Control Systems, Project 2 Report

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Heading control of a small bi-wing aircraft

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Abstract

In the scope of this project we analyzed a bi-wing aircraft system, with and without wind disturbance. We found the system to be unstable. We tried to stabilize the system using lead compensators with different desired phased margins. At the end we got four different stabilized systems, compared the step response of those systems with each other and made some conclusions.

Introduction

The heading control of the given bi-wing aircraft with appropriate TFs is shown below.

a) Determine the minimum value of gain K when Gc(s)=K, so that the steady-state effect of a unit step disturbance Td(s)=1/s is less than or equal to 5% of the unit step.

b) Is the system with the gain from part a) stable?

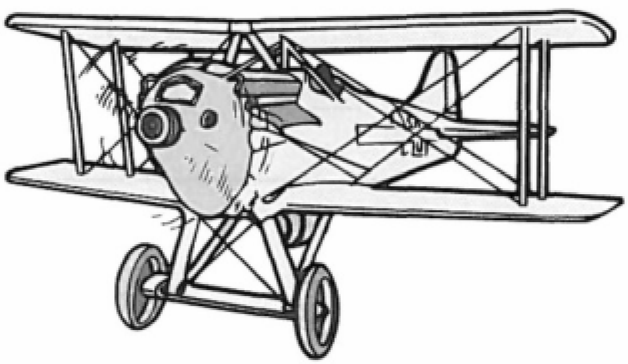
c) Design a (one-stage) lead compensator with a PM of 30°

d) Design a two-stage lead compensator with a PM of 55°

e) Compare the bandwidths of parts c) and d)

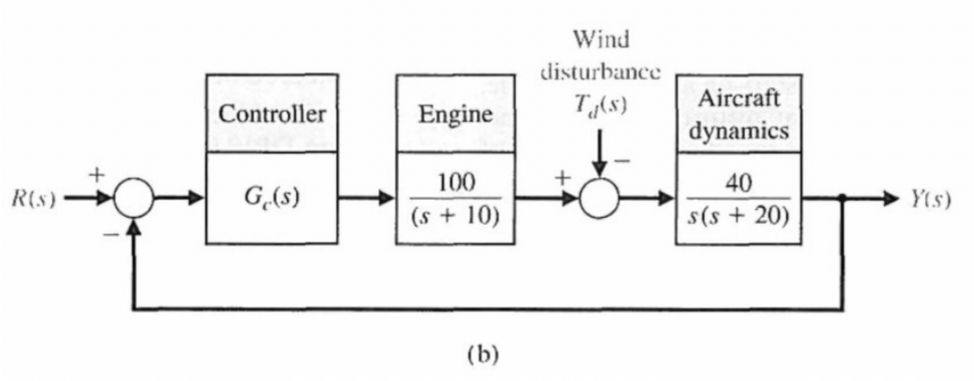
f) Plot the step responses y(t) for parts c) and d) and compare percent overshoots, settling times (2% criterion), and peak times.

Note: All the codes mentioned in the report will be submitted with the report.



Methods

We were given a block diagram representing the bi-wing aircraft model as it is flying in the air. The block diagram for this system is shown below:



Part A

To find the effect of disturbance on our system, we took our input R(s) =0. The resulting transfer function of $Y(s)/T_d(s)$ is

$$G(s) = -\frac{40}{s(s+20)} * \frac{1}{1-\frac{40*k*100}{s(s+20)(s+10)}} = \frac{-40(s+10)}{s(s+20)(s+10)-4000k}$$

Take $T_d(s) = \frac{1}{s}$, the error would be given by the following equation

$$e_{ss} = \lim_{s\to 0} s * \frac{1}{s} * G(s) = \frac{400}{4000k} \leq 0.05$$

$$\frac{1}{10k} \leq 0.05 \quad => \quad k \geq 2$$

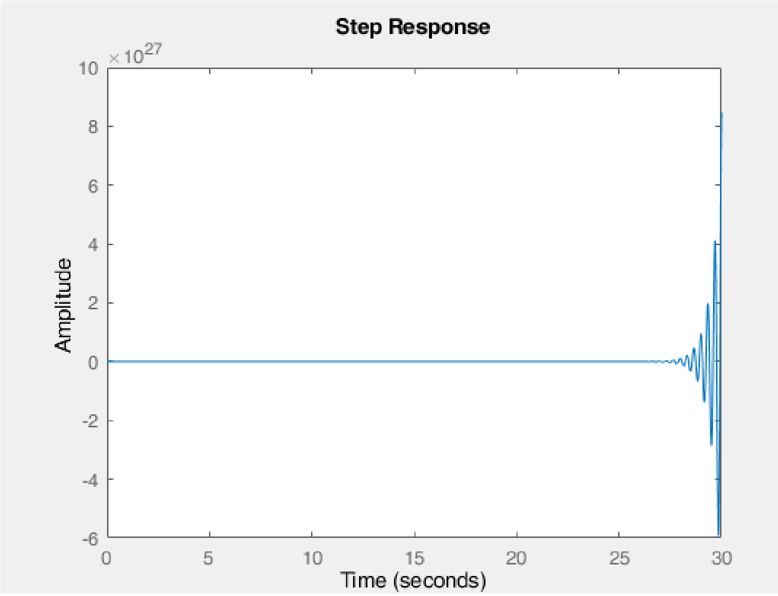
We arbitrarily chose k=3.

Part B

To check the stability of the system, we first calculate the closed-loop transfer function with disturbance $T_d(s) = \frac{1}{s}$ and gain k =3.

$$G(s) = \frac{\left(\frac{3*100}{s+10} - \frac{1}{s}\right)*\left(\frac{40}{s(s+20)}\right)}{1+\left(\frac{3*100}{s+10} - \frac{1}{s}\right)*\left(\frac{40}{s(s+20)}\right)} = \frac{11960*s-400}{s^4+30*s^3+200*s^2+11960*s-400}$$

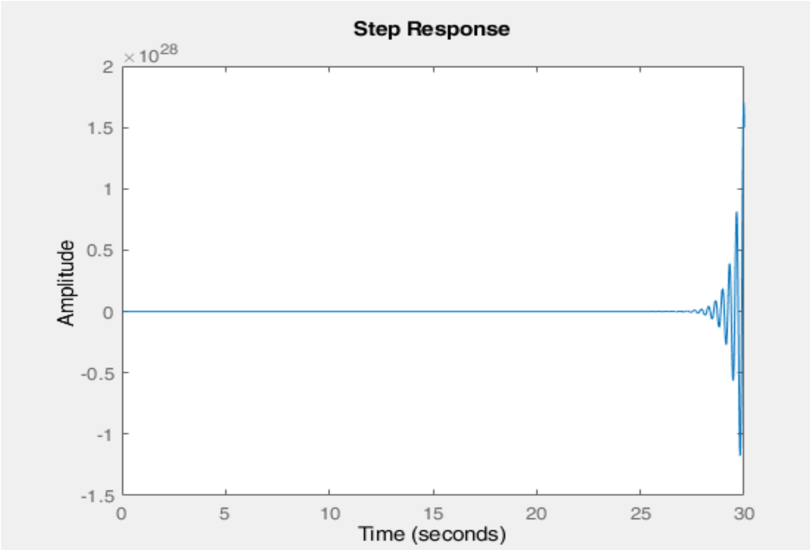
Using Matlab to plot the step response of the system, we find that the system is unstable.



We calculated the transfer function with $T_d(s)$ = 0 and the same gain

$$G(s) = \frac{\frac{3*100*40}{s(s+10)(s+20)}}{1+\left(\frac{3*100}{s+10}\right)*\left(\frac{40}{s(s+20)}\right)} = \frac{11960*s-400}{s^3+30*s^3+200*s\ +12000}$$

Again, we find that the step response of the system even without disturbance is unstable:



Part C

From the results of Part B we saw that the system is unstable, whether we have disturbance or not. Meaning, that just by choosing different gains we cannot stabilize our system. Hence, we needed to add a lead compensator. We designed the lead compensator with a PM of 30°. To achieve this we combined the block diagram into a singular plant which contained the controller, engine, wind disturbance, and aircraft dynamics under a closed-loop system.

We did two examples:

1) When the wind disturbance was equal to 0.

2) When the wind disturbance was set as a step input (1/s).

To design a lead compensator, using Matlab we found the initial PM. Taking a safety factor of 5°, we found that the angle of compensation is $\varphi = PM_{desired} - PM + 5$.

Next, we found the attenuation factor $\alpha = \frac{1+sin(\varphi)}{1-sin(\varphi)}$. After we found the corner frequency w_m at $M = 20log(\frac{1}{\sqrt{\alpha}})$, again using Matlab. We found the lead compensator corner frequency $T_{lead} = \frac{1}{w_m\sqrt{\alpha}}$. As a result, our compensator was given with the following formula:

$$G(s) = \frac{T_{lead} * \alpha * s + 1}{T_{lead} * s + 1}$$

We did this with both wind disturbance and without. All the calculations were done by using the Matlab code labeled “PartC.m” & “PartCwithDisturbance.m”.

Part D

The 1st compensator in Part D was found in the same way as the compensator in Part C. Then, we realized that with just one compensator we won’t be able to get the desired phase margin. So, we had to design a second compensator for our new block diagram, consisting of our first compensator and initial open loop transfer function. We did this using all the same steps used in Part C. All the calculations were done by using the Matlab code labeled “PartD.m” & “PartDwithDisturbance.m”.

Part E

After finding the compensators for Part C and Part D, we found the closed-loop transfer functions of the compensated systems, using Matlab’s feedback() function. After that, using Matlab’s bandwidth() function, we found the bandwidths for all compensated transfer functions (wind disturbance and no wind disturbance). The codes for finding the bandwidths are in “PartC.m”, “PartCwithDisturbance.m”, “PartD.m” & “PartDwithDisturbance.m” files.

Part F

Using Matlab’s step() and stepinfo() functions we plotted the step response of each of our compensated systems and found the overshoot and settling time with 5% percent criterion. The codes for step response are also in “PartC.m”, “PartCwithDisturbance.m”, “PartD.m” & “PartDwithDisturbance.m” files.

Results

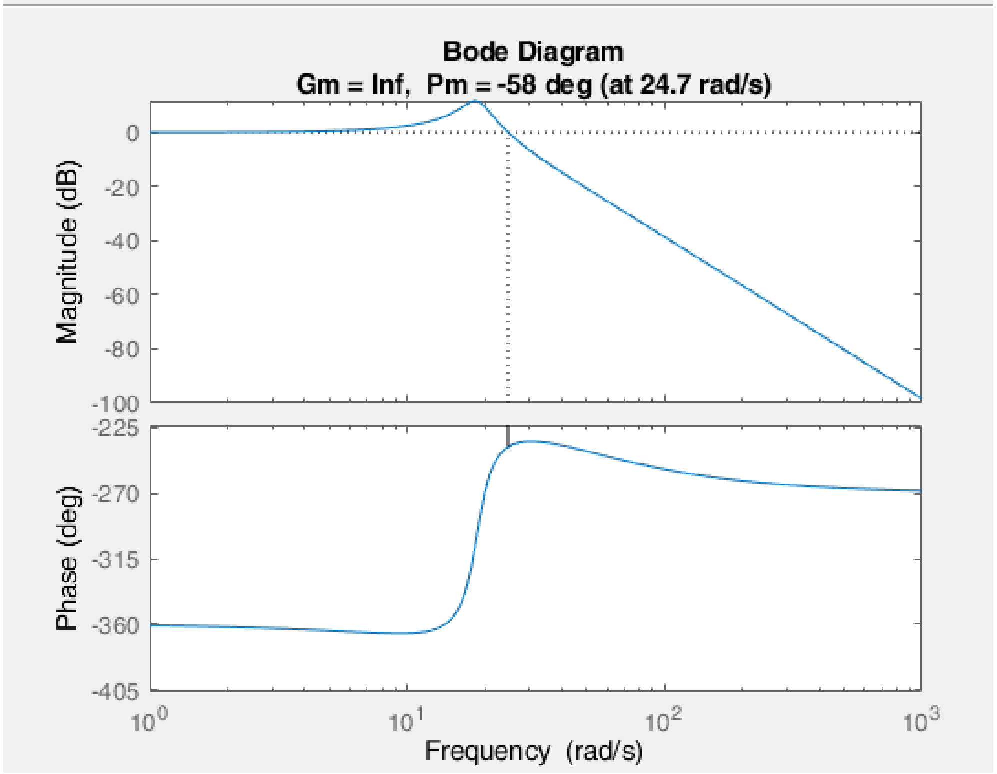
Part C.1 (wind disturbance is taken as 0)

Our code showed our lead compensator to be:

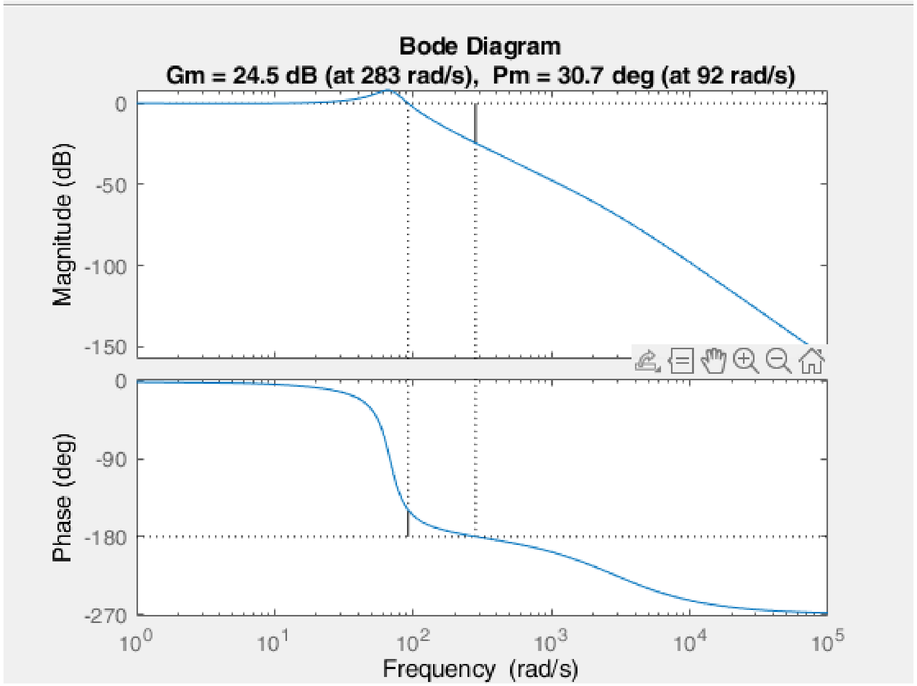
$$\frac{0.4953s+1}{0.0003505s+1}$$

We tested multiple different gains for when the wind disturbance was 0 and we found that a gain of 0.7 gave us the closest value of 30° phase margin as shown below:

Before applying lead compensation:



After applying lead compensation:



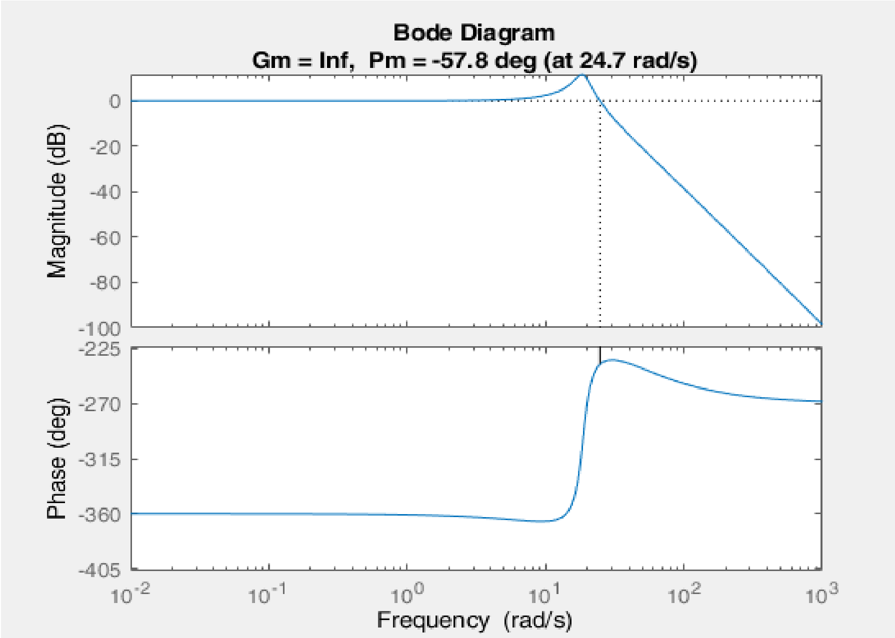
Part C.2 (wind disturbance taken as 1/s)

From our code we get the lead compensator to be:

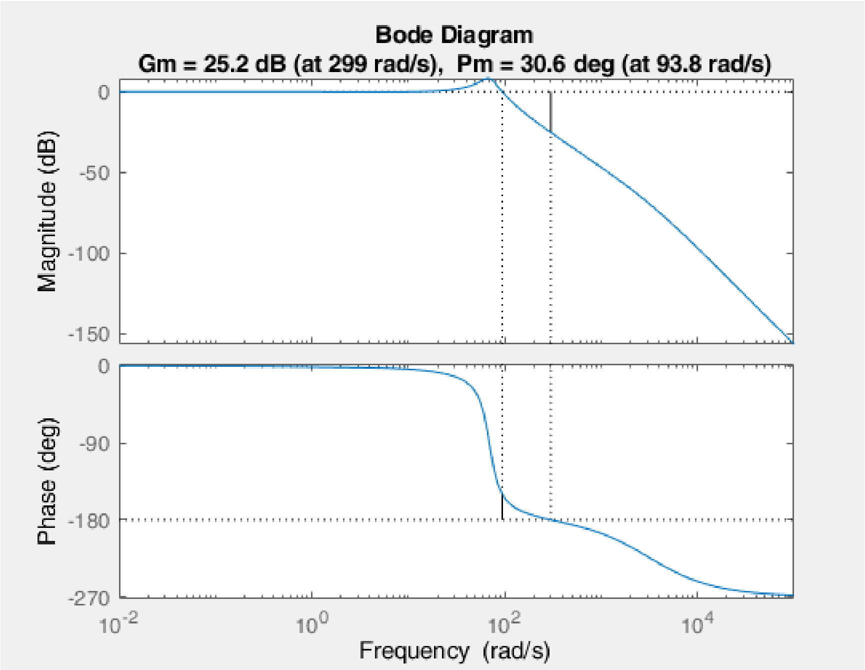
$$\frac{0.5231s+1}{0.0003142s+1}$$

After testing a few different gains we came up with the conclusion that a gain of 0.75 resulted in the closest value to our desired phase margin of 50° as shown below:

Before applying lead compensation:



After applying lead compensation:



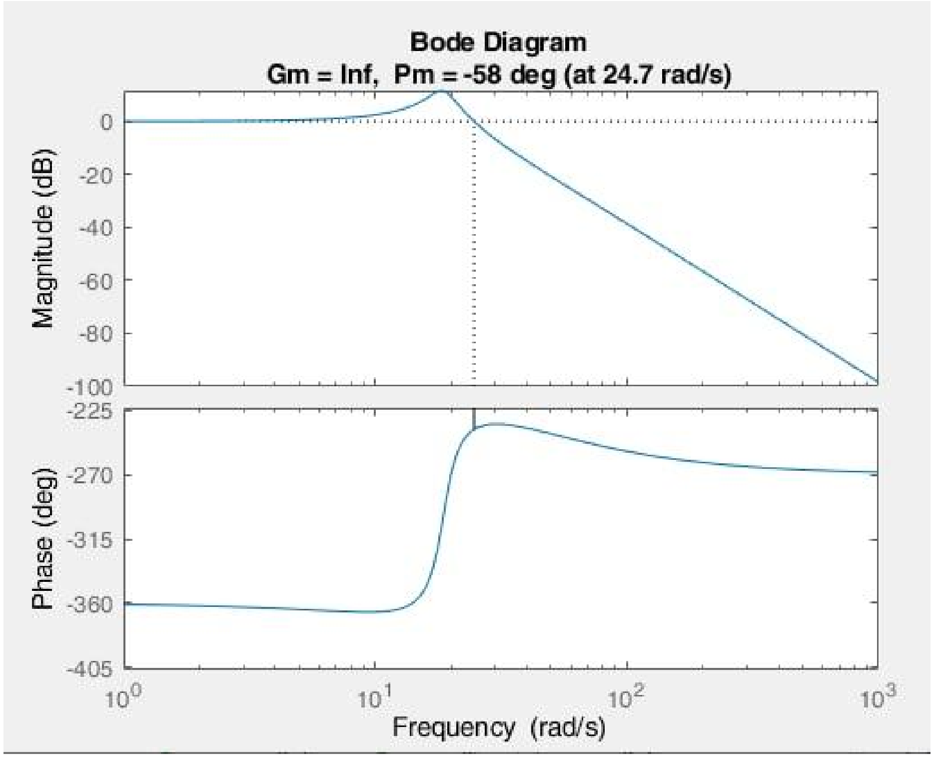
Part D.1 (wind disturbance is taken as 0)

Following the same methods we used for part C, we will apply for a phase margin of 55°. From our first example (taking the wind disturbance to be 0), our code showed the compensator to be:

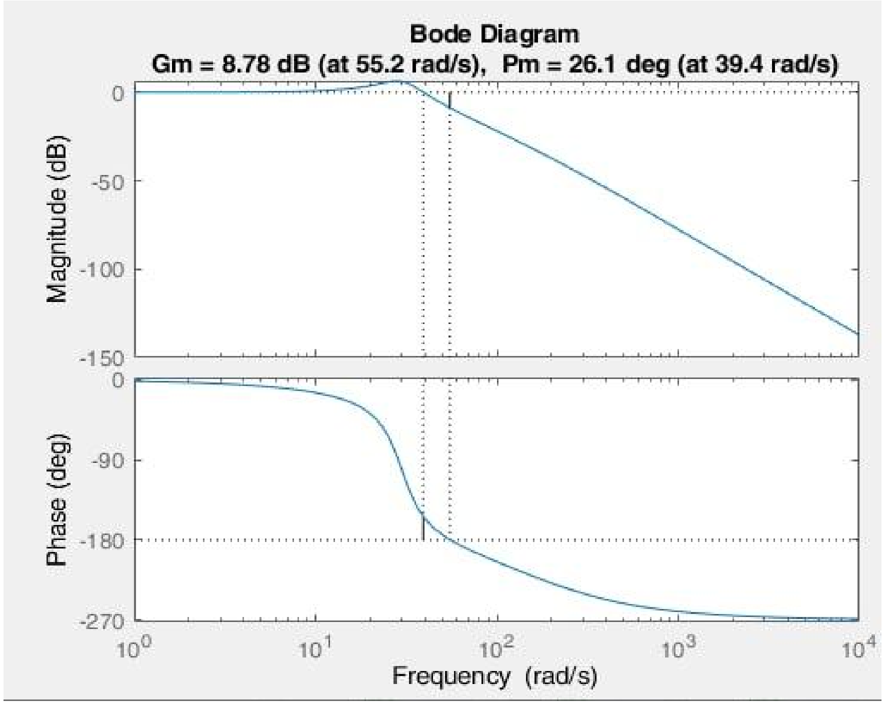
$$\frac{0.1106s+1}{0.006902s+1}$$

Applying the newly found compensator to our closed-loop system and inputting multiple different gains we came to the conclusion that whatever gain we included the result did not reach a phase of 55°, instead we took an arbitrary gain of 0.7 and concluded that we need a second lead compensator to reach our desired phase. The next page shows the phase margins for both uncompensated and compensated systems:

Bode diagram (Uncompensated)



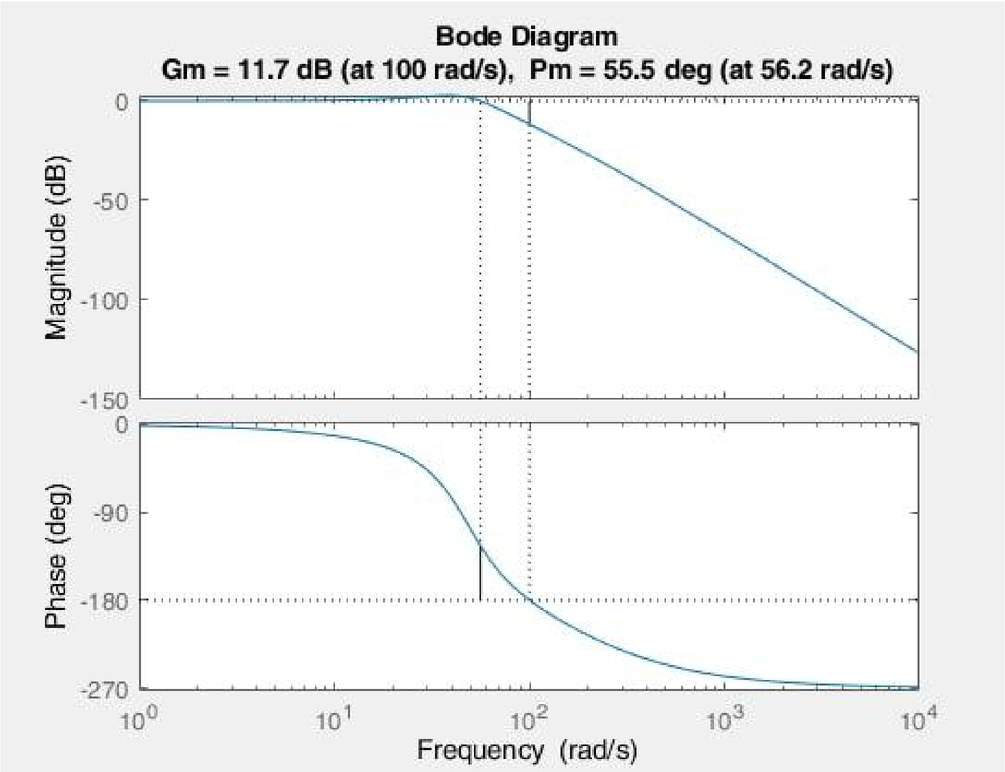
Bode diagram (Compensated)



We calculate the closed-loop transfer function of our system with the first compensator to calculate the second lead compensator. Applying the same code we used previously but for our new closed system, we get our second lead compensator to be:

$$\frac{0.03631s+1}{0.01266s+1}$$

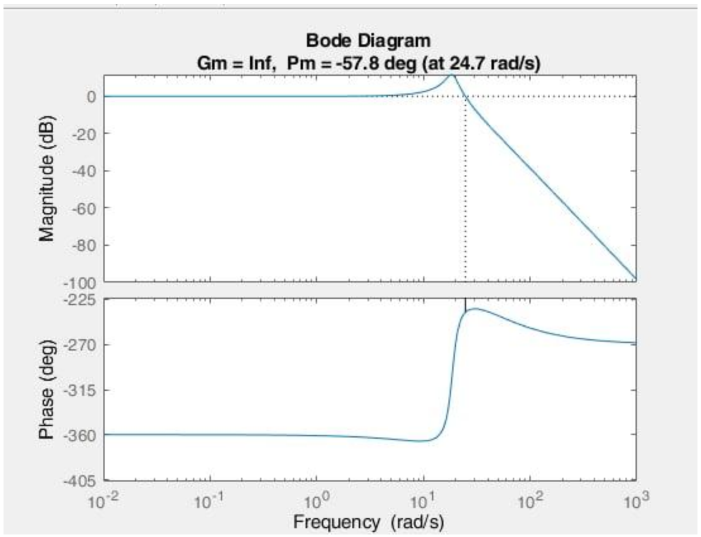
Applying different gains to our second lead compensator we conclude that a gain of 0.83 gives us a phase margin close to the desired value (55°) as shown below:



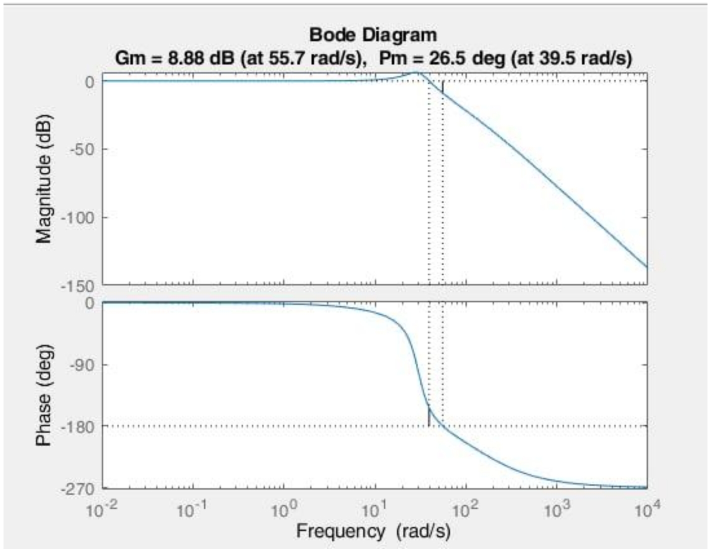
Part D.2 (wind disturbance is taken as 1/s)

When applying the same methods with disturbance our result shows as follows. 1st lead compensator: $\frac{0.1116s+1}{0.00684s+1}$

Bode diagram (uncompensated)



Bode diagram (first compensation)

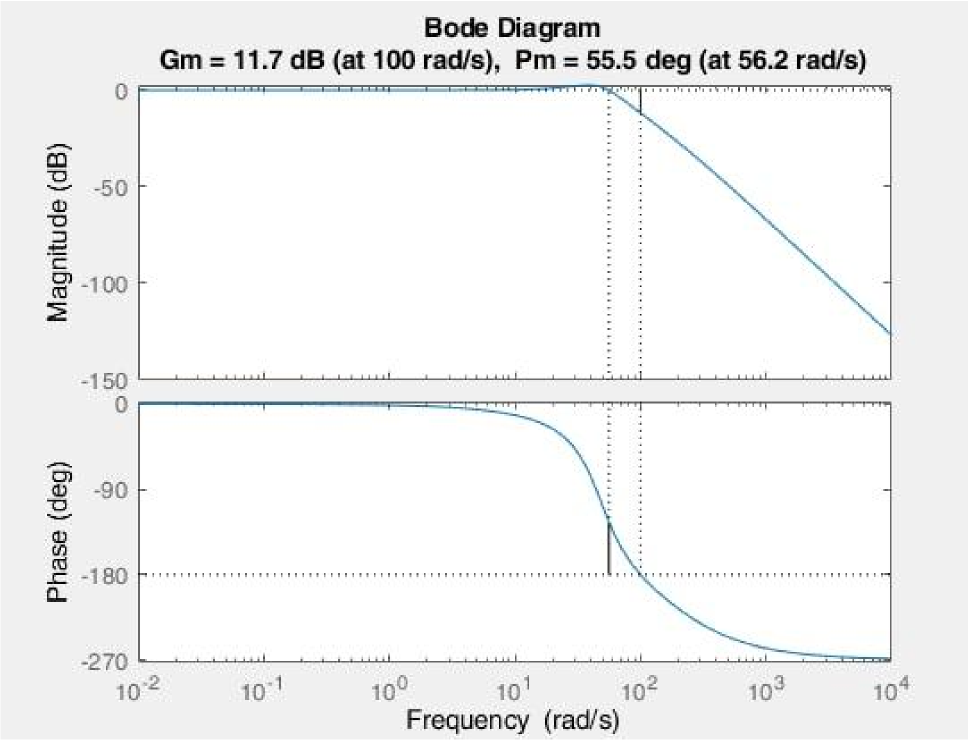


For the first lead compensation, we had the same issue as in part D.1. where changing the gain did not result in our desired phase margin. Instead, we took an arbitrary gain of 0.7, which resulted in a phase of

Applying the code for our closed-loop transfer function including the 1st lead compensator, we get our new compensator to be: $\frac{0.03601s+1}{0.01274s+1}$

Applying a gain of 0.83 we get close to our desired phase value of 55° as shown below

Bode diagram (final compensation)



Part E

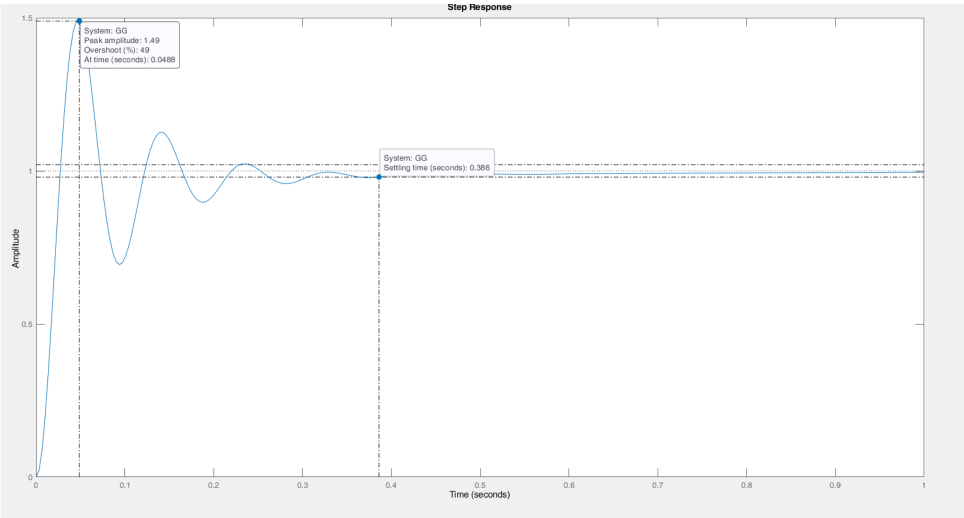
Bandwidth for

a. Compensated PM of 30° with no disturbance: 102.1410 rad/sec
b. Compensated PM of 30° with disturbance: 104.1541 rad/sec
c. Compensated PM of 55° with no disturbance: 66.4162 rad/sec
d. Compensated PM of 30° with no disturbance: 66.3001 rad/sec

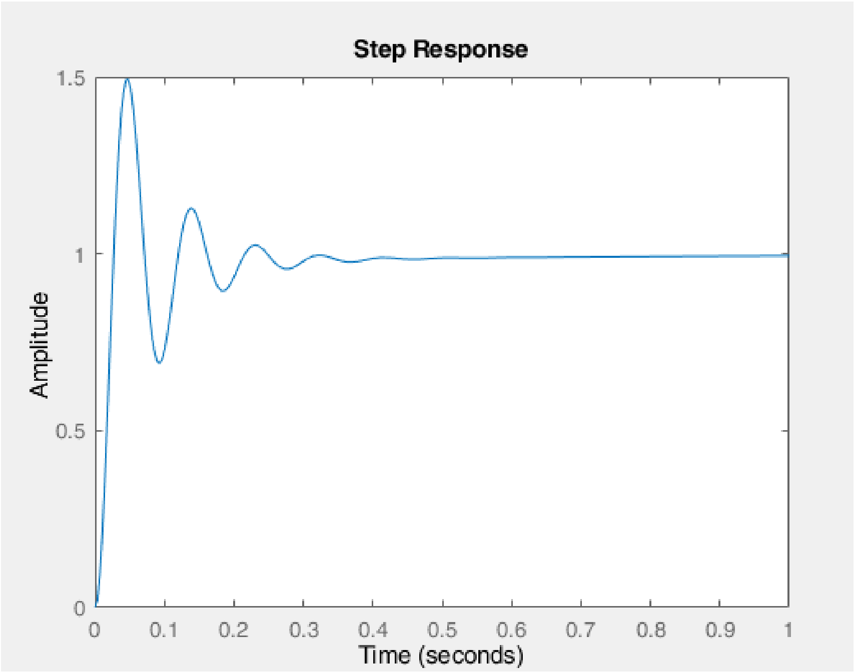
Part F.1 (continuation of Part C.1)

When applying a step input in our final compensated systems from part c we get the following results

No Wind Disturbance



With wind disturbance

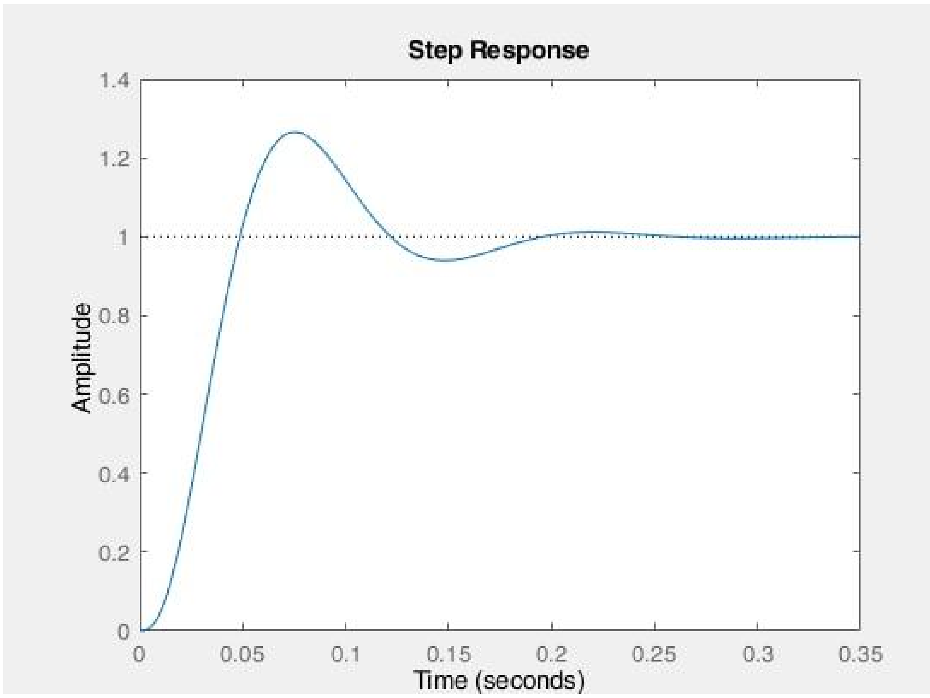



```
        RiseTime: 0.0177
   TransientTime: 0.3824
    SettlingTime: 0.3824
     SettlingMin: 0.6906
     SettlingMax: 1.4927
       Overshoot: 49.2730
      Undershoot: 0
            Peak: 1.4927
        PeakTime: 0.0449
```

Part F.2 (continuation of part C.2)

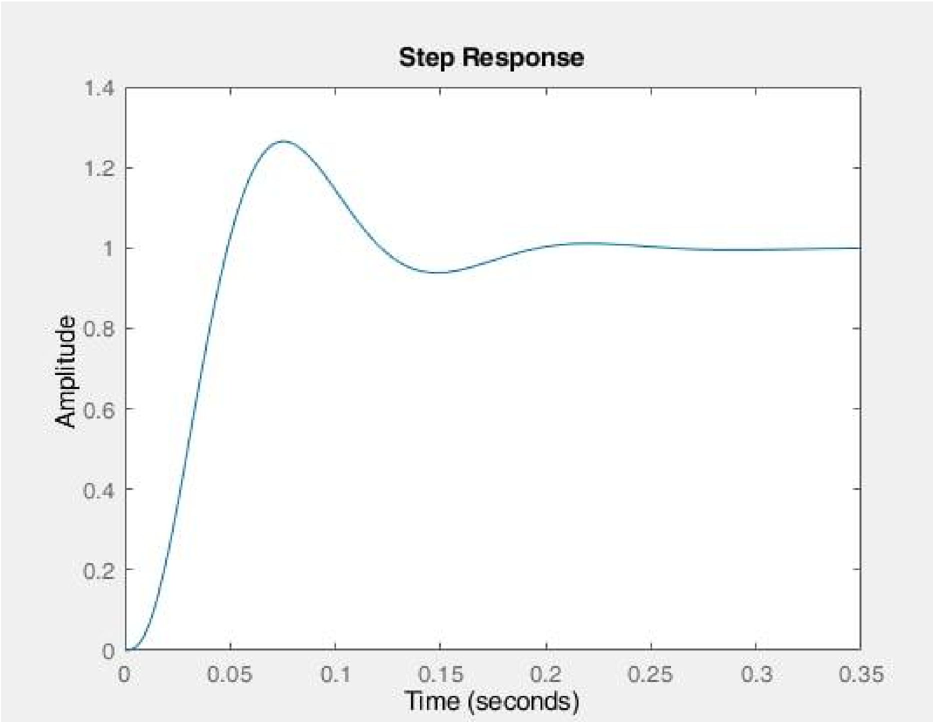
Applying a step input in our final compensated systems from part, we get the following results:

No wind disturbance




```
      RiseTime: 0.0305
 TransientTime: 0.1807
  SettlingTime: 0.1807
   SettlingMin: 0.9142
   SettlingMax: 1.2653
     Overshoot: 26.5338
    Undershoot: 0
          Peak: 1.2653
      PeakTime: 0.0762
```

Without wind disturbance




```
        RiseTime: 0.0306
   TransientTime: 0.1817
    SettlingTime: 0.1817
     SettlingMin: 0.9180
     SettlingMax: 1.2650
       Overshoot: 26.5043
      Undershoot: 0
            Peak: 1.2650
        PeakTime: 0.0743
```

We can conclude that the result is far better than the 30° counterparts, having fewer oscillations and a smaller overshoot.

Conclusion

From part E we compare the bandwidths. In general, bandwidth is a measure of how fast it responds to changing input commands. The higher the bandwidth, the more responsive the system is to changes. From the results of Part E, both with disturbance and without the system with a PM of 30° is more responsive than the system with PM of 55°.

If we were to compare the system without disturbance and with disturbance, in the case of the system with PM of 30°, the system with disturbance is more responsive. In the case of the system with PM of 55°, the system without disturbance is more responsive.

From part F.1 we can conclude a few things. First of all the system was stabilized after applying the lead compensation for both cases (with and without wind disturbance), having very few oscillations in both cases and a similar overshoot value (49% and 49.27%). However, the settling time showed a weird phenomenon, the settling time with the disturbance was faster than without the disturbance. (0.3824 and 0.386 respectively)

From part F.2 we can conclude that there was a significant difference for both cases from their F.1 counterparts. Both results had a lot fewer oscillations and their overshoots decreased significantly as well (from ~49% to ~26%) as well as the settling times (from ~ 0.38 to ~0.18). Though the strange phenomenon still exists, where the set time with the disturbance was faster (only slightly) than the settling time without the wind disturbance.