Introduction to Computer Graphics

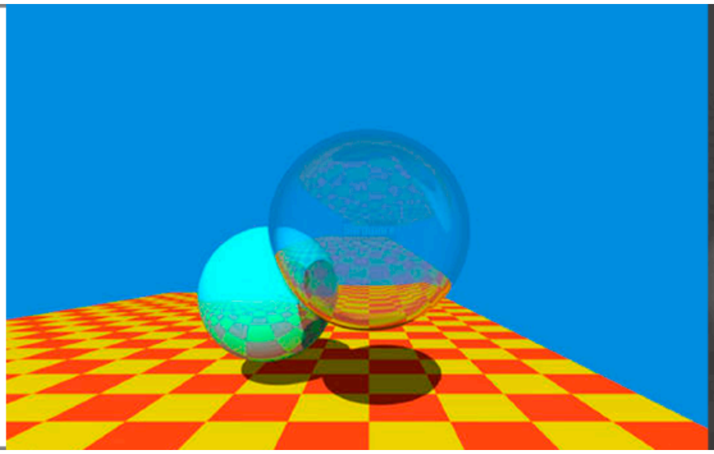
GAMES101, Lingqi Yan, UC Santa Barbara

Lecture 15:

Ray Tracing 3

(Light Transport & Global Illumination)






http://www.cs.ucsb.edu/~lingqi/teaching/games101.html

Announcements

- Homework 5 — 240 submissions so far
- Next two homeworks (1.5 weeks each)
  - Homework 6 — acceleration
  - Homework 7 — path tracing (new!)
- Course website has been updated
  - Two more lectures, one more homework (hw7)

Announcements Cont.

- Why am I always extending the lecture length
  - My CS180 was designed to last 1h to 1.25h
- On the BBS
  - I'd welcome more questions on concepts
- My real-time rendering course
  - Unfortunately has to be internal
  - But will deliver it to GAMES later (maybe summer 2020)
- Again, today's lecture won't be easy

Last Lectures

- Basic ray tracing
  - Ray generation
  - Ray object intersection
- Acceleration
  - Ray AABB intersection
  - Spatial partitions vs object partitions
  - BVH traversal
- Radiometry

Today

- Radiometry cont.
- Light transport
    - The reflection equation
    - The rendering equation
- Global illumination
- Probability review

Reviewing Concepts

Radiant energy $Q\ [\mathrm{J} = \mathrm{Joule}]$ (barely used in CG)

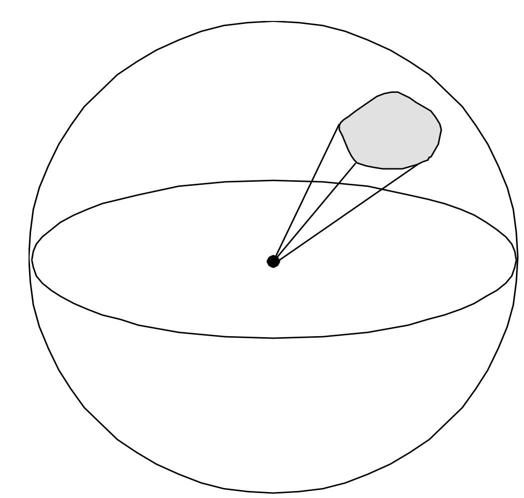
- the energy of electromagnetic radiation

Radiant flux (power) $\Phi \equiv \frac{\mathrm{d}Q}{\mathrm{d}t}\ [\mathrm{W} = \mathrm{Watt}]\ [\mathrm{lm} = \mathrm{lumen}]$

- Energy per unit time

Radiant intensity $I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$

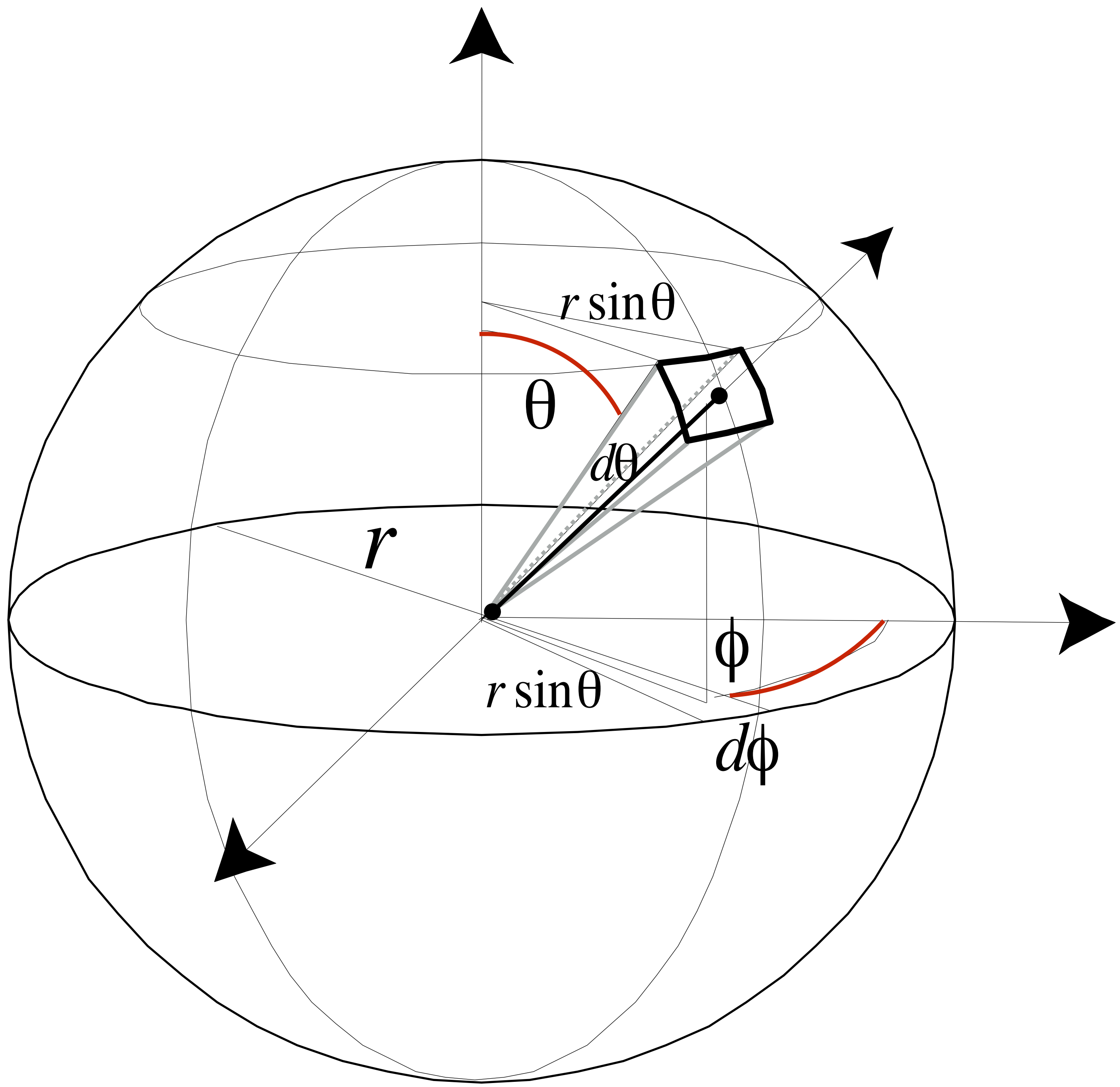
- power per unit solid angle



Solid Angle $\Omega = \frac{A}{r^2}$

- ratio of subtended area on sphere to radius squared

Differential Solid Angles



$$\begin{aligned} \mathrm{d}A &= (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi) \\ &= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \end{aligned}$$

$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

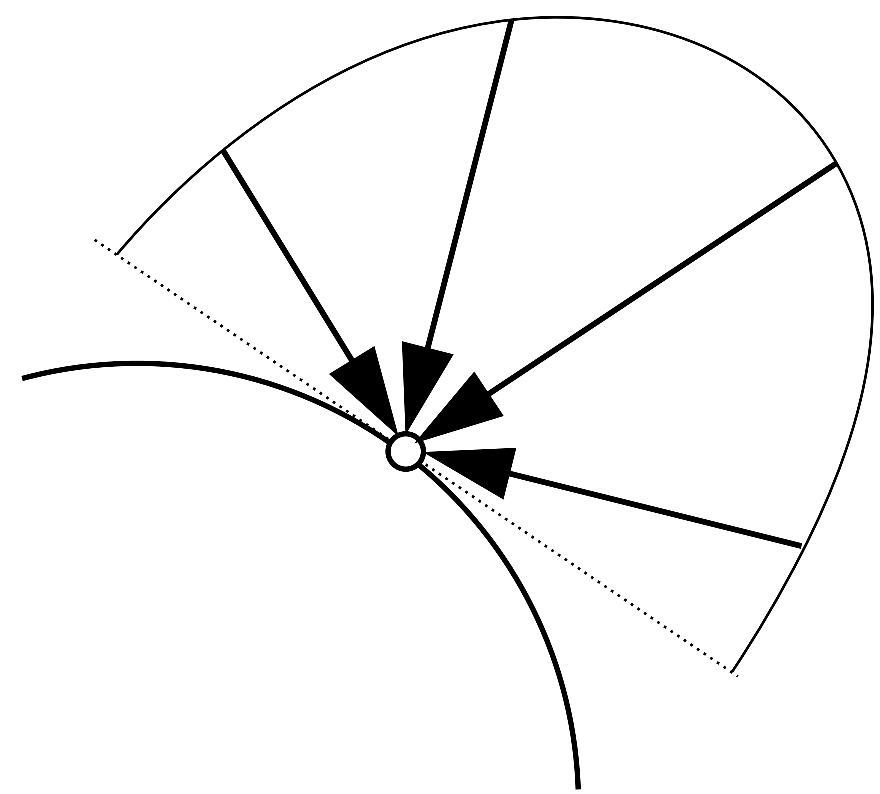
Irradiance

Irradiance

Definition: The irradiance is the power per (perpendicular/ projected) unit area incident on a surface point.

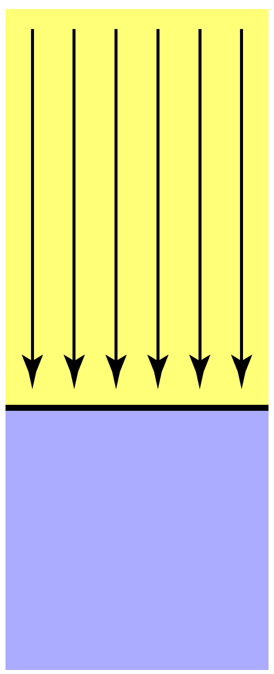
$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A}$$

$$\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \left[\frac{\mathrm{lm}}{\mathrm{m}^2} = \mathrm{lux}\right]$$



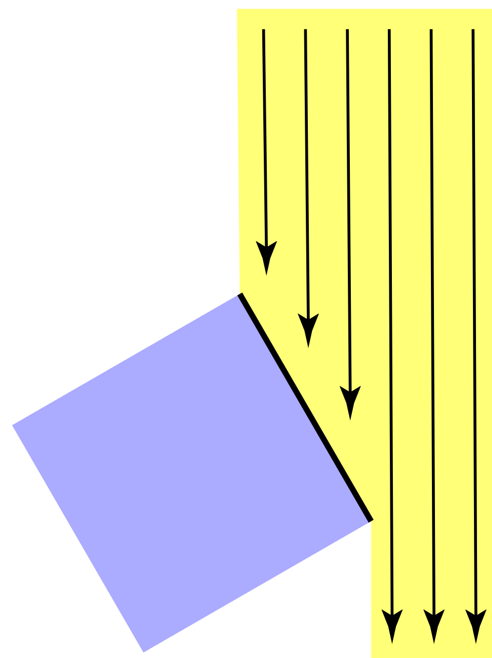
Lambert's Cosine Law

Irradiance at surface is proportional to cosine of angle between light direction and surface normal.



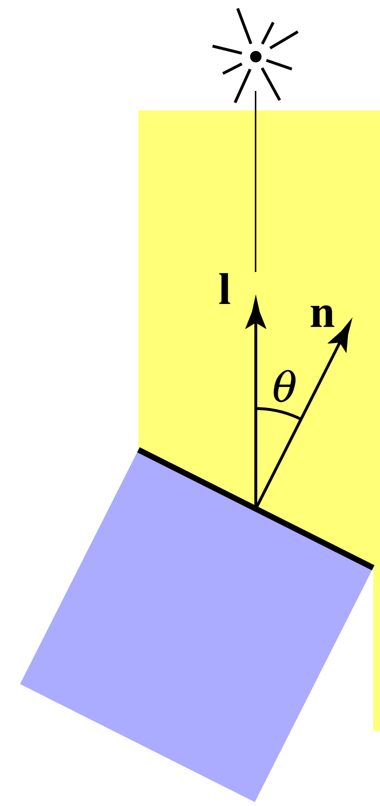
Top face of cube receives a certain amount of power

$$E = \frac{\Phi}{A}$$



Top face of 60° rotated cube receives half power

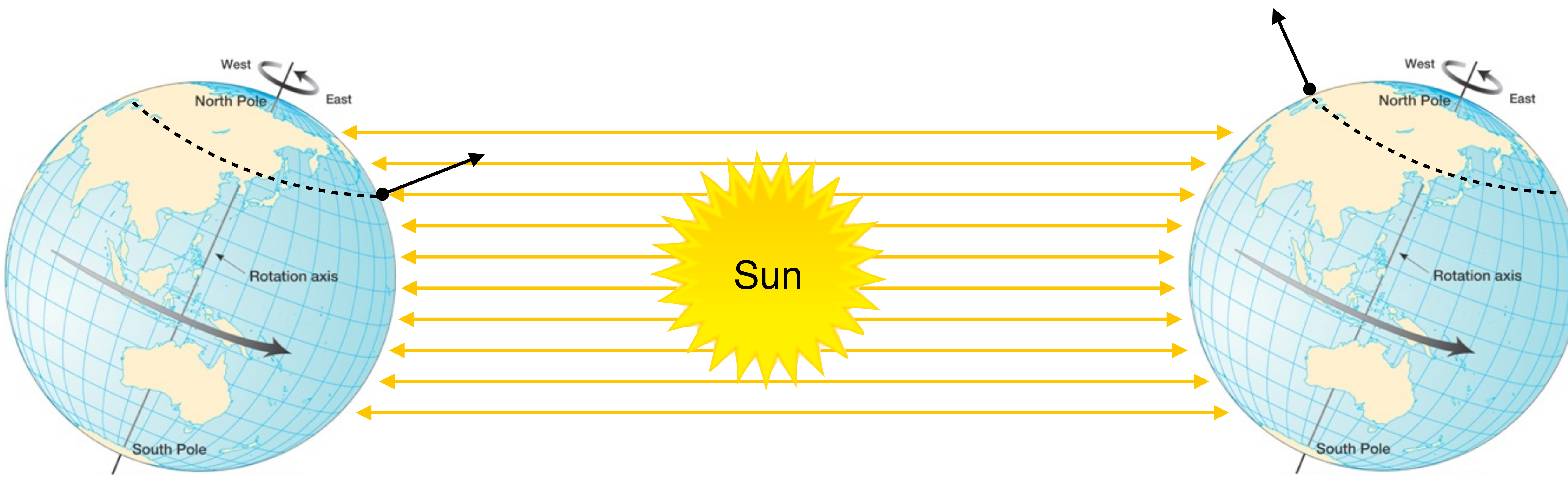
$$E = \frac{1}{2}\frac{\Phi}{A}$$



In general, power per unit area is proportional to $\cos\theta = l \cdot n$

$$E = \frac{\Phi}{A}\cos\theta$$

Why Do We Have Seasons?



Summer
(Northern hemisphere)

Winter
(Northern hemisphere)

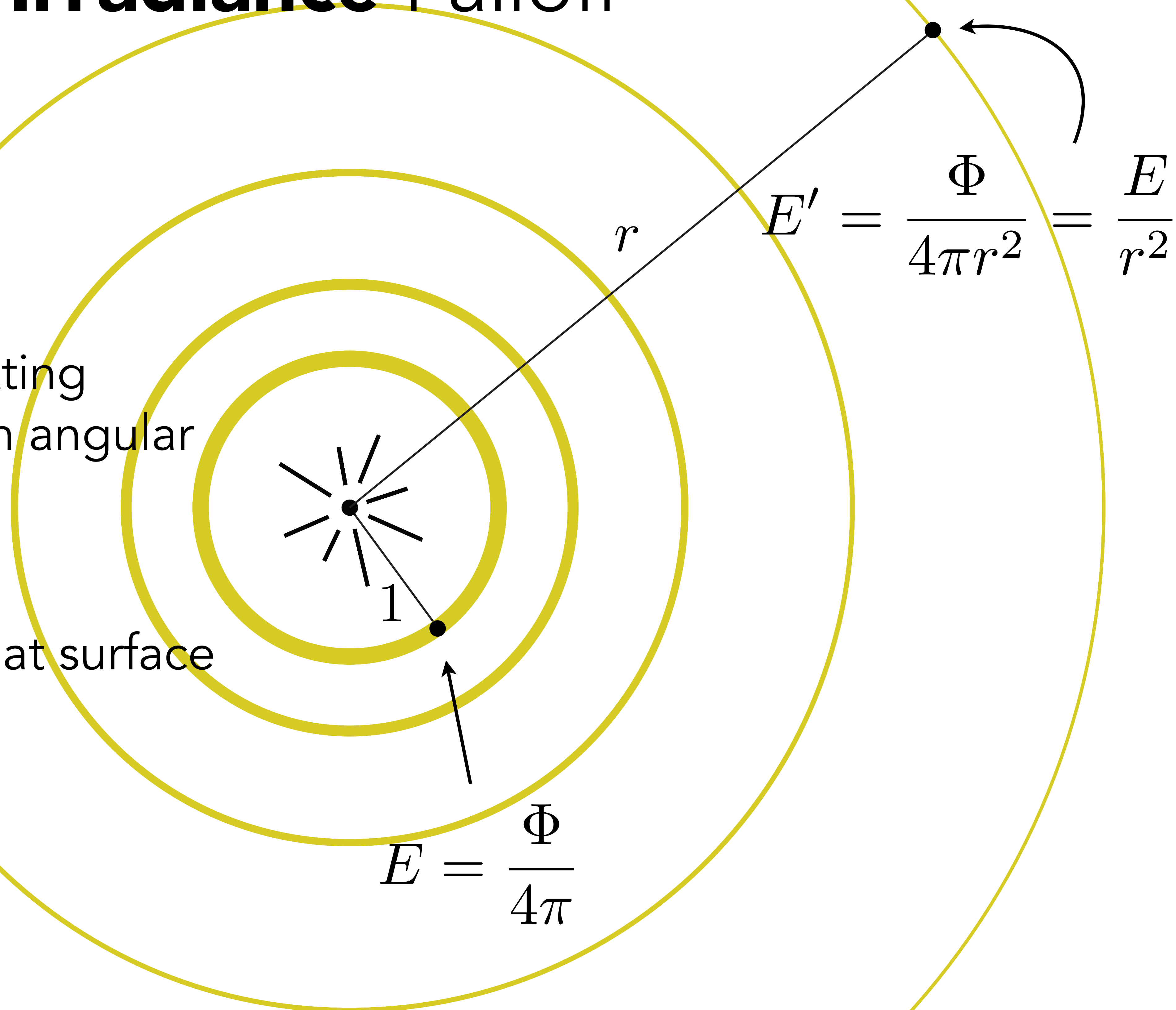
[Image credit: Pearson Prentice Hall]

Earth's axis of rotation: ~23.5° off axis

Correction: **Irradiance** Falloff

Assume light is emitting power Φ in a uniform angular distribution

Compare irradiance at surface of two spheres:



$$E' = \frac{\Phi}{4\pi r^2} = \frac{E}{r^2}$$

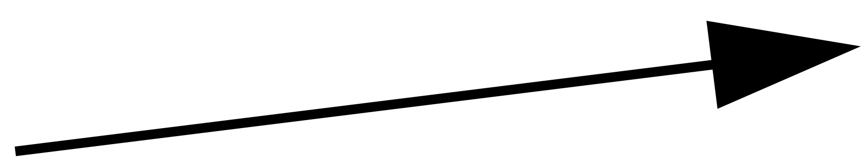
$$E = \frac{\Phi}{4\pi}$$

Radiance

Radiance

Radiance is the fundamental field quantity that describes the distribution of light in an environment

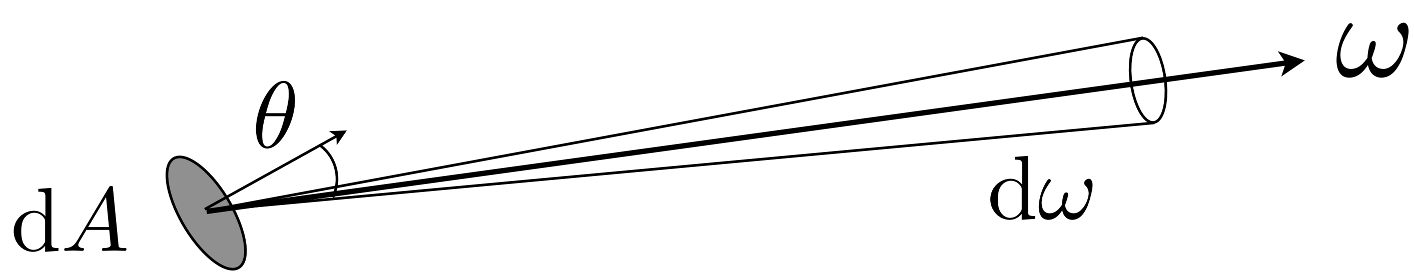
- Radiance is the quantity associated with a ray
- Rendering is all about computing radiance



Light Traveling Along A Ray

Radiance

Definition: The radiance (luminance) is the power emitted, reflected, transmitted or received by a surface, per unit solid angle, per projected unit area.



$$L(\mathrm{p}, \omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p}, \omega)}{\mathrm{d}\omega\, \mathrm{d}A \cos\theta}$$

$\cos\theta$ accounts for projected surface area

$$\left[\frac{\mathrm{W}}{\mathrm{sr}\,\mathrm{m}^2}\right] \left[\frac{\mathrm{cd}}{\mathrm{m}^2} = \frac{\mathrm{lm}}{\mathrm{sr}\,\mathrm{m}^2} = \mathrm{nit}\right]$$

Radiance

Definition: power per unit solid angle per projected unit area.

$$L(\mathrm{p}, \omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p}, \omega)}{\mathrm{d}\omega \, \mathrm{d}A \cos\theta}$$

Recall

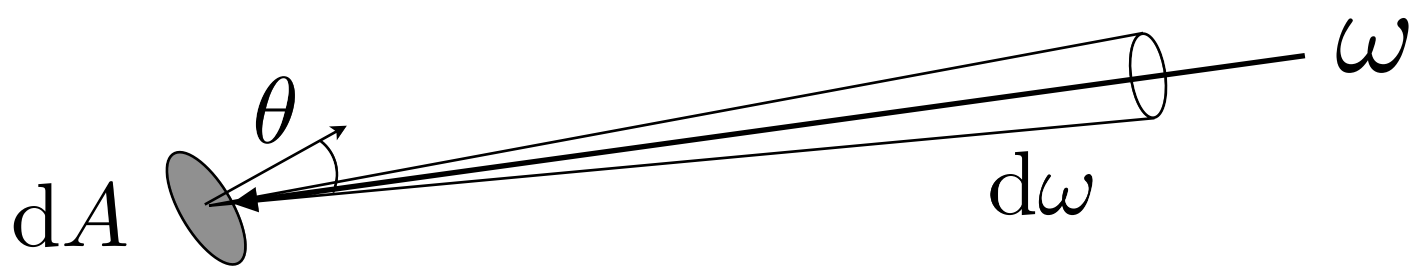
- Irradiance: power per projected unit area
- Intensity: power per solid angle

So

- Radiance: Irradiance per solid angle
- Radiance: Intensity per projected unit area

Incident Radiance

Incident radiance is the irradiance per unit solid angle arriving at the surface.

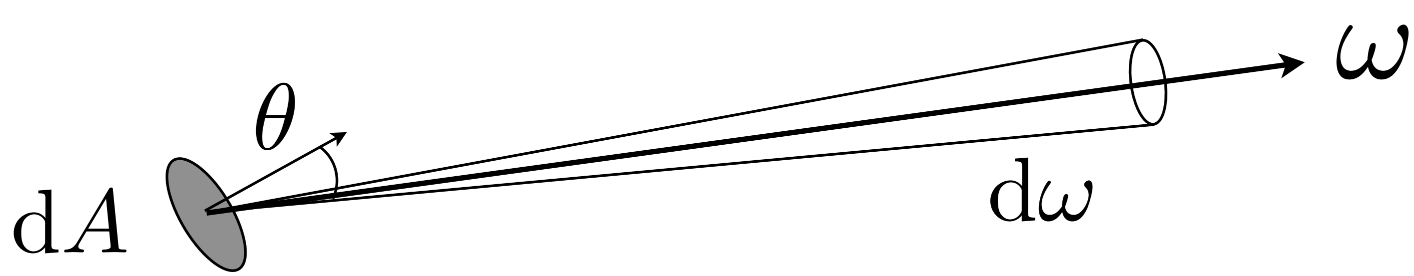


$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}E(\mathrm{p})}{\mathrm{d}\omega \cos\theta}$$

i.e. it is the light arriving at the surface along a given ray (point on surface and incident direction).

Exiting Radiance

Exiting surface radiance is the intensity per unit projected area leaving the surface.



$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}I(\mathrm{p}, \omega)}{\mathrm{d}A \cos \theta}$$

e.g. for an area light it is the light emitted along a given ray (point on surface and exit direction).

Irradiance vs. Radiance

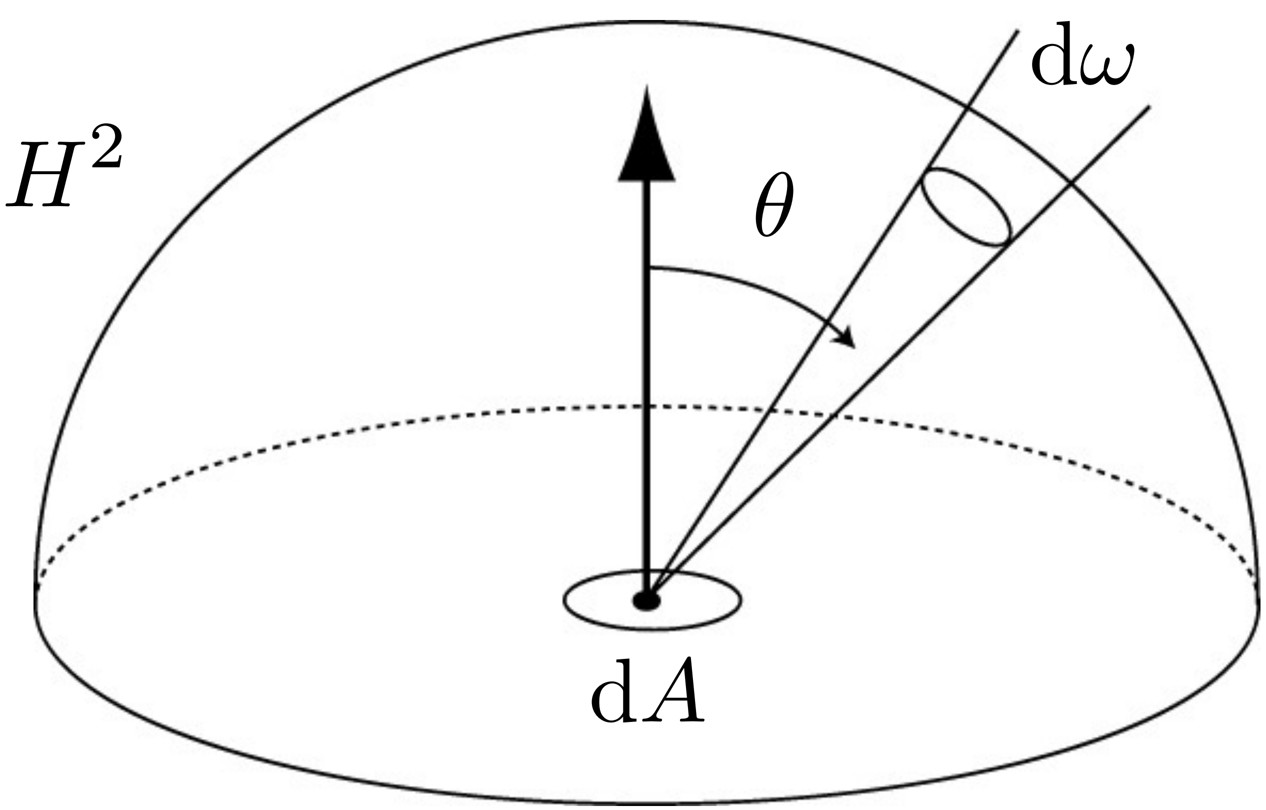
Irradiance: total power received by area dA

Radiance: power received by area dA from "direction" dω

$$dE(\mathrm{p},\omega) = L_i(\mathrm{p},\omega)\cos\theta\,\mathrm{d}\omega$$

$$E(\mathrm{p}) = \int_{H^2} L_i(\mathrm{p},\omega)\cos\theta\,\mathrm{d}\omega$$

Unit Hemisphere: H^2

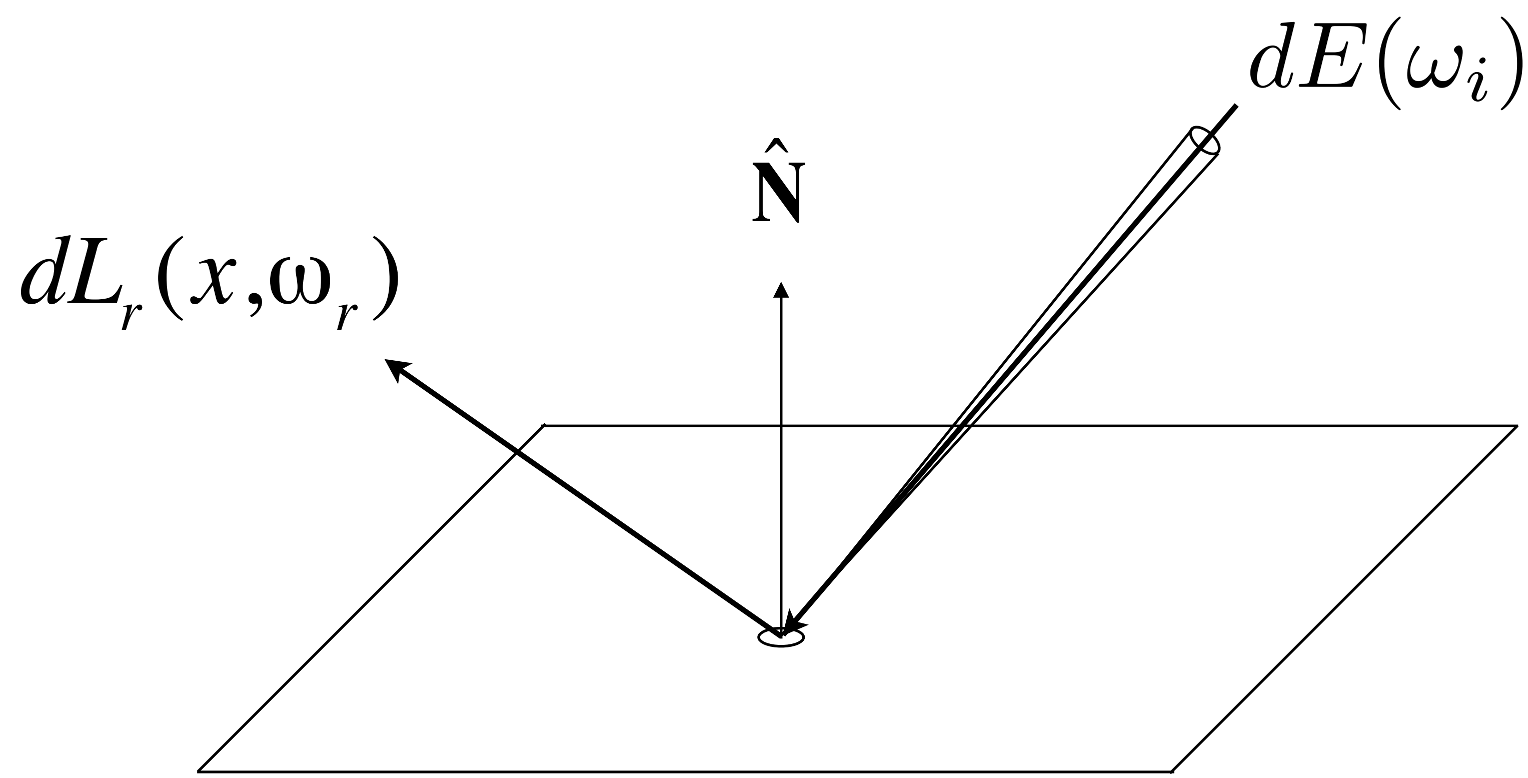


Bidirectional Reflectance Distribution Function (BRDF)

Reflection at a Point

Radiance from direction ω_i turns into the power E that dA receives

Then power E will become the radiance to any other direction ω_o

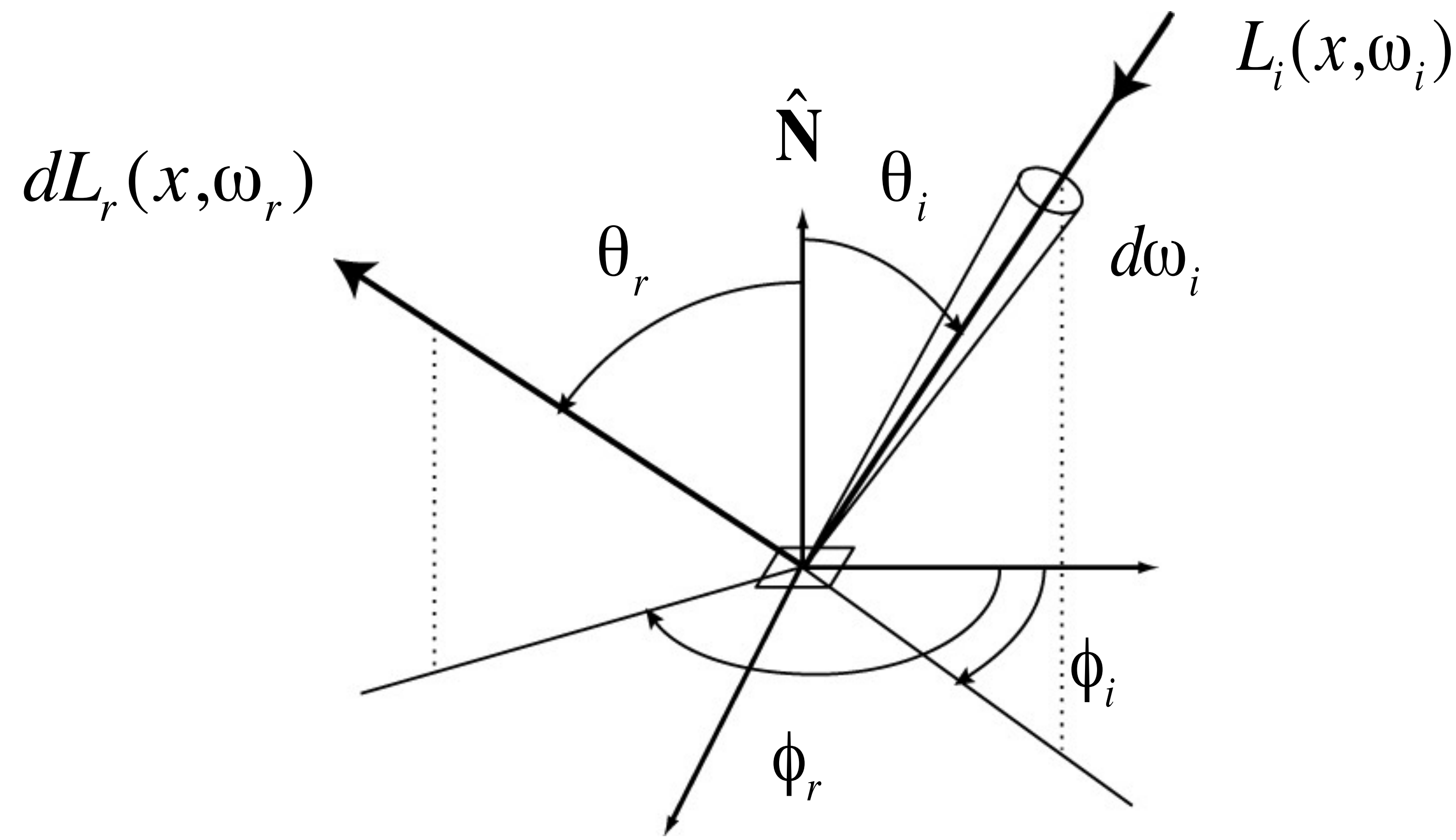


Differential irradiance incoming: $dE(\omega_i) = L(\omega_i)\, cos\, \theta_i \, d\omega_i$

Differential radiance exiting (due to $dE(\omega_i)$): $dL_r(\omega_r)$

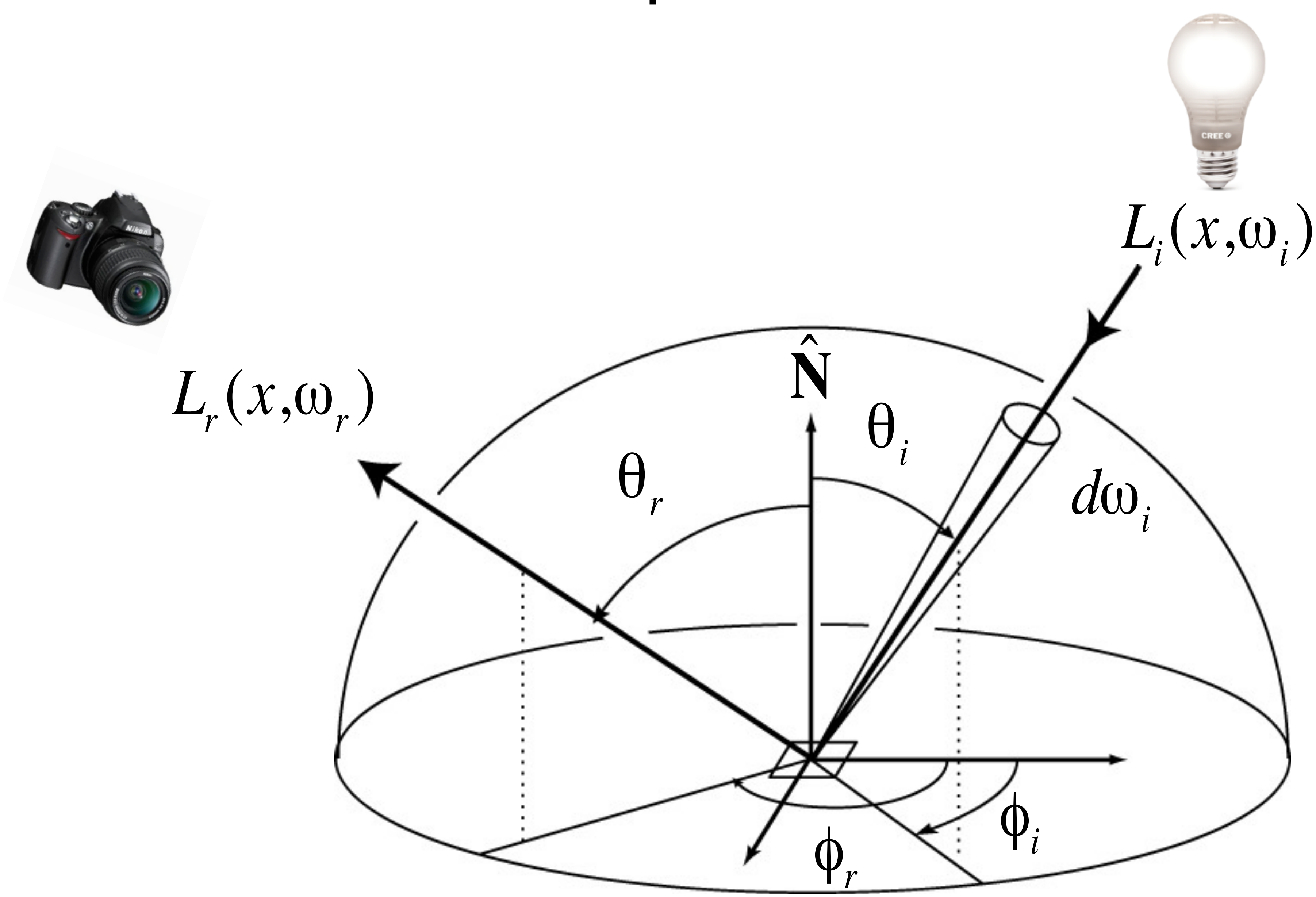
BRDF

The Bidirectional Reflectance Distribution Function (BRDF) represents how much light is reflected into each outgoing direction ω_r from each incoming direction



$$f_r(\omega_i \rightarrow \omega_r) = \frac{\mathrm{d}L_r(\omega_r)}{\mathrm{d}E_i(\omega_i)} = \frac{\mathrm{d}L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i\,\mathrm{d}\omega_i} \quad \left[\frac{1}{\mathrm{sr}}\right]$$

The Reflection Equation

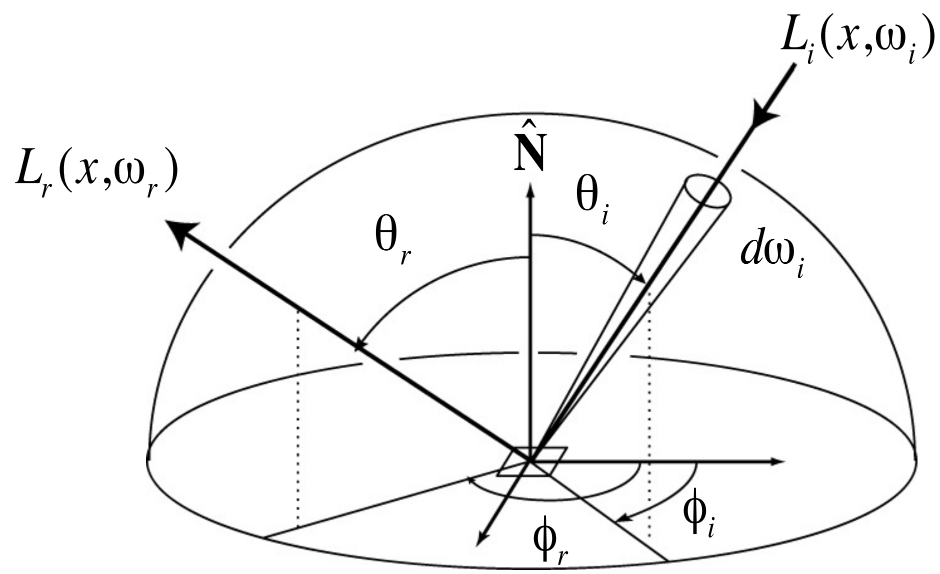


$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_r)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i \, \mathrm{d}\omega_i$$

Challenge: Recursive Equation

Reflected radiance depends on incoming radiance

$$\boxed{L_r(\mathrm{p},\omega_r)} = \int_{H^2} f_r(\mathrm{p},\omega_i \rightarrow \omega_r) \boxed{L_i(\mathrm{p},\omega_i)} \cos\theta_i \, \mathrm{d}\omega_i$$



But incoming radiance depends on reflected radiance (at another point in the scene)

The Rendering Equation

Re-write the reflection equation:

$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_r)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i \,\mathrm{d}\omega_i$$

by adding an Emission term to make it general!

The Rendering Equation

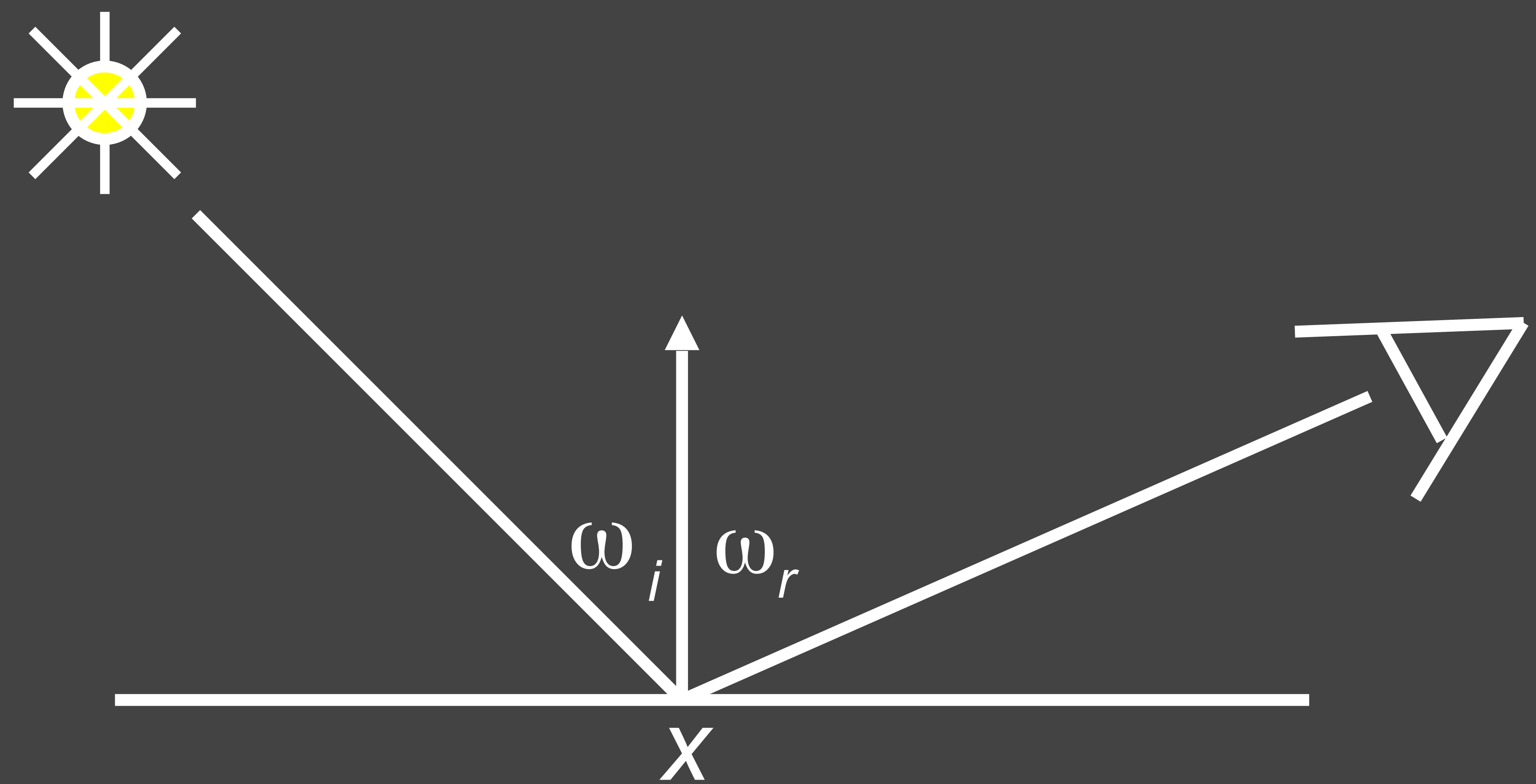
$$\boxed{L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\Omega^+} L_i(p, \omega_i) f_r(p, \omega_i, \omega_o)(n \cdot \omega_i)\,\mathrm{d}\omega_i}$$

How to solve? Next lecture!

Note: now, we assume that all directions are pointing outwards!

Understanding the rendering equation

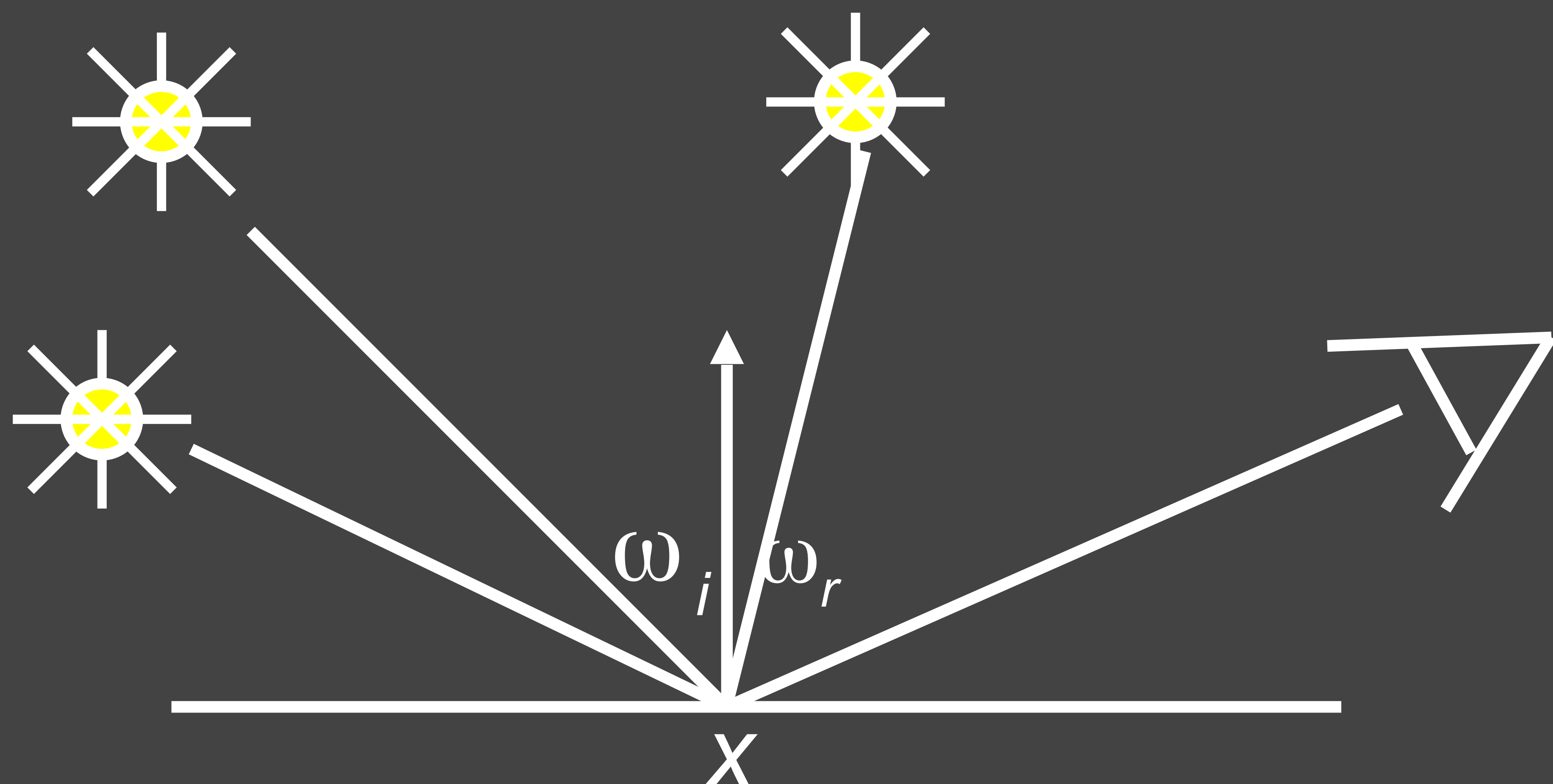
Reflection Equation



$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ (\omega_i, n)$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle

Reflection Equation

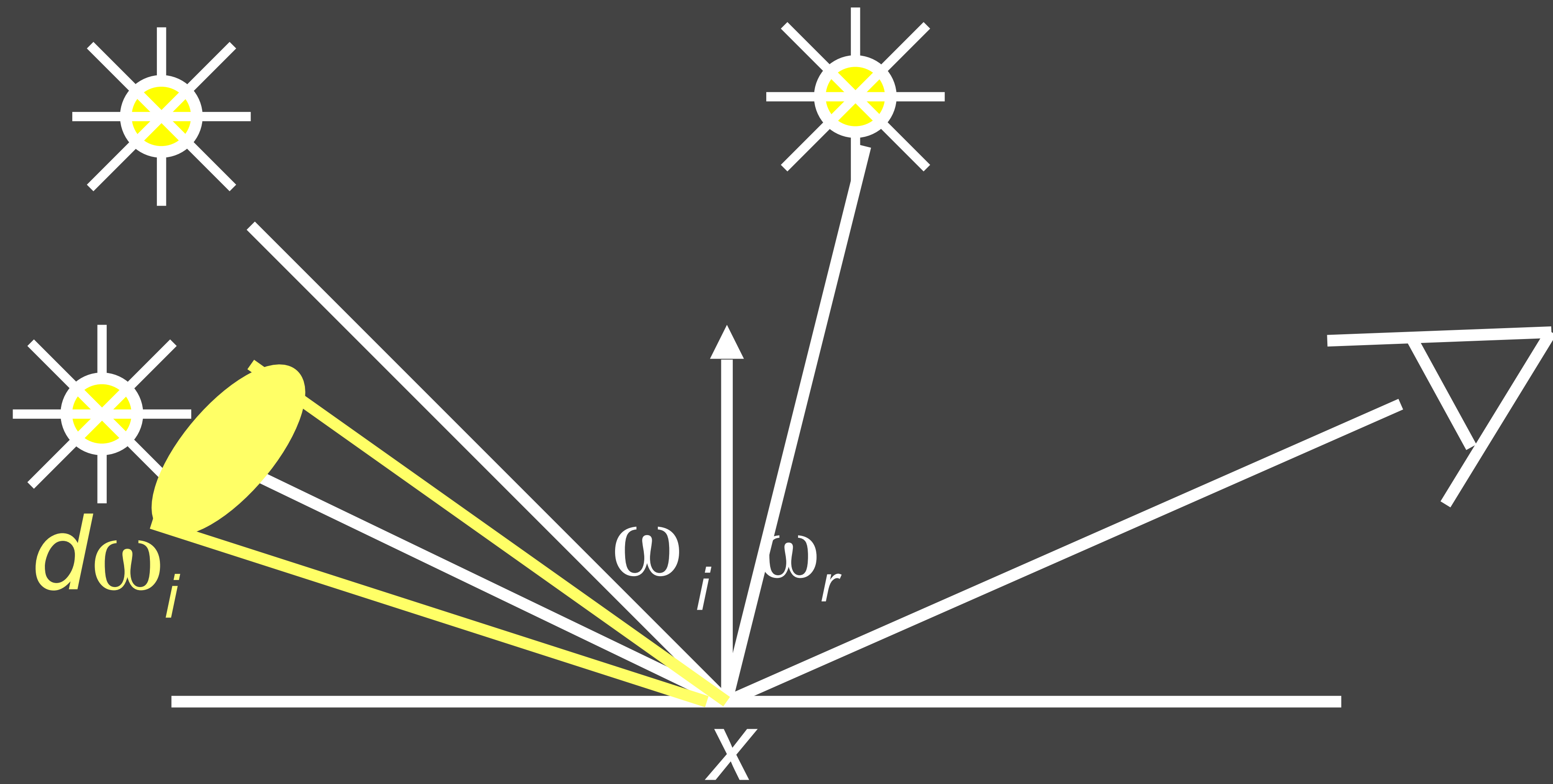


Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ (\omega_i,n)$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle

Reflection Equation



Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ \cos\theta_i\,\mathrm{d}\omega_i$$

Reflected Light (Output Image)

Emission

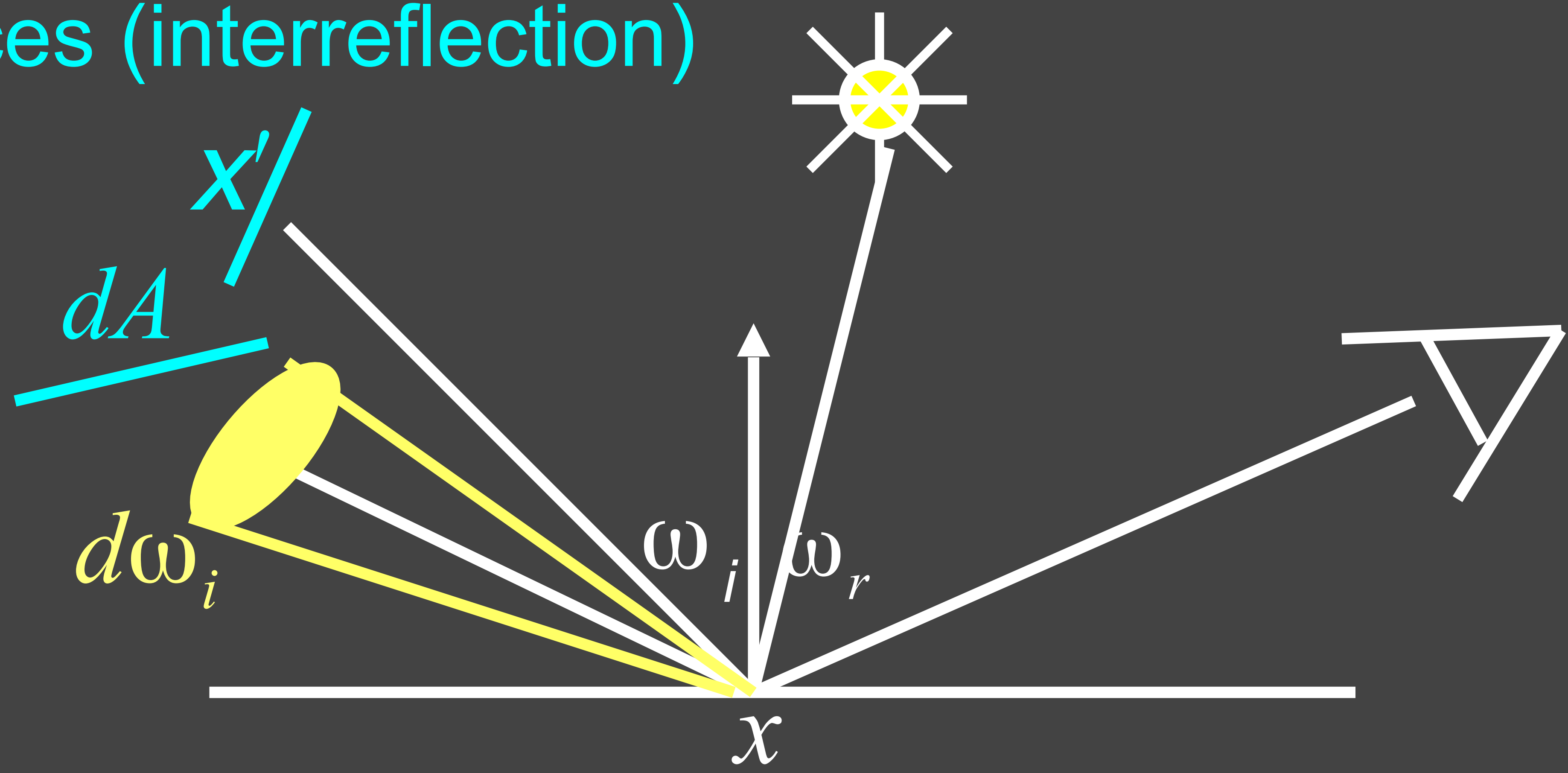
Incident Light (from light source)

BRDF

Cosine of Incident angle

Rendering Equation

Surfaces (interreflection)



$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i \, d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Rendering Equation (Kajiya 86)

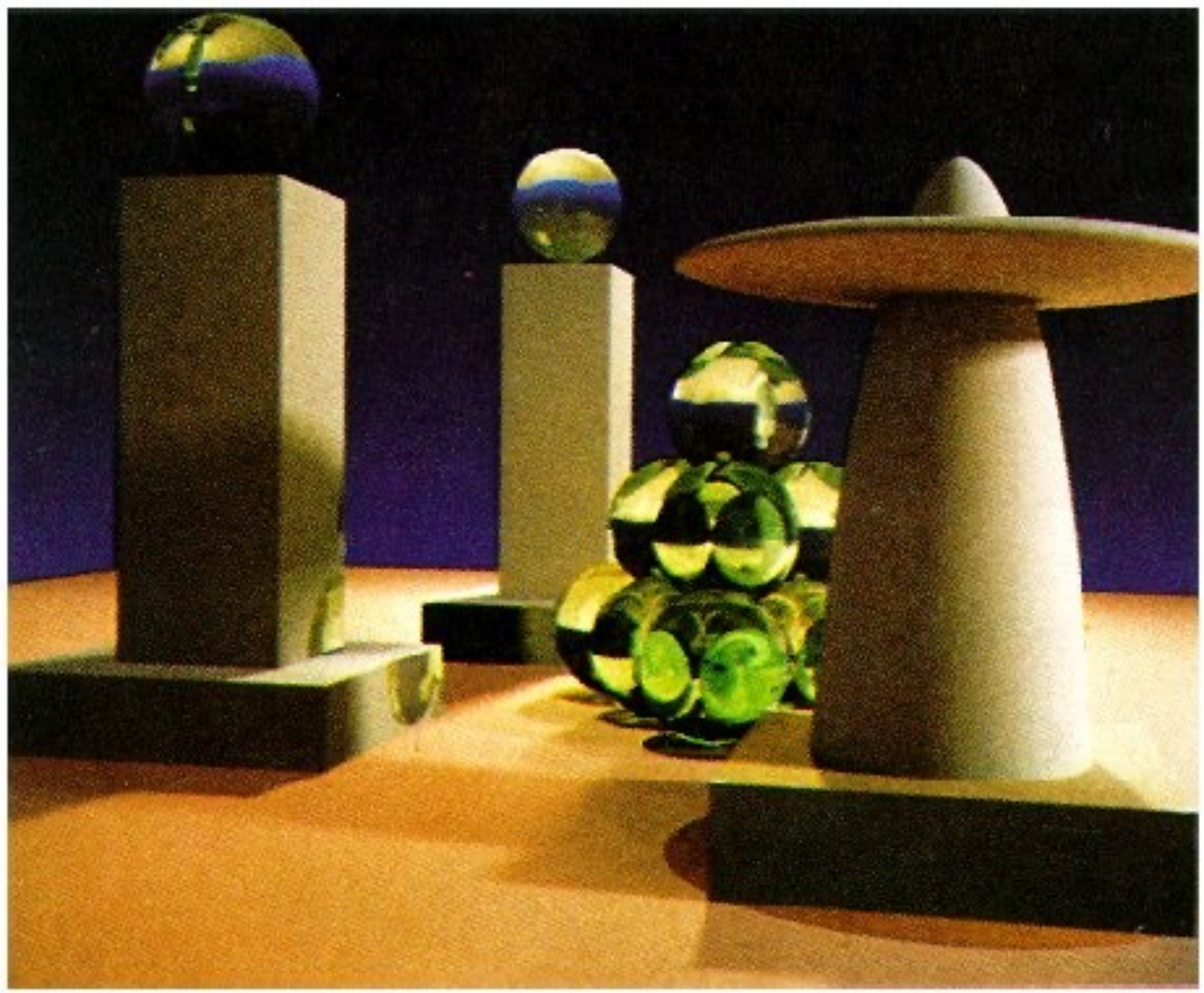


Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

Rendering Equation as Integral Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i)\boxed{f(x,\omega_i,\omega_r)\cos\theta_i d\omega_i}$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v)\boxed{K(u,v)dv}$$

Kernel of equation

Linear Operator Equation

$$l(u) = e(u) + \int l(v)\,\boxed{K(u,v)dv}$$

Kernel of equation
Light Transport Operator

$$L = E + KL$$

Can be discretized to a simple matrix equation
[or system of simultaneous linear equations]
(L, E are vectors, K is the light transport matrix)

Ray Tracing and extensions

- General class numerical Monte Carlo methods
- Approximate set of all paths of light in scene

$$L = E + KL$$

$$IL - KL = E$$

$$(I - K)L = E$$

$$L = (I - K)^{-1}E$$

Binomial Theorem

$$L = (I + K + K^2 + K^3 + ...)E$$

$$L = E + KE + K^2E + K^3E + ...$$

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

Direct Illumination
on surfaces

Indirect Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect illum.)

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

Direct Illumination
on surfaces

Indirect Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect illum.)

Shading in
Rasterization


p
Direct illumination


p
One-bounce global illumination (dir+indir)


p
Two-bounce global illumination


p
Four-bounce global illumination


p
Eight-bounce global illumination


p
Sixteen-bounce global illumination

Probability Review

Random Variables

X random variable. Represents a distribution of potential values

$X \sim p(x)$ probability density function (PDF). Describes relative probability of a random process choosing value x

Example: uniform PDF: all values over a domain are equally likely

e.g. A six-sided die

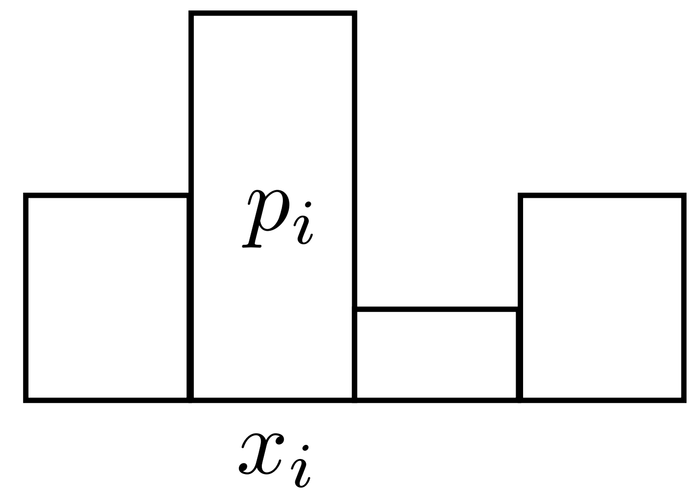
X takes on values 1, 2, 3, 4, 5, 6

$$p(1) = p(2) = p(3) = p(4) = p(5) = p(6)$$

Probabilities

n discrete values x_i

With probability p_i



Requirements of a probability distribution:

$$p_i \geq 0$$

$$\sum_{i=1}^{n} p_i = 1$$

Six-sided die example: $p_i = \frac{1}{6}$

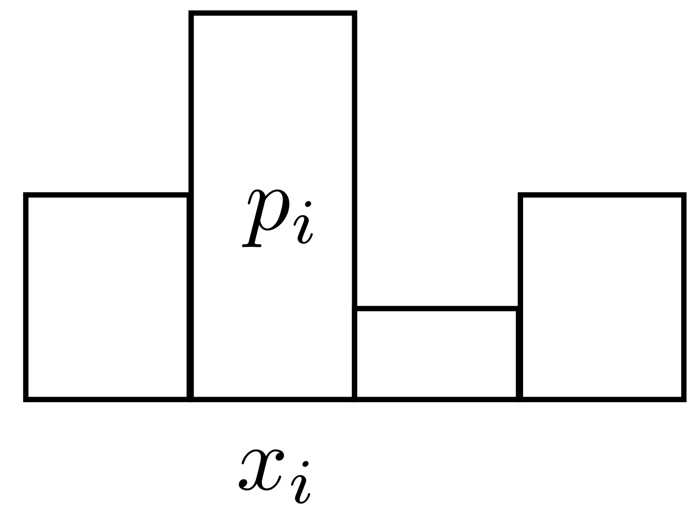
Expected Value of a Random Variable

The average value that one obtains if repeatedly drawing samples from the random distribution.

X drawn from distribution with

n discrete values x_i

with probabilities p_i



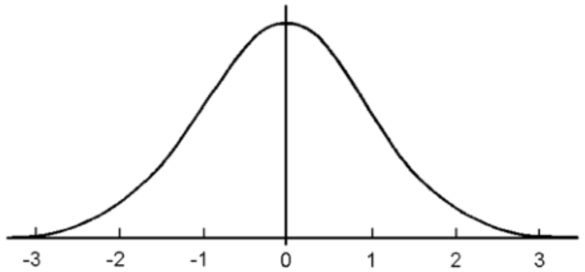
Expected value of X: $E[X] = \sum_{i=1}^{n} x_i p_i$

Die example: $E[X] = \sum_{i=1}^{n} \frac{i}{6}$

$$= (1 + 2 + 3 + 4 + 5 + 6)/6 = 3.5$$

Continuous Case: **Probability Distribution Function (PDF)**

$$X \sim p(x)$$



A random variable X that can take any of a continuous set of values, where the relative probability of a particular value is given by a continuous probability density function p(x).

Conditions on p(x): $p(x) \geq 0 \text{ and } \int p(x)\, dx = 1$

Expected value of X: $E[X] = \int x\, p(x)\, dx$

Function of a Random Variable

A function Y of a random variable X is also a random variable:

$$X \sim p(x)$$
$$Y = f(X)$$

Expected value of a function of a random variable:

$$E[Y] = E[f(X)] = \int f(x)\, p(x)\, dx$$

Thank you!

(And thank Prof. Ravi Ramamoorthi and Prof. Ren Ng for many of the slides!)