Sample Questions : Part !

Q 4-(a)

$m_t = c_0 + c_1 t$ show that $\sum_{j=-q}^{q} a_j m_{t-j} = m_t$, $a_j = \frac{1}{2q+1}$

$$\sum_{j=-q}^{q} a_j m_{t-j} = \sum_{j=-q}^{q} \frac{1}{2q+1} (c_0 + c_1(t-j))$$

$$= \frac{1}{2q+1} \sum_{j=-q}^{q} (c_0 + c_1 t - c_1 j)$$

$$= \frac{1}{2q+1} \left(\sum_{j=-q}^{q} c_0 + \sum_{j=-q}^{q} c_1 t - \sum_{j=-q}^{q} c_1 j \right)$$

$$= \frac{1}{2q+1} \Big[c_0(2q+1) + c_1 t(2q+1)$$

$$- c_1 \underbrace{((-q) + (-q+1) + \cdots + (-1) + 0 + 1 + \cdots + q-1 + q)}_{0} \Big]$$

$$= \frac{1}{2q+1} \left[(c_0 + c_1 t)(2q+1) \right] = c_0 + c_1 t = m_t \checkmark$$

Q 4-(b)

$$A_t = \sum_{j=-q}^{q} a_j Z_{t-j} \qquad Z_t \sim \text{iid nois}(0, \sigma^2)$$

$$E(A_t) = E \sum_{j=-q}^{q} a_j Z_{t-j} = \sum_{j=-q}^{q} a_j E(Z_{t-j}) = 0$$

$$Var(A_t) = Var \sum_{j=-q}^{q} a_j Z_{t-j} = \sum_{j=-q}^{q} a_j^2 Var(Z_{t-j})$$

by independence

$$= \sigma^2 \sum_{j=-q}^{q} a_j^2$$

$$= \sigma^2 \frac{1}{(2q+1)^2} \sum_{j=-q}^{q} 1$$

$$= \frac{\sigma^2}{2q+1} \checkmark$$

Q5-(a)

$$x_t = a + bt + s_t + y_t \qquad s_t = s_{t+12}$$

$$Z_t = (1-B)(1-B^{12})x_t = (1-B)(x_t - x_{t-12})$$

$$= x_t - x_{t-12} - x_{t-1} + x_{t-13}$$

$$= (a+bt+s_t+y_t) - (a+b(t-12)+s_{t-12}+y_{t-12})$$
$$- (a+b(t-1)+s_{t-1}+y_{t-1})$$
$$+ (a+b(t-13)+s_{t-13}+y_{t-13})$$

note that $s_t = s_{t-12}$, $s_{t-1} = s_{t-13}$

$$= y_t - y_{t-1} - y_{t-12} + y_{t-13}$$

$$E(Z_t) = E(y_t - y_{t-1} - y_{t-12} + y_{t-13}) = 0 \qquad \text{since } E(y_t) = 0$$

$$Cov(Z_{t+h}, Z_t) = Cov(y_{t+h} - y_{t+h-1} - y_{t+h-12} + y_{t+h-13}, y_t - y_{t-1} - y_{t-12} + y_{t-13})$$

$$= \gamma_y(h) - \gamma_y(h+1) - \gamma_y(h+12) + \gamma_y(h+13)$$
$$- \gamma_y(h-1) + \gamma_y(h) + \gamma_y(h+11) - \gamma_y(h+12)$$

$$- \gamma_y(h-12) + \gamma_y(h-11) + \gamma_y(h) - \gamma_y(h+1)$$
$$+ \gamma_y(h-13) - \gamma_y(h-12) - \gamma_y(h-1) + \gamma_y(h)$$

$$= 4\gamma_y(h) - 2\gamma_y(h+1) - 2\gamma_y(h-1) + \gamma_y(h+11)$$
$$+ \gamma_y(h-11) - 2\gamma_y(h+12) - 2\gamma(h-12) + \gamma_y(h+13)$$
$$+ \gamma_y(h-13)$$

$\Rightarrow$ $E(Z_t) = 0$ and $\mathrm{cov}(Z_{t+h}, Z_t) = \gamma_Z(h)$ just depends on h $\Rightarrow$ Z_t is stationary

Q5-(b)

$$x_t = (a+bt)s_t + y_t$$

$$Z_t = \nabla_{12}^2 x_t = (1-B^{12})^2 x_t$$

$$= (1-B^{12})(x_t - x_{t-12}) = x_t - 2x_{t-12} + x_{t-24}$$

$$= (a+bt)s_t + y_t - 2\left[(a+b(t-12))s_{t-12} + y_{t-12}\right]$$

$$+ (a+b(t-24))s_{t-24} + y_{t-24}$$

$$= a(s_t - 2s_{t-12} + s_{t-24}) + bt(s_t - 2s_{t-12} + s_{t-24})$$

$$+ 24b\,s_{t-12} - 24b\,s_{t-24} + y_t - 2y_{t-12} + y_{t-24}$$

note that

$s_t = s_{t-12} = s_{t-24}$

$$\Rightarrow Z_t = y_t - 2y_{t-12} + y_{t-24}$$

$$E(Z_t) = E(y_t - 2y_{t-12} + y_{t-24}) = 0$$

$$\text{cov}(Z_{t+h}, Z_t) = \text{cov}(y_{t+h} - 2y_{t+h-12} + y_{t+h-24}, y_t - 2y_{t-12} + y_{t-24})$$

$$= \gamma_y(h) - 2\gamma_y(h+12) + \gamma_y(h+24)$$
$$- 2\gamma_y(h-12) + 4\gamma_y(h) - 2\gamma_y(h+12)$$
$$+ \gamma_y(h-24) - 2\gamma_y(h-12) + \gamma_y(h)$$

$$= 6\gamma_y(h) - 4\gamma_y(h+12) - 4\gamma_y(h-12)$$
$$+ \gamma_y(h+24) + \gamma_y(h-24)$$

$$\Rightarrow E(Z_t) = 0 , \quad \text{cov}(Z_{t+h}, Z_t) = \gamma_z(h)$$

depends just on $h \Rightarrow Z_t$ is stationary