

N.1a Collatz Conjectures

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Conjectures:

1. There are more even terms than odd terms.
2. For some $b \in \mathbb{Z}$, an initial term in the format of 2^b creates b number of even terms, excluding the last term, 1.
3. For some even integers, there exist an even and an odd integer that create the integer and therefore creates two sequences to the same even integer.

Proof of 1:

Proof. This will be a proof by cases. First, let $a_0 = 2k$ be an even integer for some $k \in \mathbb{Z}$. Then the next term is,

$$\begin{aligned} a_{1n} &= \frac{a_0}{2} \\ &= \frac{2k}{2} \\ &= k \end{aligned}$$

Thus, a_{1n} is either even or odd. Secondly, let $a_0 = 2m + 1$ be an odd integer for some $m \in \mathbb{Z}$. Then the next term is,

$$\begin{aligned} a_{2n} &= 3(a_0) + 1 \\ &= 3(2m + 1) + 1 \\ &= 6m + 4 \\ &= 2(3m + 2) \\ &= 2f \end{aligned} \qquad f = 3m + 2$$

Thus, a_{2n} is always even. Therefore, there are more even terms than odd terms. □