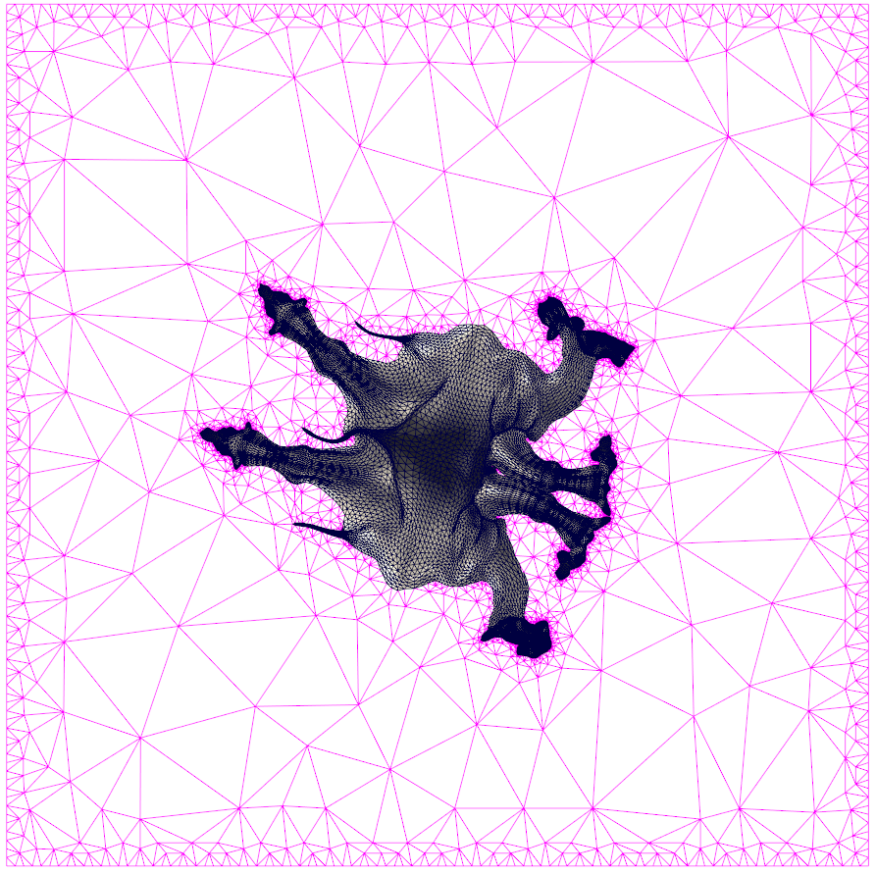





Scaffold structure

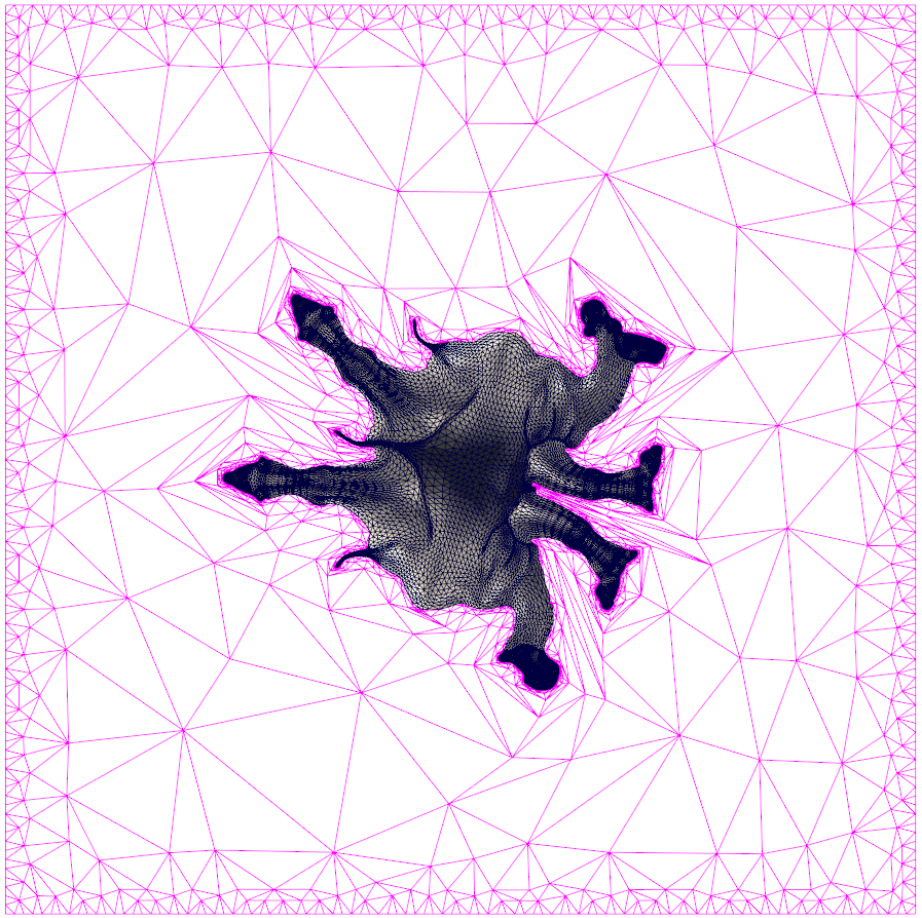
$$\min\ E_M + E_S$$



Fix boundary
Locally injective



Remesh



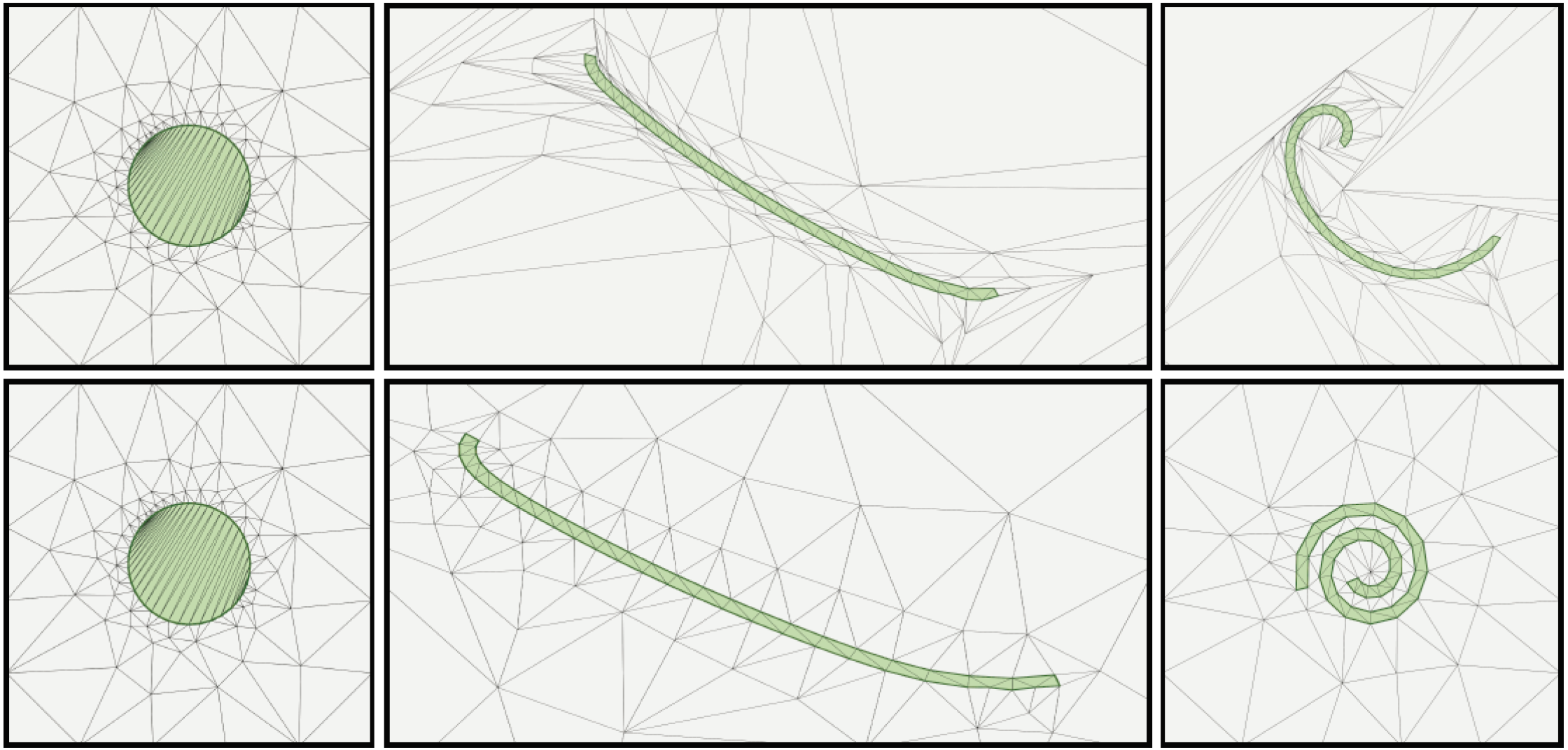
$$\min \; E_M + E_S$$

Hessian's sparse structure changes

Solving sparse equations

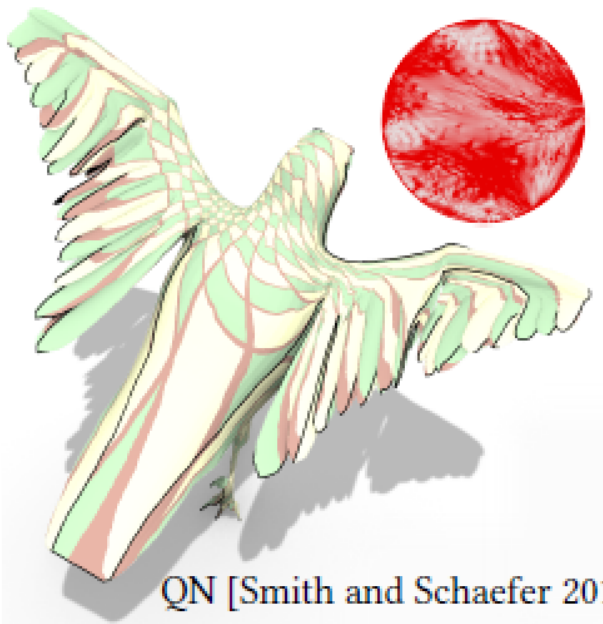
- Symbolic phase (nonzero structure) (0.071)
- Numerical phase (value) (0.012)
- Solve phase (value)

Remesh

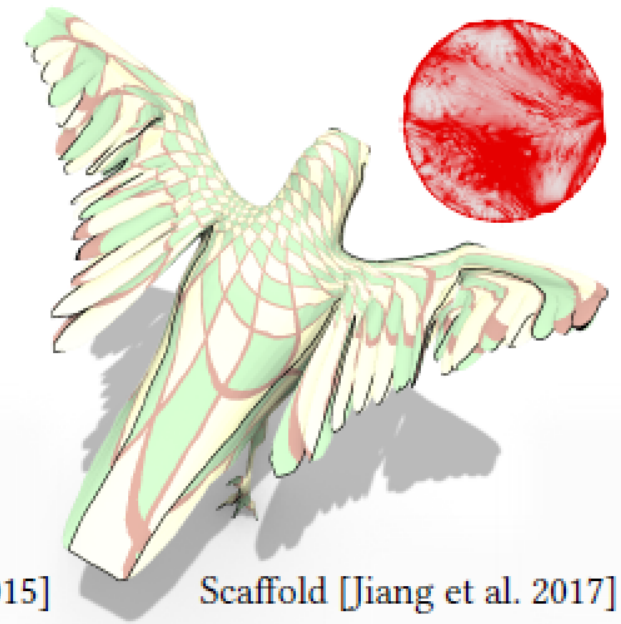


Efficient Bijective Parameterizations

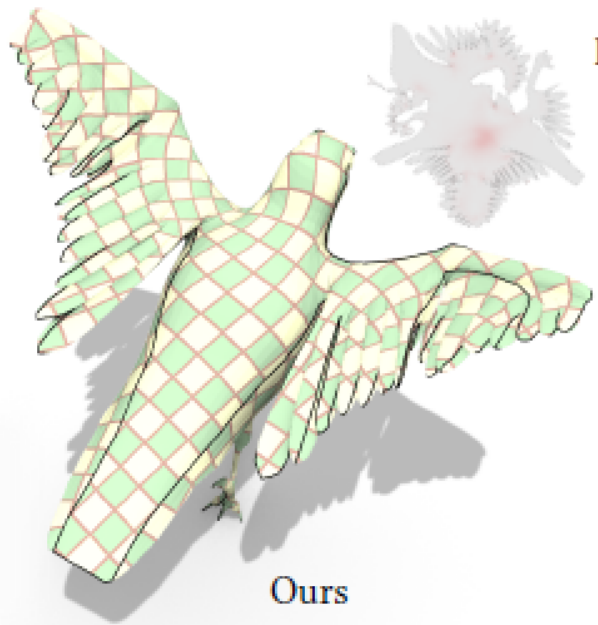
Jian-Ping Su , Chunyang Ye, Ligang Liu, Xiao-Ming Fu



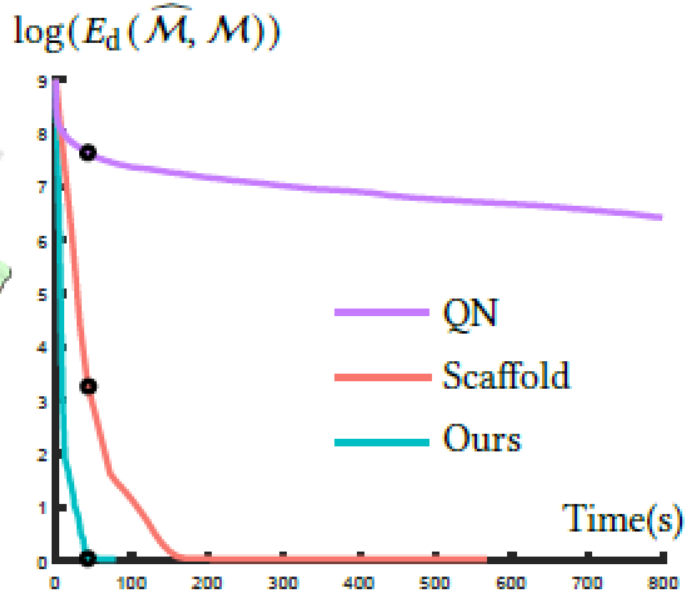
QN [Smith and Schaefer 2015]



Scaffold [Jiang et al. 2017]



Ours

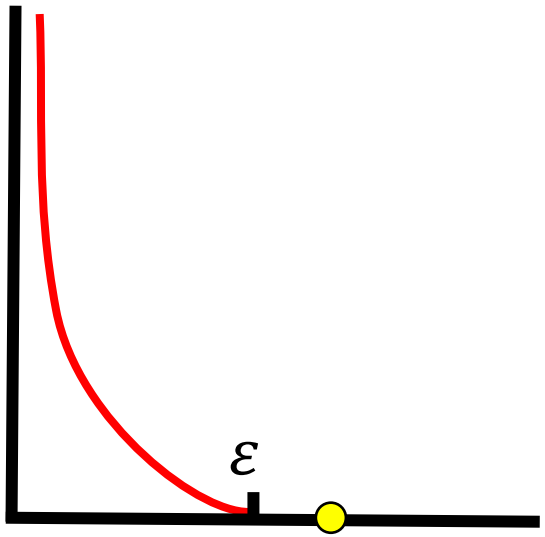


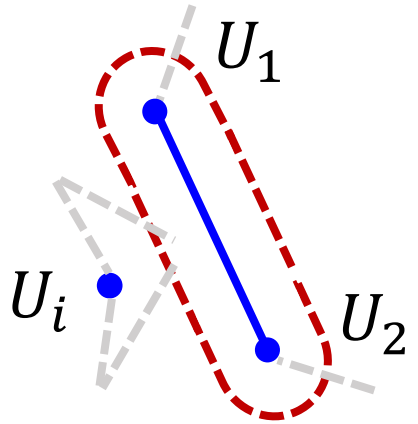
Fixed nonzero structure

distortion barrier

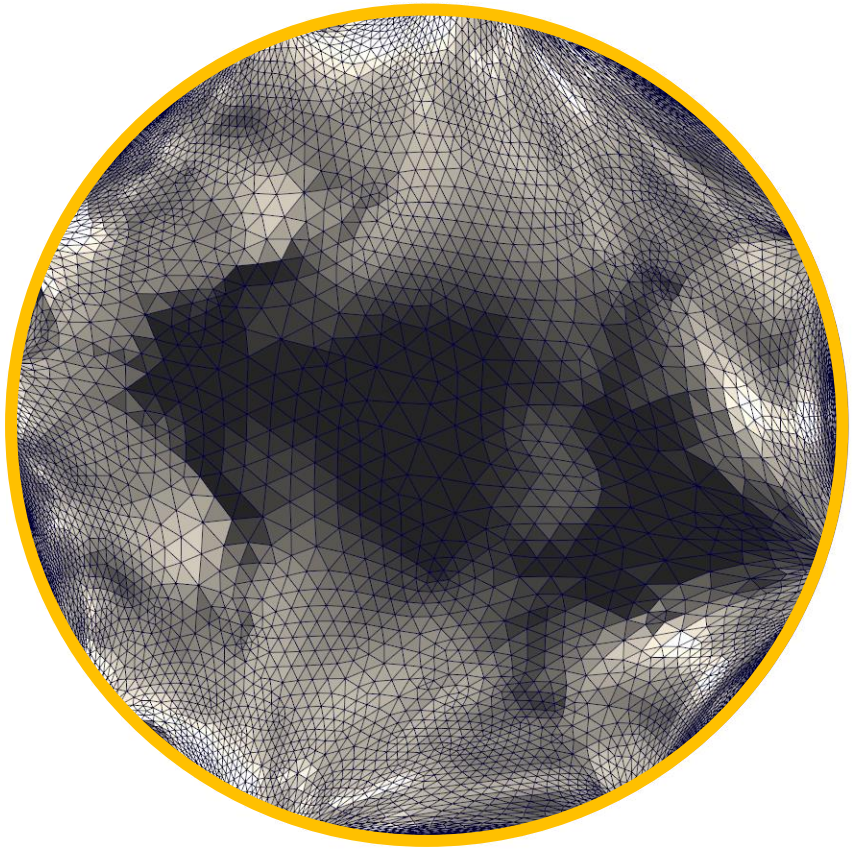
$$\min \boxed{E_D} + \boxed{E_B}$$

$$E_B = \max\left(0, \frac{\varepsilon}{dist(U_1, U_2, U_i)} - 1\right)^2$$





High density of matrix

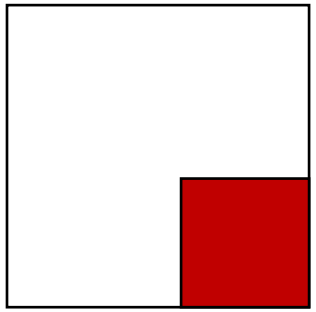


each boundary edge and any boundary vertex

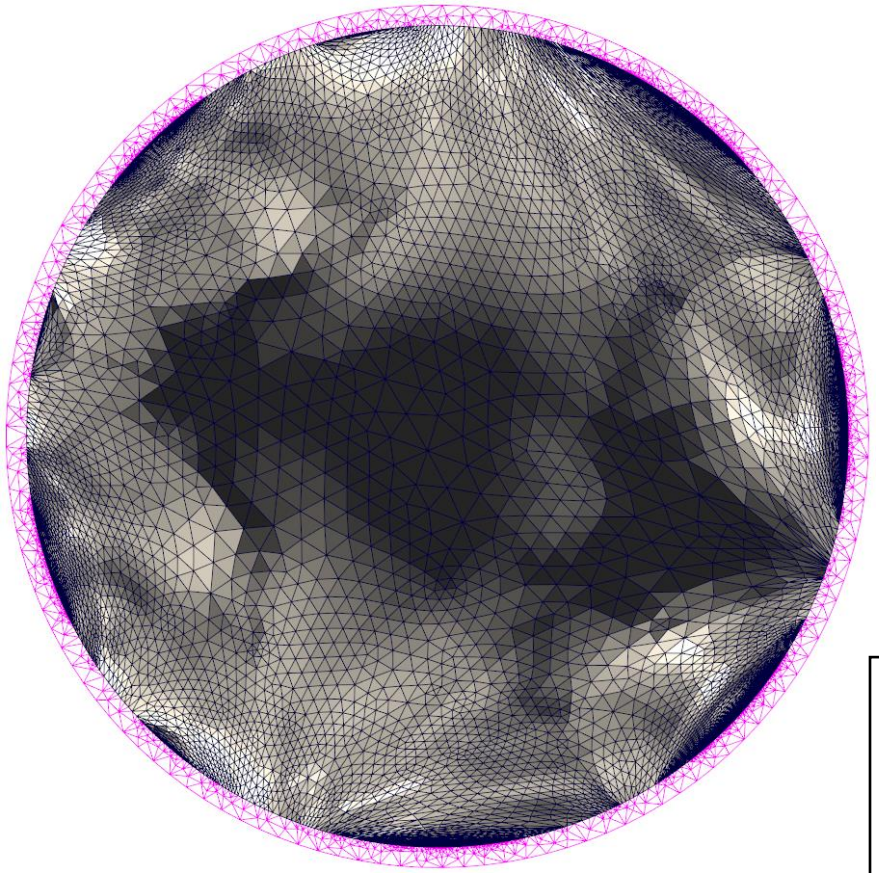
Hessian’s sparse structure is fixed

B_N: the number of boundary vertices

Hessian is dense



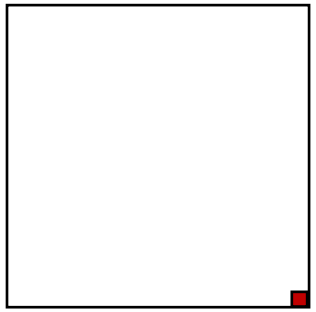
Coarse shell



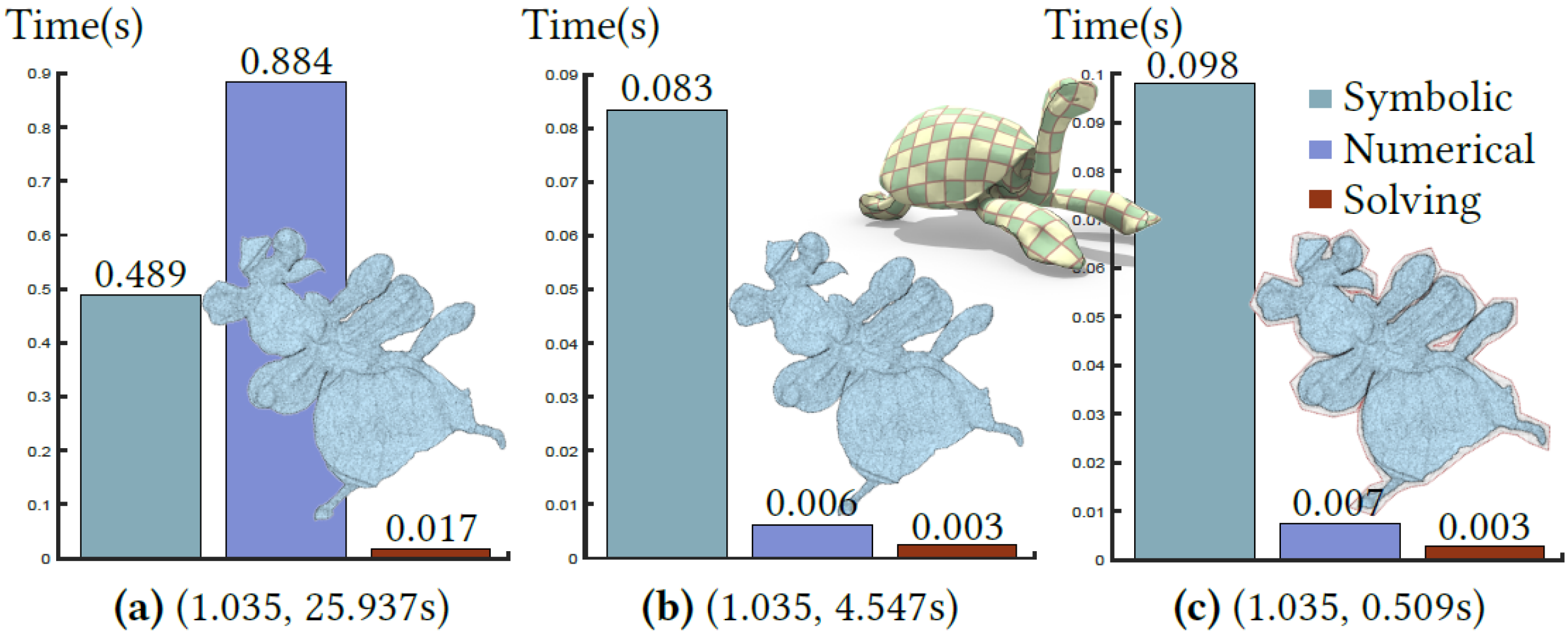
C_N: the number of cage boundary vertices

C_N<<B_N

Hessian is sparse



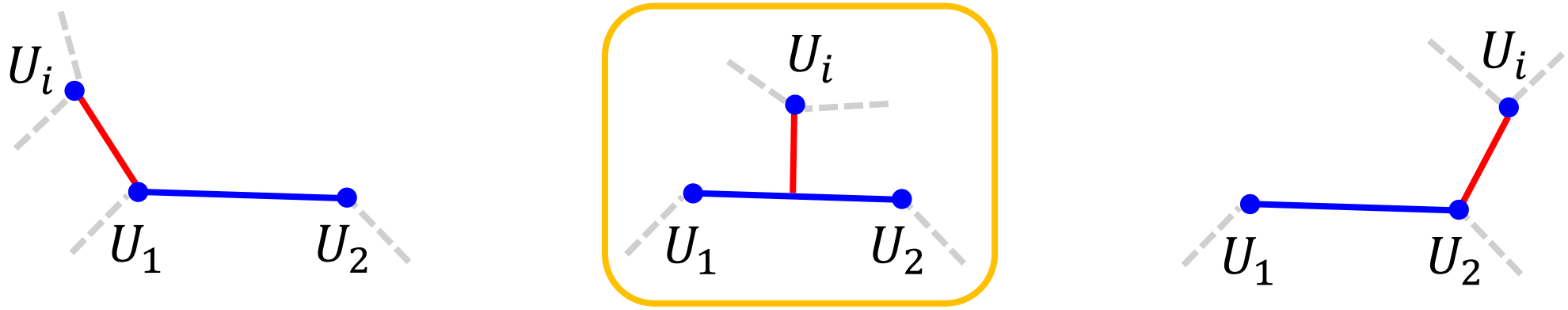
Coarse shell



(a) (1.035, 25.937s) **(b)** (1.035, 4.547s) **(c)** (1.035, 0.509s)

Distance in [Smith et al. 2015]

- Distance is not C^2.
- Hessian of barrier function is difficult to compute.
- Convex-concave decomposition is not easy.



Distance based on triangle inequality

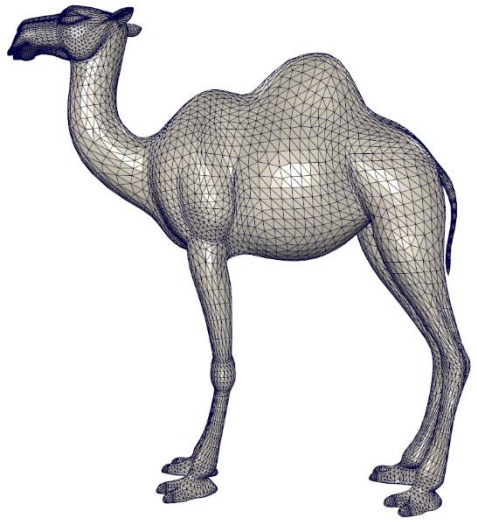
- Infinitely differentiable.
- Analytical second order approximation.

convex concave

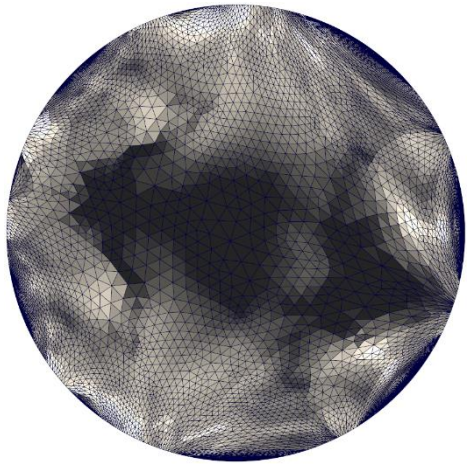
$$dist(U_1, U_2, U_i) = \|U_1U_i\| + \|U_2U_i\| - \|U_1U_2\|$$

$$E_B = \max\left(0, \frac{\varepsilon}{dist(U_1, U_2, U_i)} - 1\right)^2$$

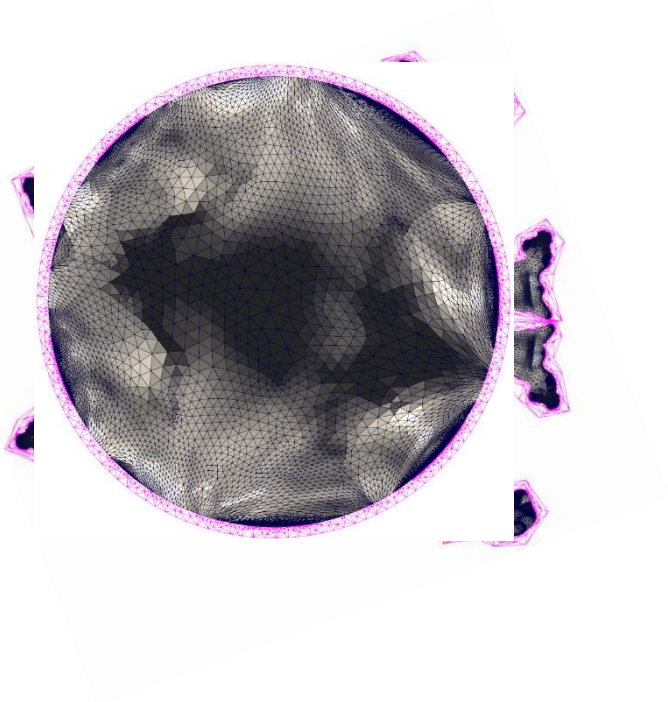
convex $f = \left(\frac{\varepsilon}{g} - 1\right)^2, g = dist(U_1, U_2, U_i)$



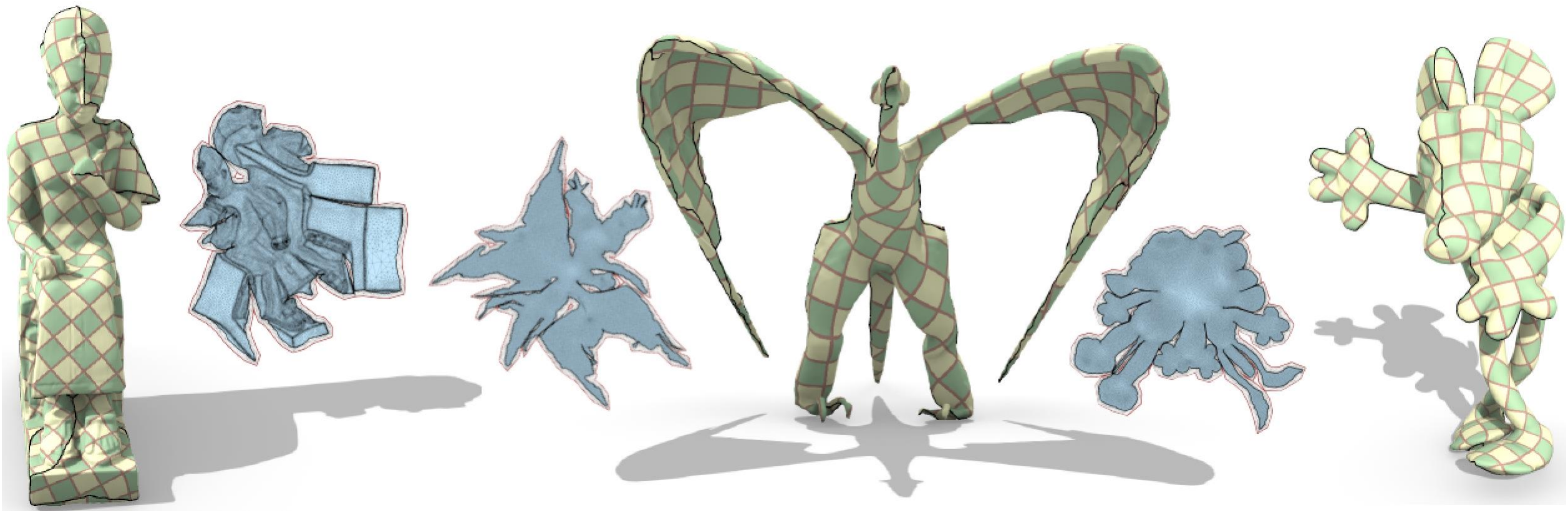
Tutte initialization



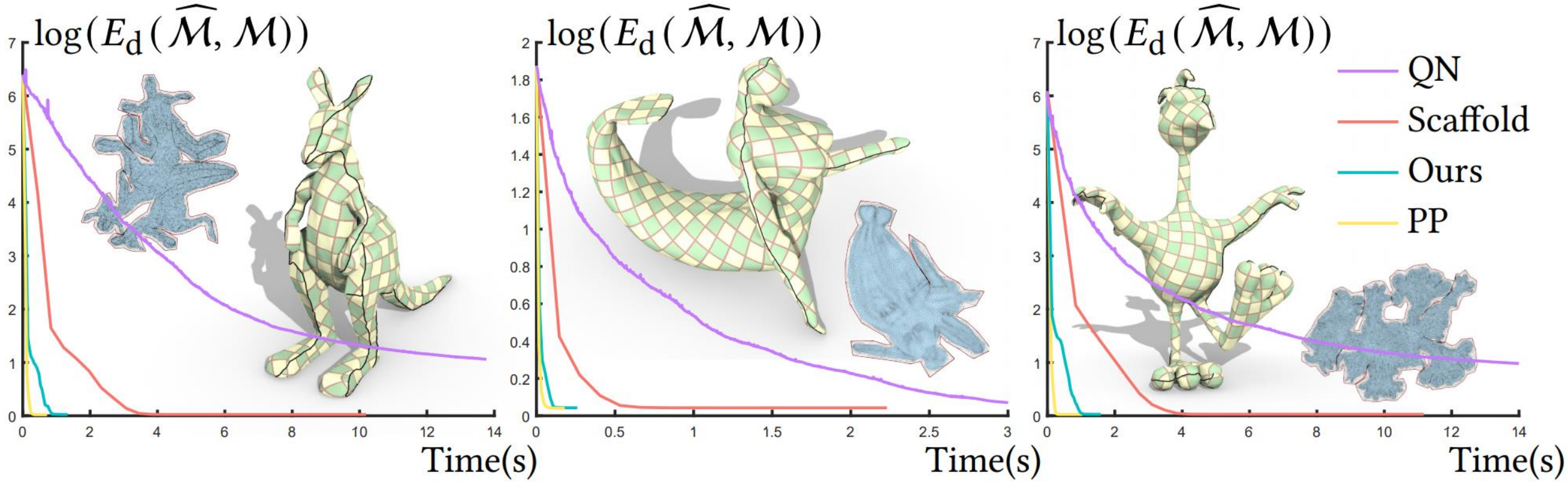
Shell



Result

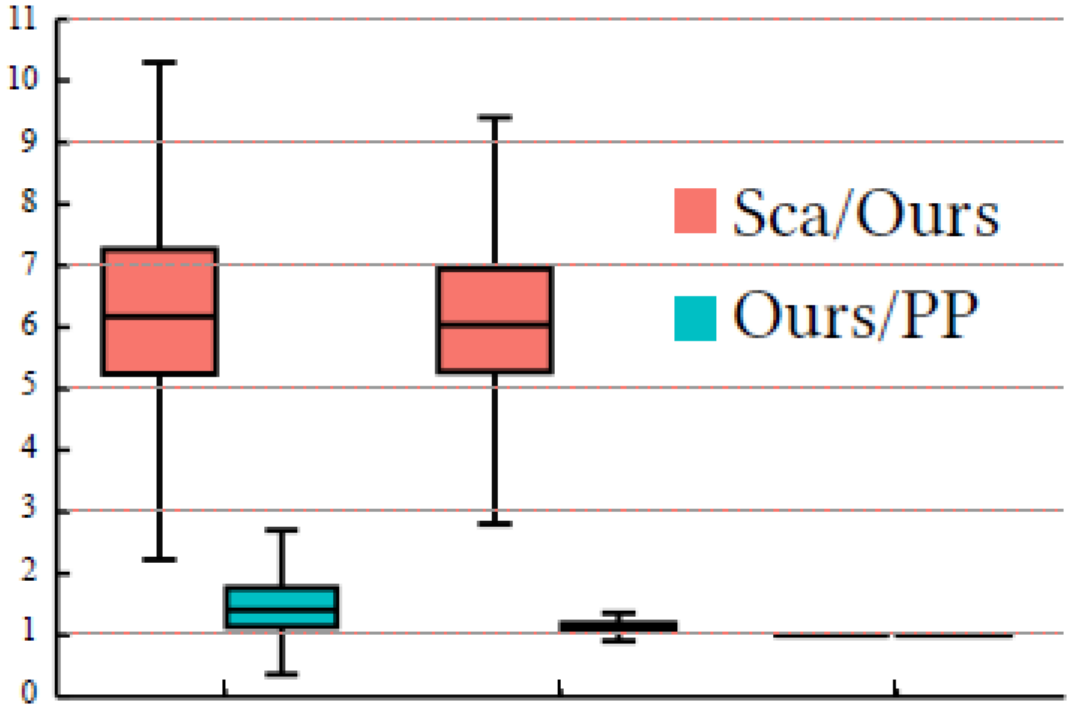


Result



Result

	QN	Sca	PP	Ours
Order	$1^{st}+$	2^{nd}	2^{nd}	2^{nd}
Symbolic	-	×	√	√
Bijection	√	√	×	√



running time rations

per iteration running time rations

energy