# COMP9414: Artificial Intelligence

# Lecture 2b: Informed Search

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## This Lecture

- Heuristics
- Informed Search Methods
  - Best-First Search
  - Greedy Search
  - A$^*$ Search
  - Iterative Deepening A$^*$ Search



## Informed (Heuristic) Search

- Uninformed methods of search are capable of systematically exploring the state space in finding a goal state
- However, uninformed search methods are very inefficient
- With the aid of problem-specific knowledge, informed methods of search are more efficient
- All implemented using a priority queue to store frontier nodes



## Heuristics

- Heuristics are “rules of thumb”
- Heuristics are criteria, methods or principles for deciding which among several alternative courses of action promises to be the most effective in order to achieve some goal. “Heuristics” (Pearl 1984)
- Can make use of heuristics in deciding which is the most “promising” path to take during search
- In search, heuristic must be an underestimate of actual cost to get from current node to any goal – an admissible heuristic
- Denoted $h(n)$; $h(n) = 0$ whenever $n$ is a goal node

## Heuristics – Example

- 8-Puzzle – number of tiles out of place

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 8 | 3 | | | 1 | 2 | 3 |
| 1 | 6 | 4 | $\longrightarrow$ | | 8 | | 4 |
| | 7 | 5 | | | 7 | 6 | 5 |

- Therefore $h(n) = 5$





## Heuristics – Example

- 8-Puzzle – Manhattan distance (distance tile is out of place)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 8 | 3 | | | 1 | 2 | 3 |
| 1 | 6 | 4 | $\longrightarrow$ | | 8 | | 4 |
| | 7 | 5 | | | 7 | 6 | 5 |

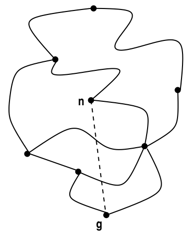
- Therefore $h(n) = 1 + 1 + 0 + 0 + 0 + 1 + 1 + 2 = 6$





## Heuristics – Example

- Another common heuristic is the straight-line distance ("as the crow flies") from node to goal




- Therefore $h(n) =$ distance from $n$ to $g$

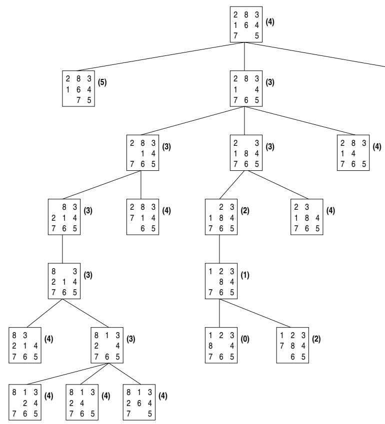




## Greedy Search

- **Idea:** Expand node with the smallest estimated cost to reach the goal
- Use heuristic function $h(n)$ to order nodes on frontier, i.e. choose node for expansion with lowest $h(n)$
- Analysis
  - Similar to depth-first search; tends to follow single path to goal
  - Not optimal, incomplete
  - Time $O(b^m)$; Space $O(b^m)$
  - However, good heuristic can reduce time and space complexity significantly

# Greedy Search









# $A^*$ Search

- **Idea:** Use both cost of path generated and estimate to goal to order nodes on the frontier
- $g(n)$ = cost of path from start to $n$; $h(n)$ = estimate from $n$ to goal
- Order priority queue using function $f(n) = g(n) + h(n)$
- $f(n)$ is the estimated cost of the cheapest solution extending this path
- Expand node from frontier with smallest $f$-value
- Essentially combines uniform-cost search and greedy search






# $A^*$ Algorithm

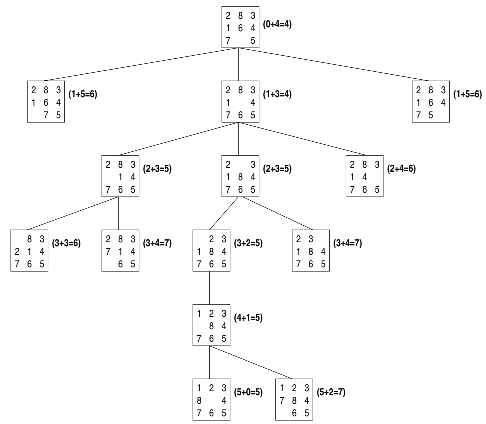
OPEN – nodes on frontier; CLOSED – expanded nodes

OPEN $= \{\langle s_0, nil\rangle\}$ where $s_0$ is the initial state
**while** OPEN is not empty
  remove from OPEN a node $n = \langle s, p\rangle$ with minimal $f(n)$
  place $n$ on CLOSED
  **if** $s$ is a goal state **return** success (with path $p$)
  **for** each edge $e$ connecting $s$ and a successor state $s'$ with cost $c$
    **if** $\langle s', p'\rangle$ is on CLOSED **then if** $cost(p \oplus e) = cost(p) + c < cost(p')$
      **then** remove $\langle s', p'\rangle$ from CLOSED and put $\langle s', p \oplus e\rangle$ on OPEN
    **else if** $\langle s', p'\rangle$ is on OPEN **then if** $cost(p \oplus e) = cost(p) + c < cost(p')$
      **then** replace $\langle s', p'\rangle$ by $\langle s', p \oplus e\rangle$ on OPEN
    **else if** $\langle s', p''\rangle$ is not on OPEN **then** put $\langle s', p \oplus e\rangle$ on OPEN
**return** failure






# $A^*$ Search



# A* Search – Analysis

Subject to conditions on next slide:

- Optimal (and optimally efficient)
- Complete
- Number of nodes searched (and stored) still exponential in worst case
  - Unless the error in the heuristic grows no faster than the log of the actual path cost $h^*(n)$ of reaching a goal from $n$:

$$|h(n) - h^*(n)| \leq O(\log h^*(n))$$

  - Which almost never happens: for many heuristics, this error is at least proportional to the path cost





# A* Search – Optimality

- Conditions on state space graph
  - Each node has a finite number of successors
  - Every arc in the graph has cost greater than some $\varepsilon > 0$
- Condition on heuristic function $h(n)$: admissibility
  - For every node $n$, the heuristic never overestimates the actual cost $h^*(n)$ of reaching a goal from $n$, i.e. $h(n) \leq h^*(n)$





# A* Search – Optimal Efficiency

- A* is optimally efficient for a given heuristic: of the optimal search algorithms that expand search paths from the root node, there is no other optimal algorithm that expands fewer nodes in finding a solution
- Monotonic heuristic – along any path, the $f$-cost never decreases
  - Follows from triangle inequality

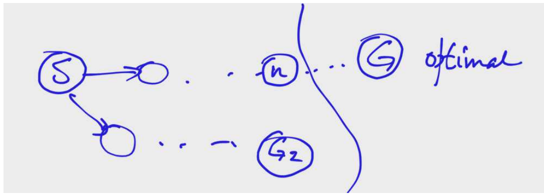
$$h(n) \leq cost(n, n') + h(n')$$

- Common property of admissible heuristics
  - If this holds, don't need CLOSED set – big saving
  - If not, the path cost for $n'$ connected to $n$ can be set to:
    (Pathmax Equation) $f(n') = \max(f(n),\ g(n') + h(n'))$





# Proof of the Optimality of the A* Algorithm




$G$: optimal goal node; $G_2$: another goal node selected by A*
$n$: node on frontier on optimal path to $G$; $h^*(n)$: true cost to goal from $n$
Suppose A* chose $G_2$ rather than $n$
Then: $g(G_2) = f(G_2) \leq f(n)$ since $G_2$ is a goal node and A* chose $G_2$
$= g(n) + h(n)$ by definition
$\leq g(n) + h^*(n)$ by admissibility
$\leq g(G)$ since $G$ is a goal node on a path from $n$
and that path is an optimal path to $G$

This means $G_2$ is also optimal, and hence, so is any node returned by A*

## Heuristics – Properties

- $h_2$ dominates $h_1$ iff $h_2(n) \geq h_1(n)$ for any node $n$
- A$^*$ expands fewer nodes on average using $h_2$ than $h_1$
  - Every node for which $f(n) < f^*$ is expanded
    So $n$ is expanded whenever $h(n) < f^* - g(n)$
    So any node expanded using $h_2$ is expanded using $h_1$
  - Always better to use an (admissible) heuristic with higher values
- Suppose there are a number of admissible heuristics for a problem $h_1(n),\ h_2(n),\ \ldots,\ h_k(n)$
  - Then $max_{i \leq k} h_i(n)$ is a more powerful admissible heuristic
  - Therefore can design a range of heuristics for special cases



## Generating Heuristics

- Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then #tiles-out-of-place gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then Manhattan distance gives the shortest solution
- For TSP: let path be any structure that connects all cities
  $\Longrightarrow$ minimum spanning tree heuristic



## Iterative Deepening A$^*$ Search

- IDA$^*$ performs repeated depth-bounded depth-first searches as in Iterative Deepening, however the bound is based on $f(n)$
- Start by using $f$-value of initial state
- If search ends without finding a solution, repeat with new bound of minimum $f$-value exceeding previous bound
- IDA$^*$ is optimal and complete with the same provisos as A$^*$
- Due to depth-first search, space complexity = $O(\frac{bf^*}{\delta})$ (where $\delta =$ smallest operator cost and $f^* =$ optimal solution cost) – often $O(bd)$ is a reasonable approximation
- Another variant – SMA$^*$ (Simplified Memory-Bounded A$^*$) – makes full use of memory to avoid expanding previously expanded nodes



## Conclusion

- Informed search makes use of problem-specific knowledge to guide progress of search
- This can lead to a significant improvement in performance
- Much research has gone into admissible heuristics
- Even on the automatic generation of admissible heuristics