

The program works as follows: user inputs the masses of the three objects (M_1 , M_2 , M_3), the coefficients of friction of these objects and initial positions as written in the problem description. Also, user inputs the time intervals with the corresponding external force's values. Then the program outputs three accelerations: first and third bodies' (as they are equal), second body's and the third body's vertical acceleration. This information is not required but is useful to understand the movements in the system. Most importantly the changes in the positions of each body for all given time moments are outputted.

The calculations of the acceleration are done based on the equations obtained in below picture.

1) N_1 , F , F' , f_1 , f_2 , N_2 , m_1g

2) N_2 , f_2 , T , m_2g

3) f_3 , T , F' , m_3g

Coordinate system: y (vertical), x (horizontal)

Equations:

$$\begin{cases} m_1 a_1 = f_1 + f_2 - F' - F - T \\ m_2 a_2 = T - f_2 \\ m_3 a_3 = F' \\ m_3 a_{3y} = T + f_3 - m_3 g \\ f_2 = \mu_2 m_2 g \\ f_3 = \mu_3 F' \\ f_1 = \mu_1 (T + m_1 g + m_2 g + f_3) \\ a_1 = a_{3x} \\ a_{3y} = a_1 - a_2 \end{cases} \Rightarrow \begin{cases} a_1 (m_1 - \mu_1 m_3 + m_3) = \mu_1 g M - a_2 (m_2 + \mu_1 m_3) - F \\ a_2 (m_3 - \mu_3 m_3) = a_2 (m_2 + m_3) + \mu_2 m_2 g - m_3 g \\ M = m_1 + m_2 + m_3 \end{cases}$$

Boxed formulas:

$$a_1 = \frac{m_2 m_3 g (\mu_1 - \mu_1 \mu_2) + g (\mu_1 m_3^2 - \mu_2 m_2^2) - (m_2 + m_3) (F - \mu_1 g M)}{2 m_2 m_3 - m_3^2 (\mu_1 - \mu_1 \mu_2) + m_2 m_3 (\mu_1 + \mu_3) - m_1 (m_2 + m_3)}$$

$$a_2 = \frac{a_1 (m_3 - \mu_3 m_3) - \mu_2 m_2 g + m_3 g}{m_2 + m_3}$$

The positions then are calculated based on $X = X_0 + V_0 t + at^2/2$ formula. I have assumed that there is no movement in the system at the time moment t_0 hence the initial velocity V_0 is 0.

The first experiment was to try and obtain the same results as in Quiz 02 where the system is frictionless and the external force is 0. Here are the inputs I used: “M1 M2 M3 n1 n2 n3 x1 x2 x3H x3V” = “1 9 5 0 0 0 1 2 2 1” where “n”s are the coefficients of friction and “x”s are initial positions. The result was the same as in Quiz02 as expected.

Another interesting experiment is if we input “10 1 1 0 0 0 1 1 1 1” and “0 10 3 50 6 30” as t0 F0 t1 F1 t3 F3. We obtain *positions of the first body*: 1.0 22.0 35.0, *positions of the second body*: 1.0 67.0 125.0, *horizontal positions of the third body*: 1.0 22.0 35.0 and *vertical positions of the third body*: 1.0 -44.0 -89.0. We see that the position of the first body doesn’t change much in the first-time moment and it is obvious as the mass is much bigger compared to other bodies and the external force is not big enough to move it much. However, we can see big change in positions of second and third bodies as the force increases.

Note that the vertical position of third body is negative as in is moving down and the y-axis are selected to be vertically up (see the picture). Also, the second body is moving along the x-axis to the right and hence the position is positive. This is also logical as second and third bodies are connected with the rope and when 3rd goes down 2nd must go right and vice versa.

There are more experiments done the inputs and outputs of which we can observe below:

Experiment #	1	2	3	4	5
M1 M2 M3 n1 n2 n3 x1 x2 x3H x3V	10 2 4 0.1 0.3 0.2 0 1 2 1	1 1 1 0 0 0 2 1 1 3	10 5 5 0 0 0 2 3 2 4	10 2 3 0.5 0.5 0.5 1 2 2 1	7 7 7 0.1 0.3 0.1 6 2 3 4
t0 F0 t1 F1 ...	0 50 3 70 4 80	0 100 5 120 6 20	0 0 5 300	0 0 5 0 80 130	0 10 5 15 10 60
Accelerations(last time moment)	7.36 69.57 -62.20	10.0 20.0 -10.0	18.57 142.86 -124.29	7.09 50.64 -43.55	6.44 131.57 -125.13
1st positions	0.0 30.0 33.0	2.0 627.0 632.0	2.0 234.0	1.0 89.0 120.0 208.0	6.0 52.0 132.0
2nd positions	1.0 304.0 338.0	1.0 751.0 761.0	3.0 1788.0	2.0 634.0 861.0 1493.0	2.0 1435.0 3079.0
3rd horix. positions	2.0 32.0 35.0	1.0 626.0 631.0	2.0 234.0	2.0 90.0 121.0 209.0	3.0 49.0 129.0
3rd vertical positions	1.0 -272.0 -303.0	3.0 -122.0 -127.0	4.0 -1549.0	1.0 -543.0 -738.0 -1282.0	4.0 -1382.0 -2946.0