

COMP9334

Capacity Planning of Computer Systems and Networks

Week 1A: Revision problems

Question 1

- An important part of performance analysis is to model the workload. In this question, you will look at a very simple model and we will generalise it to a very well known model in performance analysis in the lecture in Week 2.
- Consider a user who may send HTTP requests to a web server. In the time interval $[k\delta, (k+1)\delta)$ where k is a non-negative integer, there is a probability of p that this user will send an HTTP request to a web server and there is a probability of $(1-p)$ that this user will not send. Assuming that the probability the user sends (or not send) in each time interval is independent. Assuming that the current time is 10δ , what is the probability that this user will not send an HTTP request to the web server before 30δ ?

Question 2

- This is a revision question on probability distribution which you should be able to solve if you have the pre-requisites.
- Consider a continuous probability distribution with sample space is $[1, \infty)$ and probability density function
 - $f(x) = a / x^3$ for $x \geq 1$
- What is the value of a in order that $f(x)$ be a valid probability density function?
- What is the probability the probability that a number drawn from this distribution is exactly 10?
- Given this probability density function, what is the probability that a number drawn from this distribution has a value greater than 10?

Question 1 - Answers

Prob (the user will not send before 30δ)
= Prob (the user will not send in $[10\delta, 11\delta)$) x
Prob (the user will not send in $[11\delta, 12\delta)$) x
Prob (the user will not send in $[29\delta, 30\delta)$)
(note: the probability to send is independent for each time)
= $(1-p)^{20}$

Question 2 – Answers (Page 1)

- In order that the probability density function be valid, the probability that the number is drawn between $[1, \infty)$ is 1.

$$\int_1^{\infty} \frac{a}{x^3} = 1 \Rightarrow \left[\frac{ax^{-2}}{-2} \right]_1^{\infty} = 1 \Rightarrow \frac{a}{2} = 1 \Rightarrow a = 2$$

- Probability that a number drawn is exactly 10 is zero
 - Explanation: The numbers that can come from this distribution is in the range $[1, \infty)$ and there are infinite numbers, hence the probability of getting just one number (in this case 10), is zero.

Question 2 – Answers (Page 2)

- Probability that a number drawn is greater than 10 =

$$\int_{10}^{\infty} \frac{2}{x^3} = \left[\frac{2x^{-2}}{-2} \right]_{10}^{\infty} = 0.01$$

- Note: The probability distribution that you've worked with is called a Pareto distribution. It has what is known as a heavy tail properties. This probability distribution appears very often in modern computer performance analysis.