Liya Li

Course: STA 106

Instructor: Erin Melcon

Project #1 Report

1. Introduction

This report summarizes the statistical Single Factor ANOVA model and analysis results associated with the Kent island Sparrow-and-Nest study. The observed data are used to analyze whether different size nests attracted different size sparrows on Kent Island. The purpose of this data analysis is to compare and understand the nests choosing preferences trend of Kent island sparrows from a statistical aspect, as well as to possibly further give reference to Kent Island residents for sparrow population balance control. During the statistical analyses, the group mean model of Single Factor ANOVA has been applied.

2. Data Summary

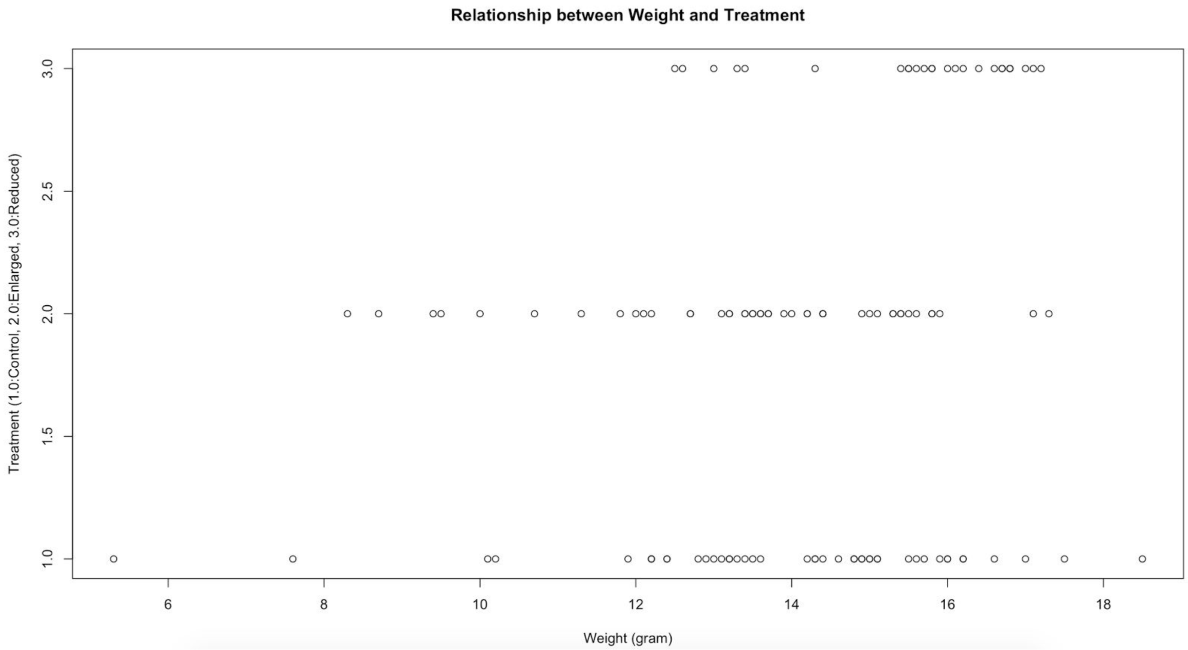
In total, there are 116 observations of two variables in the sample, where one is numerical variable named “Weight” and the other one is categorical variable named “Treatment” with three factor levels “Control”, “Enlarged” and “Reduced”. “Weight” are the measures of the observed sparrow weights on Kent Island, while “Treatment” stand for the types of nests. Respectively, three factor levels of “Treatment” refer to “not manipulated”, “manipulated to be a larger nest than normal” and “manipulated to be a smaller nest than normal”. Table 2.1 summarizes the sample values, including the sample sizes (n.i), sample means (mu.i), sample standard deviation (sd.i), and sample variances (var.i).

Table 2.1 Sample Values

groups	n.i	mu.i	sd.i	var.i
control	45	13.92444	2.419631	5.854616
enlarged	45	13.51556	2.103996	4.426798
reduced	26	15.56923	1.459252	2.129415

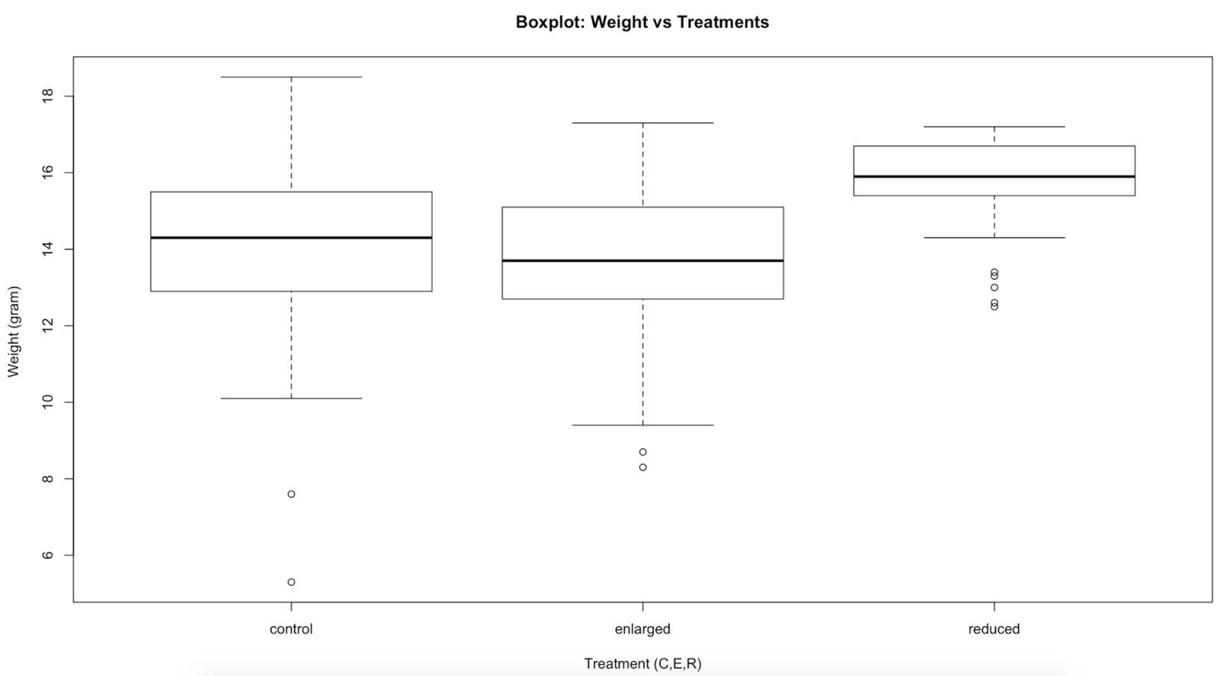
Table 2.1 shows that the three groups have different group sizes, sample means and sample variances, which might violate the null hypothesis of the group mean model of Single Factor ANOVA and the equal variance assumption. Keeping the above guesses in mind, the next step is to plot the data and obtain an approximate trend of them. Figure 2.2 is a data plot on “Weight” and “Treatment”. Figure 2.3 is a boxplot on “Weight” and “Treatment”.

Figure 2.2 Data Plot: Relation Between Weight And Treatment



From this plot, Group Reduced tends to have more data points fall within the range of weight 15 to 17 grams, while Group Enlarged data concentrate more from weight 12 to 16 grams and Group Control from weight 13 to 16 grams.

Figure 2.3 Boxplot: Relation Between Weight And Treatment



From this box plot, Group Reduced box shape is narrower than the other two groups, meaning the sample data in this group are more concentrated while the other two have spread-out data. Also, its mean line lies higher in the Weight axis, meaning this group has a higher sample mean. These indicates that Group Reduced has a significant group mean difference compared to Group Control and Group Enlarged. Moreover, although each group box shows outliers within the groups, further testings are needed to determine whether there are significant outliers since boxplot are subjective.

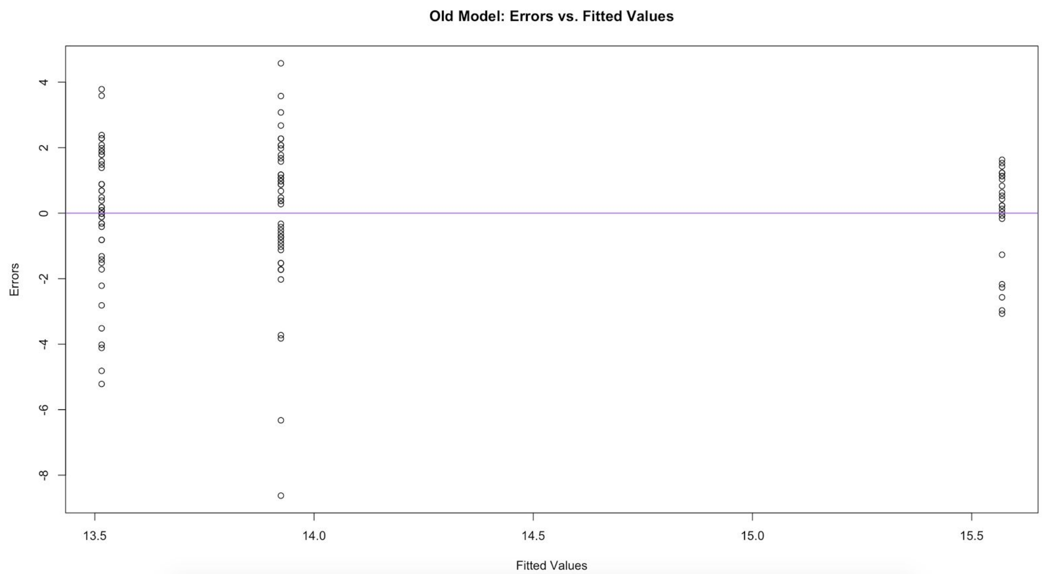
In order to determine whether there exist outliers violating the Single Factor ANOVA, two methods can be used to find the outliers: via semi-studentized/standardized residuals, and, via studentized/standardized residuals. After fitting the sample data into a linear model, both methods show that the data value in row 45 indicates it to be a outlier.

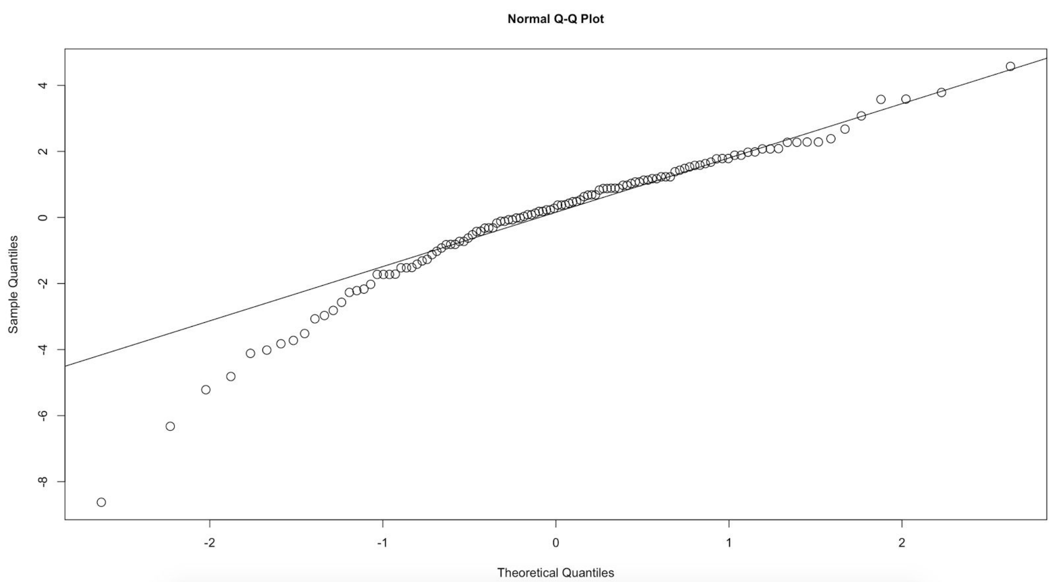
However, unless this outlier violates the model assumptions and removing it is a must, we generally don't remove the outlier in risk of losing essential information. Therefore, in the next section, it's necessary to compare the normality and equal variance assumptions of Single Factor ANOVA model before and after removing this outlier.

3. Diagnostics

Using the model which contains data on row 45 and a new model which doesn't contain it, Figure 3.1, 3.2, 3.3 and 3.4 show the results about the normality and equal variance assumptions with diagnostics plots.

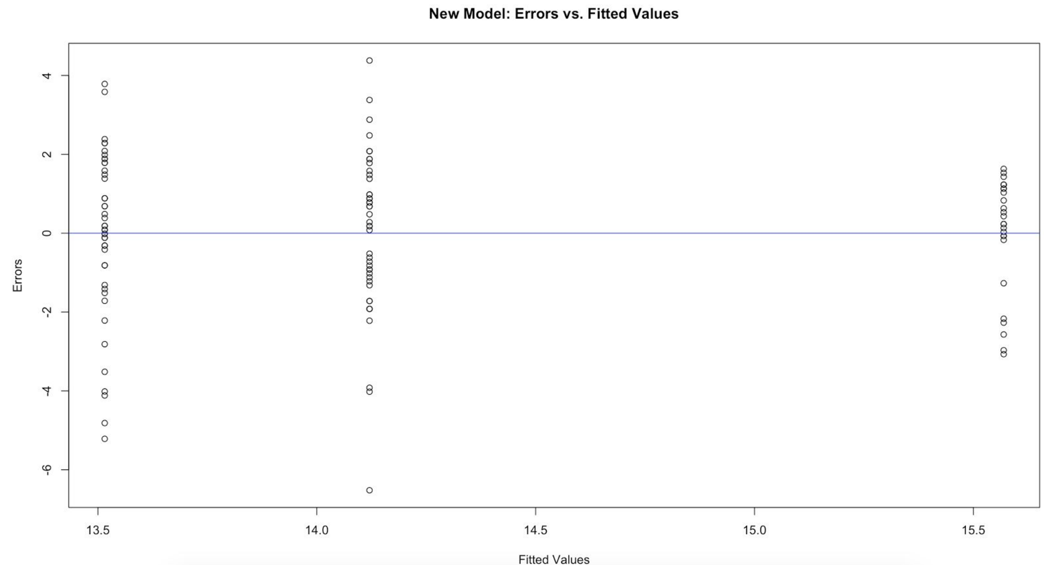
Figure 3.1, 3.2 Diagnostics Plots Of Old Model

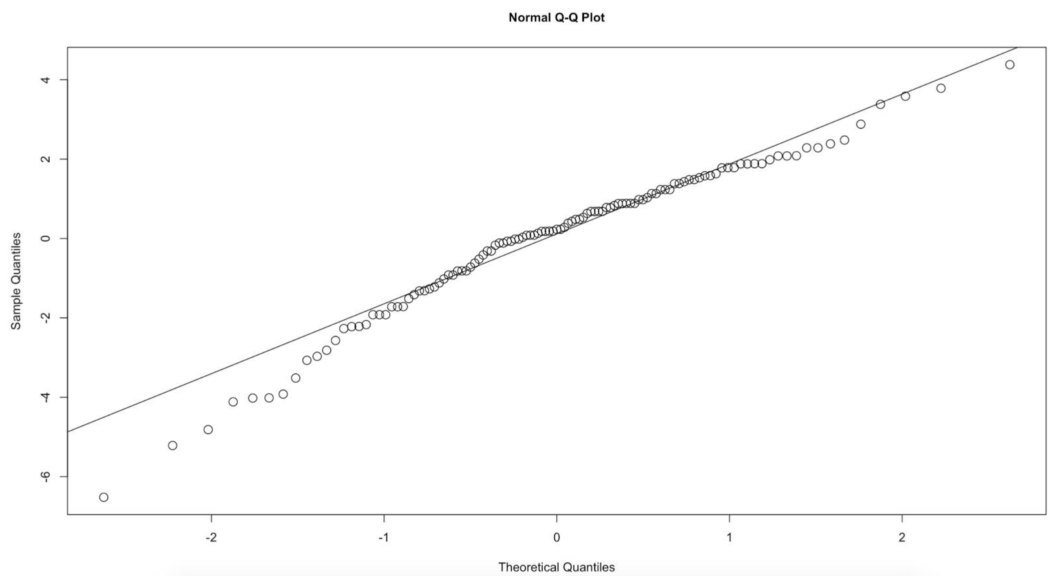




In Figure 3.1, the Errors vs Fitted Value plot for old model (with the outlier) suggests roughly equal vertical spread for each group, meaning that equal variance assumption for this model holds. In Figure 3.2, the Normal QQ plot for old model (with the outlier) suggests that most data points approximately fall around the normal line except on the left tail, partial data points are way off the normal line. This means that the normality assumption for this model doesn't hold.

Figure 3.3, 3.4 Diagnostics Plots Of New Model





Then, let's check the same assumptions for the new model. In Figure 3.3, the Errors vs Fitted Value plot for new model (without the outlier) suggests roughly equal vertical spread for each group as in Figure 3.1, meaning that equal variance assumption for the new model also holds. In Figure 3.4, the Normal QQ plot for new model (without the outlier) suggests that most data points approximately fall around the normal line except on the left tail, partial data points are still off the normal line even though compared to the Normal QQ plot for old model, data points are now closer to the line. This means that the normality assumption for the new model doesn't hold too.

In order to double check with these conclusion, Shapiro-Wilks Test and Brown-Forsythe Test are used. Significance level alpha is 0.05. Results are shown in Figure 3.5.

Figure 3.5 Shapiro-Wilks and Brown-Forsythe Tests Results

rownames	old	new
Shapiro.pval	6.244921e-05	0.003714486
BF.pval	1.785612e-01	0.229294606
Alpha	5.000000e-02	0.050000000

Figure 3.5 shows that using the same alpha value 0.05, the old model and new model both have smaller P value from Shapiro-Wilks test than alpha and larger P value from Brown-Forsythe test than alpha, which leads to the conclusions respectively that the null hypothesis in Shapiro-Wilks test is rejected and the null hypothesis in Brown-Forsythe test is not rejected. That is, the data in this sample is not normally distributed, and, all groups have the equal variance.

Thus, from the above comparison, no matter whether the outlier is taken out, the equal variance assumption for the Single Factor ANOVA model holds, but the normality assumption doesn't. Since taking away this outlier won't change the fact that the normality assumption is violated, and, it's risky to take away a data which might contains important information, the final decision for the outlier is to keep it in the dataset.

4. Analysis and Interpretation

Now, assuming the normality and equal variance assumptions hold, in this Single Factor ANOVA Group Mean model, the null hypothesis is that Group Control, Group Enlarged and Group Reduced have the same true population averages in attracted sparrow weight, and the alternative hypothesis is that at least one Group has a different true population average in attracted sparrow weight.

Figure 4.1 ANOVA Table

```
Analysis of Variance Table

Response: Weight
           Df Sum Sq Mean Sq F value    Pr(>F)
Treatment   2  72.74  36.372  8.1288 0.0005031 ***
Residuals 113 505.62   4.474
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Shown in Figure 4.1, the test-statistics is 8.1288, following an F distribution with degrees of freedoms 2 and 113 under the null hypothesis.

The P value in this hypothesis test is 0.0005031, which indicates that if the three nes groups are equally attractive for sparrows on Kent Island, we would expect to observe the sample data with a probability 0.0005031. And, the P value is smaller than alpha 0.05, meaning to reject the null hypothesis at the significance level 0.05. That is, we are 95% confident with significant statistical evidence that the at least one of the three groups has a different true population average in attracted sparrow weight. Also, the value of power is computed to be 0.9547259 which also indicates the probability of rejecting the null hypothesis (three nest groups have equal true average weight of attracted sparrows on Kent Island) when it is false is 0.9547259. That is, we have a very high probability to get the “correct conclusion”.

Using the sample data and the old model which contains the outlier, three confidence intervals are constructed. Interval bounds are shown in Figure 4.2.

Figure 4.2 Three Interested Confidence Interval

	mu.Reduced	mu.Control.Enlarged	mu.Control.Reduced
Estimate	15.56923	0.4088889	-1.6447863
Lower Bound	14.74735	-0.6041455	-2.8285137
Upper Bound	16.39111	1.4219233	-0.4610589

The 95% confidence interval for the true average attracted sparrow weight of Group Reduced is (14.74735, 16.39111). Thus, with 95% confidence, the true average weight of sparrows in Kent Island attracted by the manipulated smaller than normal nest is between 14.74735 grams and 16.39111 grams.

The 95% confidence interval for the true difference of average attracted sparrow weight of Group Control and Group Enlarged is (-0.6041455, 1.4219233). Thus, with 95% confidence, there is no significant difference between the true average weight of sparrows in Kent Island attracted by the not manipulated nest and the manipulated larger than normal nest.

The 95% confidence interval for the true difference of average attracted sparrow weight of Group Control and Group Reduced is (-2.8285137, -0.4610589). Thus, with 95% confidence, the true average weight of sparrows in Kent Island attracted by the not manipulated nest is less than the manipulated smaller than normal nest by between 0.4610589 grams and 2.8285137 grams.

5. Conclusion

With the above exploring, plotting and testing the observed data on attracted sparrow weight and nest manipulation levels, firstly, we find that the three types of nests, not manipulated, manipulated to be larger than normal and manipulated to be smaller than normal, are not as the same as effective in attracting different sizes of sparrows on Kent Island. Secondly, nests that are manipulated to be smaller than normal ones are likely more attractive for larger sparrows on Kent Island. And, between the not manipulated nest and the manipulated to be larger than normal nest, different sizes of sparrows on Kent Island tend to choose either one as their attractive target nest, which indicates that the sparrows are likely to have no preference between these two nest types. If all three types of nests are the same effective to attract the different sizes of sparrows on Kent Island, we would expect to observe the sample sparrows weights along with these three types of nests with a very low probability.