| Value | Probability | |
|---|---|---|
| 1 | 0.1 | |
| 2 | 0.1 | |
| 3 | 0.2 | |
| 4 | 0.1 | |
| 5 | 0.3 | |
| 6 | 0.2 | |
| | mwan | Sum of the above |

Sum {}

where x runs over all possible event

Roll 120 times
1, 6, 2, 5, 4, 2, 3, 4, 5, …. 6

Roughly how often will you get a 1? 20
2? 20

(20 * 1 + 20 * 2 + 20 * 3 +. + 20 * 6) /
120
= 1 * 1/6 + 2 * 1/ 6 + … + 6 * 1/6
= 3.5

| Value | Probability | |
|---|---|---|
| 1 | 0.1 | 1 * 0.1 |
| 2 | 0.1 | 2 * 0.1 |
| 3 | 0.2 | 3 * 0.2 |
| 4 | 0.1 | 4 *.0.1 |
| 5 | 0.3 | 5 * 0.3 |
| 6 | 0.2 | 6 * 0.2 |
| | mwan | Sum of the above |

# Sum over all possible outcomes x {x * Prob(x)}

where x runs over all possible event

Roll 1000 times
1, 6, 2, 5, 4, 2, 3, 4, 5, …. 6

Roughly how often will you get a 1? 100
2? 100
3? 200

(100 * 1 + 100 * 2 + 200 * 3 + … + 200 * 6)/ 1000 =
1 * 0.1 + 2 * 0.1 + 3 * 0.2 … +6 * 0.2

| | **Meaning** |
|---|---|
| $\lambda$ | Arrival rate |
| $\mu$ | service rate<br>Mean service time = $1 / \mu$ |
| $\boldsymbol{\rho}$ | Utilisation |
| m | Number of servers |
| n | Number of holding slots |

For M/M/1 queue:

$\boldsymbol{\rho} = \lambda / \mu$

Procedure:

- Draw a diagram with the states
- Add arcs between states with transition rates
- Derive flow balance equation for each state, i.e.
  - Rate of entering a state = Rate of leaving a state
- Solve the equation for steady state probability

| Value | Probability | |
|---|---|---|
| 1 | 0.1 | 1 * 0.1 |
| 2 | 0.1 | 2 * 0.1 |
| 3 | 0.2 | 3 * 0.2 |
| 4 | 0.1 | 4 *.0.1 |
| 5 | 0.3 | 5 * 0.3 |

| Number of jobs k | Probability P_k | |
|---|---|---|
| 0 | P_0 | 0 * P_0 |
| 1 | P_1 | 1 * P_1 |
| 2 | P_2 | 2 * P_2 |
| 3 | P_3 | 3 * P_3 |
| 4 | P_4 | |
| 5 | P_5 | |
| 6 | P_6 | |
| | | |

Sum (k * Prob[k jobs])

$= 0 * (1-\boldsymbol{\rho}) + 1 * (1-\boldsymbol{\rho})\boldsymbol{\rho} + 2 * (1-\boldsymbol{\rho})\boldsymbol{\rho}^2 + 3 * (1-\boldsymbol{\rho})\boldsymbol{\rho}^3 + \ldots.$

$=$

$(1-\boldsymbol{\rho})[1 * \boldsymbol{\rho} + 2 * \boldsymbol{\rho}^2 + 3 * \boldsymbol{\rho}^3 + \ldots.]$

Use the formula on p.20.

p = ?. q = ?. x = ?.

For $0 \leq x < 1$,

$$p + x(p+q) + x^2(p+2q) + x^3(p+3q) + \ldots = \frac{p}{1-x} + \frac{xq}{(1-x)^2}$$

| | **Meaning** |
|---|---|
| $\lambda$ | Arrival rate |
| $\mu$ | service rate<br>Mean service time = $1 / \mu$ |
| $\boldsymbol{\rho}$ | Utilisation |
| m | Number of servers |
| n | Number of holding slots |

For M/M/1 queue:
$\boldsymbol{\rho} = \lambda / \mu$

Need : $\lambda < \mu$

Throughput

= min(offered load, maximum processing rate)
= min($\lambda$ , $\mu$)
= $\lambda$

Probability [ a job will finish its service in next $\delta$ seconds ] = $\mu \delta$
Probability [ a job will NOT finish its service in next $\delta$ seconds ] = $1 - \mu \delta$

Prob(Job 1 will finish in the next $\delta$ seconds OR
Job 2 will finish in the next $\delta$ seconds)
= Prob(Job 1 will finish in the next $\delta$ seconds) +
Prob(Job 2 will finish in the next $\delta$ seconds)
-
Prob(Job 1 will finish in the next $\delta$ seconds AND
Job 2 will finish in the next $\delta$ seconds)

$= \mu \delta + \mu \delta - (\mu \delta)^2$
$= 2 \mu \delta$

Prob(A or B) = Prob(A) + Prob(B) – Prob(A and B)