GPGN328: Physics of the Earth 1 - Assignment 5

Due: 11.59pm on Tuesday, November 11

### Your name:

Student #:

Instructions

- The Assignment 5 problem set below consists of **four** questions on materials discussed in course Modules 7 and 8.
- **You are required to answer all questions.**
- Please provide complete answers that clearly show your calculations steps and ensure that you use correct units.
- At the end of each full question (not each subquestion!) include a short statement outlining the importance or significance of your result.
- It is expected (and encouraged) that you will work on this assignment in small groups; however, **please ensure that your write up is your own**.

Questions (Module 7)

Q7-1: Interface Conditions. At the interface between two layers with electrical resistivities ρ_1 and ρ_2 as in the figure below, the electrical boundary conditions are: (1) the component of current density j_z *normal* to the interface is continuous; and (2) the component of electric field e_x *tangential* to the interface is continuous. A current flow-line makes angles θ_1 and θ_2 before and after refraction, respectively. Derive the electrical "law of refraction" given by

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\rho_2}{\rho_1}$$
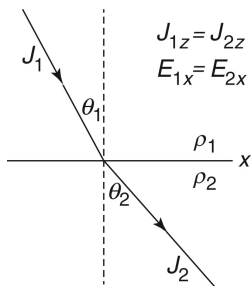
Figure 1. Illustration the interface discussed in this problem, where J and E shown here are the current density **j** and electric field **e** from the Module 7 course notes.

Q7-2: Effective resistivity. What is the **effective resistivity** of a slab of thickness L composed of two half-slabs each of thickness $L/2$ and resistivities 2ρ and $\rho/2$, respectively, as in the diagram below?
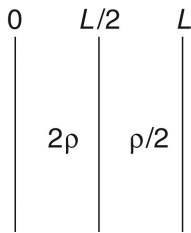

Figure 2. Illustration of the problem discussed in Q7-2.

Questions (Module 8)

Q8-1: Modified Schlumberger Array. Equations 13 and 14 of the Module 8.3 notes discuss the generalized arrangement for four electrodes; however, we did not discuss specific electrode geometries. In the Schlumberger resistivity method the separation of the current electrodes L is much larger than the separation a of the voltage electrodes. Suppose that the mid-point of the voltage pair is displaced by a distance x from the mid-point of the current electrode pair (see sketch below). Show that, when assuming $(L-2x) >> a$, the apparent resistivity ρ_a is given by

$$\rho_a = \frac{\pi}{4} \frac{\Delta\phi}{I} \frac{\left(L^2 - 4x^2\right)^2}{a\left(L^2 + 4x^2\right)}$$

where I is current and $\Delta\phi$ is the potential difference (also commonly given by V).
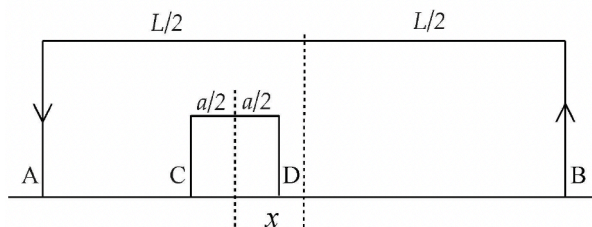
Figure 3. Sketch of the electrode geometry used in the Schlumberger resistivity method.

Q8-2: Downhole electrode array. We start by plotting the measured voltage for a four-electrode configuration when the positions of three electrodes are fixed, and the position of the fourth electrode is allowed to vary along a profile at the surface of the Earth. We will see that this apparently simple task may already lead to numerical problems.

In [1]:
```python
import numpy as np
import matplotlib.pyplot as plt
```

In [3]:
```python
# X-positions of the current electrodes [m]
xA=-10.0
xB=10.0

# X-position of detection electrode C [m]
xC=2.0

# X-position of detection electrode D is variable [m]
xD=np.linspace(-20.0,20.0,101)+0.2

# Measured voltage (assuming rho*I/(2pi)=1)
```
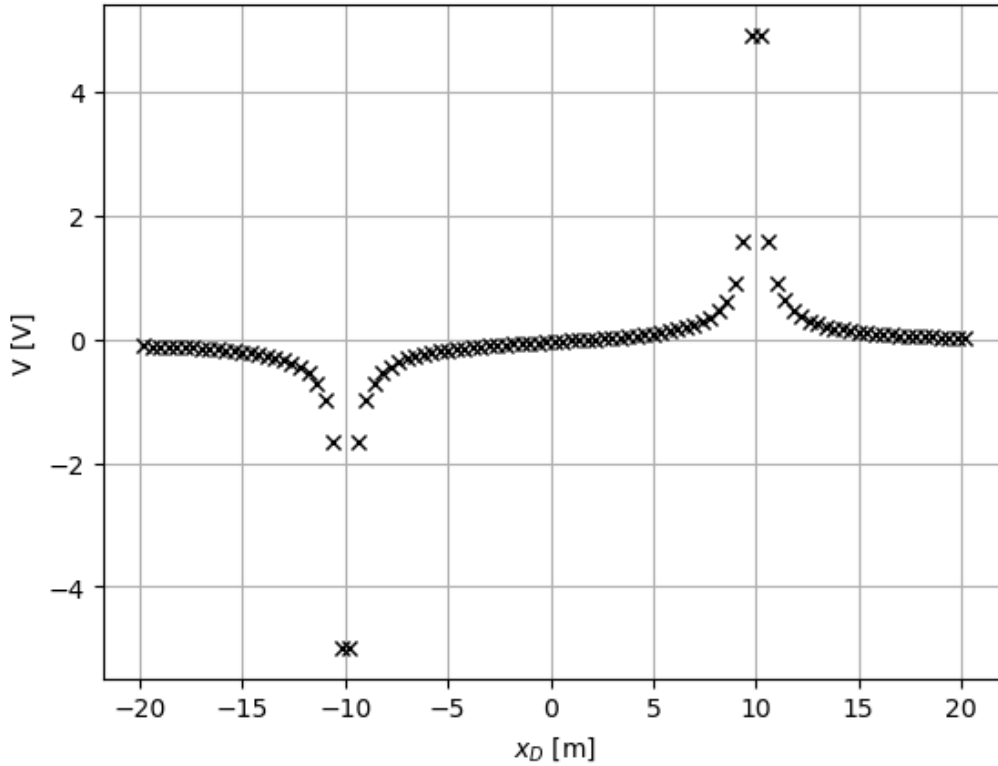
```
V=(1.0/np.abs(xA-xC)-1.0/np.abs(xC-xB))-(1.0/np.abs(xA-xD)-1.0/np.abs(xD-xB))

# Plot
plt.plot(xD,V,'kx')
plt.xlabel(r'$x_D$ [m]')
plt.ylabel('V [V]')
plt.grid()
plt.show()
```


(a) When plotting the voltage V as a function of the detection electrode position x_D, we do not consider values where x_D equals either x_A or x_B. Why is is this done?

(b) Consider a new configuration where the detection electrode D moves **down into a borehole** instead of moving across the surface. For this, keep $x_C = 2$ m, $z_C = 0$ m, and set $x_D = -2$ m. Then let z_D be a variable. Plot the measured voltage as a function of depth z_D by modifying the above calculations.

(c) Display the potential U

$$U = \frac{\rho I}{2\pi} \left(\frac{1}{r_{AC}} - \frac{1}{r_{CB}} \right)$$

as a function of x_C and z_C in a two-dimensional plot. Remember what you learned from (a).

(d) Using your results from (c), compute the vertical electric field e_z using a finite-difference approximation:

$$e_z(x, z) = -\frac{\partial U}{\partial z} \approx -\frac{1}{\Delta z}[U(x, z + \Delta z) - U(x, z)] \, ,$$

with some suitable increment Δz in z-direction. Check your result against an analytic differentiation (i.e., by hand) of U.