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HW3

Computer Graphics

① 2D translations commute.

show: $T_1 \cdot T_2 = T_2 \cdot T_1$

prove: $T_1(a,b) = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$

$T_2(c,d) = \begin{bmatrix} 1 & 0 & c \\ 0 & 1 & d \\ 0 & 0 & 1 \end{bmatrix}$

$$T_1(a,b) \cdot T_2(c,d) = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & c \\ 0 & 1 & d \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a+c \\ 0 & 1 & b+d \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2(c,d) \cdot T_1(a,b) = \begin{bmatrix} 1 & 0 & c \\ 0 & 1 & d \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a+c \\ 0 & 1 & b+d \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, we have that $T_1 \cdot T_2 = T_2 \cdot T_1$

② 2D Rotations commute

To show: $R_1 \cdot R_2 = R_2 \cdot R_1$

prove: $R_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix}$

$R_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}$

$$\because R_1 \cdot R_2 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \cdot \sin\theta_2 & -\cos\theta_1 \cdot \sin\theta_2 - \sin\theta_1 \cos\theta_2 \\ \sin\theta_1 \cdot \cos\theta_2 + \cos\theta_1 \cdot \sin\theta_2 & -\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2 \end{bmatrix}$$

$$\because R_2 \cdot R_1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_2 \cdot \cos\theta_1 - \sin\theta_1 \sin\theta_2 & -\cos\theta_2 \cdot \sin\theta_1 - \sin\theta_2 \cos\theta_1 \\ \sin\theta_2 \cdot \cos\theta_1 + \cos\theta_2 \cdot \sin\theta_1 & -\sin\theta_1 \cdot \sin\theta_2 + \cos\theta_1 \cdot \cos\theta_2 \end{bmatrix}$$

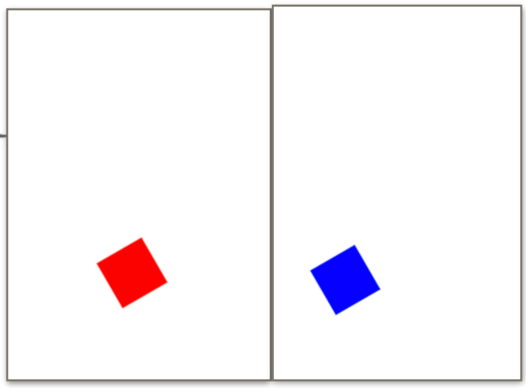
$\therefore R_1 \cdot R_2 = R_2 \cdot R_1$

③ 2D Translations don't commute with rotations.

$TR = RT$ is not always true.

we are going to prove by counterexample.

Assume $T_{(30,40)} = \begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 40 \\ 0 & 0 & 1 \end{bmatrix}$



$$R = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$TR = \begin{bmatrix} 0.5 & -0.87 & 30 \\ 0.87 & 0.5 & 40 \\ 0 & 0 & 1 \end{bmatrix}$$

$$RT = \begin{bmatrix} 0.5 & -0.87 & -19.8 \\ 0.87 & 0.5 & 46.1 \\ 0 & 0 & 1 \end{bmatrix}$$

$TR \neq RT$. we have shown an counterexample for that statement.

④ 2D Translations don't commute with scales.

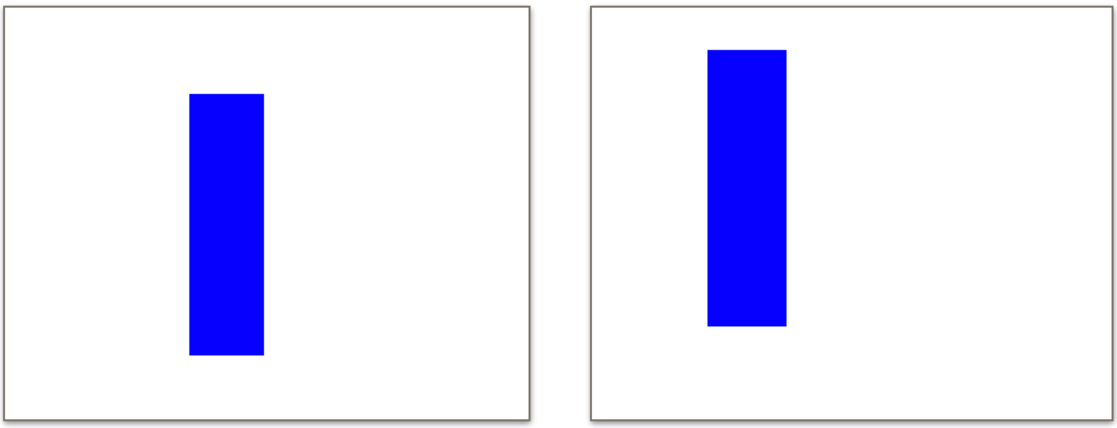
$TS = ST$ is not always true.

we are going to prove by counterexample.

$$S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad T_{[100,10]} = \begin{bmatrix} 1 & 0 & 100 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

$$ST = \begin{bmatrix} 2 & 0 & 200 \\ 0 & 7 & 70 \\ 0 & 0 & 1 \end{bmatrix} \quad TS = \begin{bmatrix} 2 & 0 & 100 \\ 0 & 7 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore $ST \neq TS$.



7) Rotations don't commute with scaling.

$RS = SR$ is not always true.

To show: $RS \mathrel{!}= SR$.

$$R = \begin{bmatrix} \cos 28.65 & -\sin 28.65 & 0 \\ \sin 28.65 & \cos 28.65 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$RS = \begin{bmatrix} -1.9 & 2.6 & 100 \\ -0.7 & -6.5 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

$$SR = \begin{bmatrix} -1.9 & 0.7 & 200 \\ -2.6 & -6.5 & 70 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore. $RS \neq SR$.