



**UNSW**  
SYDNEY

MATH1081 – Discrete Mathematics

Topic 1 – Set theory and functions

Lecture 1.03 – Venn diagrams and set operations

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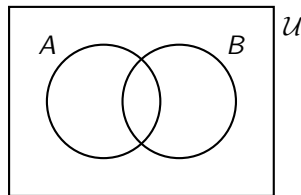
# Venn diagrams

**Definition.** When working with related sets, it can be useful to define a **universal set** which contains all elements relevant to these sets. The universal set is usually denoted by  $\mathcal{U}$  or just  $U$ . In particular, all the related sets are subsets of  $\mathcal{U}$ .

For example, if our sets are MATH1081 tutorials, then an appropriate universal set might be the set of all MATH1081 students. If our sets are intervals on the real number line, then the universal set would be  $\mathbb{R}$ .

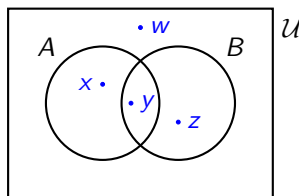
**Definition.** A **Venn diagram** is a diagrammatic tool for visualising relationships between related sets. Sets are represented as overlapping closed figures (usually circles) within a larger figure (usually a rectangle) representing the universal set. Elements can be represented as points placed within different sections of the diagram to show which sets they belong to.

For example, here is a general Venn diagram for two sets  $A$  and  $B$  with universal set  $\mathcal{U}$ :



## Venn diagram example

**Example.** Consider the following Venn diagram:



For each of the elements  $w, x, y, z \in \mathcal{U}$ , we can make the following conclusions:

In order to more easily refer to the different sections in a Venn diagram, we shall introduce new notation that allows us to perform operations on sets.

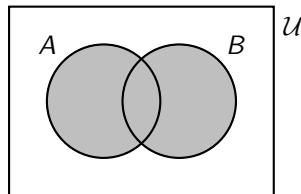
# Union

**Definition.** The **union** of two sets  $A$  and  $B$ , written as  $A \cup B$ , is the set of all elements in  $A$  or  $B$  (or both). That is,

$$A \cup B = \{x \in \mathcal{U} : x \in A \text{ or } x \in B\}.$$

(Note: The word “or” in mathematics is always treated as inclusive.)

In a Venn diagram,  $A \cup B$  is represented by this shaded region:



**Example.** Given that  $R = \{a, c, e\}$ ,  $S = \{b, d\}$ , and  $T = \{d, e, f\}$ , find the following:

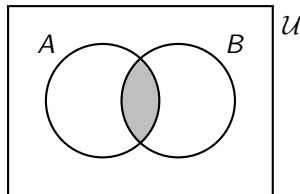
- $R \cup S =$
- $R \cup T =$
- $S \cup T =$
- $R \cup S \cup T =$

# Intersection

**Definition.** The **intersection** of two sets  $A$  and  $B$ , written as  $A \cap B$ , is the set of all elements in both  $A$  and  $B$ . That is,

$$A \cap B = \{x \in \mathcal{U} : x \in A \text{ and } x \in B\}.$$

In a Venn diagram,  $A \cap B$  is represented by this shaded region:



**Example.** Given that  $R = \{a, c, e\}$ ,  $S = \{b, c, d\}$ , and  $T = \{a, b, d, e\}$ , find the following:

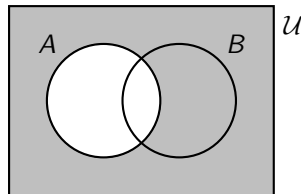
- $R \cap S =$
- $R \cap T =$
- $S \cap T =$
- $R \cap S \cap T =$

# Complement

**Definition.** The **complement** of a set  $A$ , written as  $A^c$  (or sometimes  $\overline{A}$ ), is the set of all elements in the universal set that are not in  $A$ . That is,

$$A^c = \{x \in \mathcal{U} : x \notin A\}.$$

In a Venn diagram,  $A^c$  is represented by this shaded region:



**Example.** Given that  $\mathcal{U} = \{a, b, c, d, e\}$ ,  $R = \{a, c, e\}$ ,  $S = \{b\}$ , and  $T = \{\}$ , find the following:

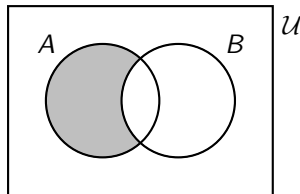
- $R^c =$
- $S^c =$
- $T^c =$

# Set difference

**Definition.** The **difference** of two sets  $A$  and  $B$ , written as  $A - B$  (or sometimes  $A \setminus B$ ), is the set of all elements in  $A$  which are not in  $B$ . That is,

$$A - B = \{x \in \mathcal{U} : x \in A \text{ and } x \notin B\}.$$

In a Venn diagram,  $A - B$  is represented by this shaded region:



**Example.** Given that  $R = \{a, b, c, d\}$ ,  $S = \{b, c, d\}$ , and  $T = \{b, d, e\}$ , find the following:

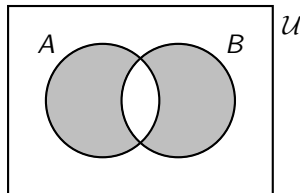
- $R - S =$
- $R - T =$
- $S - T =$
- $S - R =$  (Notice that in general,  $A - B \neq B - A$  for sets  $A$  and  $B$ .)

# Symmetric difference

**Definition.** The **symmetric difference** of two sets  $A$  and  $B$ , written as  $A \ominus B$  (or sometimes  $A \triangle B$  or  $A \oplus B$ ), is the set of all elements in  $A$  or  $B$ , but not both. That is,

$$A \ominus B = \{x \in \mathcal{U} : x \in A \cup B \text{ and } x \notin A \cap B\}.$$

In a Venn diagram,  $A \ominus B$  is represented by this shaded region:



**Example.** Given that  $R = \{a, b, c\}$ ,  $S = \{b\}$ , and  $T = \{b, c, d\}$ , find the following:

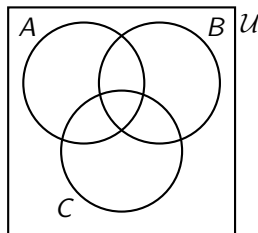
- $R \ominus S =$
- $R \ominus T =$
- $S \ominus T =$



# Combining set operations

A general Venn diagram for three sets  $A$ ,  $B$ , and  $C$  with universal set  $\mathcal{U}$  is shown to the right.

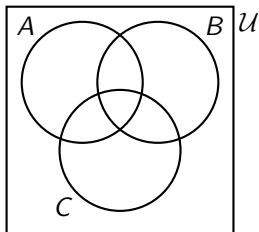
Venn diagrams for four or more sets can become unwieldy or impossible to draw, so they are normally avoided.



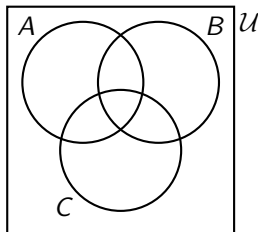
Set operations can be applied to multiple sets, so long as the order of operations is clearly indicated by the use of brackets.

**Example.** Shade the regions indicated by the following set expressions.

$$(A \cup B) \cap C$$



$$(A \cap B)^c - C$$



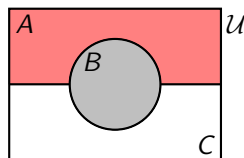
# Disjoint sets

**Definition.** Two sets  $A$  and  $B$  are **disjoint** if their intersection is empty, that is, if  $A \cap B = \emptyset$ .

**Definition.** The sets  $A_1, A_2, A_3, \dots, A_k$  are **pairwise disjoint** if every pair of sets is disjoint, that is, if  $A_i \cap A_j = \emptyset$  for all  $i \neq j$ .

**Definition.** The sets  $A_1, A_2, A_3, \dots, A_k$  **partition** the set  $B$  if they are pairwise disjoint and  $A_1 \cup A_2 \cup A_3 \cup \dots \cup A_k = B$ .

For example, in the Venn diagram on the right,  $\mathcal{U}$  is partitioned by the sets  $A$ ,  $B$ , and  $C$ .



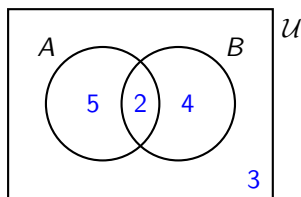
**Example.** Complete the following sentences regarding general sets  $A$  and  $B$ :

- $A$  and  $A^c$  partition
- $A$  and  $B - A$  partition
- $A \cap B$  and  $A \ominus B$  partition
- $A \cap B$  and  $A - B$  partition

# Cardinalities in Venn diagrams

Another way to represent information in a Venn diagram is by writing the cardinality of each individual section in the diagram. Cardinalities can be represented as numbers placed within each section (without points, to distinguish set cardinalities from set elements).

**Example.** Consider the following Venn diagram:



Find each of the following:

- $|A \cap B| =$
- $|A| =$
- $|A \cup B| =$
- $|\mathcal{U}| =$

## Example – Finding cardinalities with a Venn diagram

**Example.** 30 people were asked to select their preferred ice-cream flavours from a list, where more than one selection was allowed. 16 people selected chocolate, 16 selected vanilla, and 16 selected strawberry. 12 people selected both chocolate and vanilla, 8 people selected both chocolate and strawberry, and 7 people selected both vanilla and strawberry. 5 people selected all three flavours. How many surveyed people selected none of the three flavours?

**Solution.**

# Inclusion-exclusion principle

**Theorem.** (Inclusion-exclusion principle)

The cardinality of a union of sets can be expressed in terms of cardinalities of their intersections. For example,

- For any two sets  $A$  and  $B$ , we have

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

- For any three sets  $A$ ,  $B$ , and  $C$ , we have

$$|A \cup B \cup C| = |A| + |B| + |C| - (|A \cap B| + |A \cap C| + |B \cap C|) + |A \cap B \cap C|.$$

**Proof (sketch).**

**Example.** Solve the previous problem using the inclusion-exclusion principle.

**Solution.**