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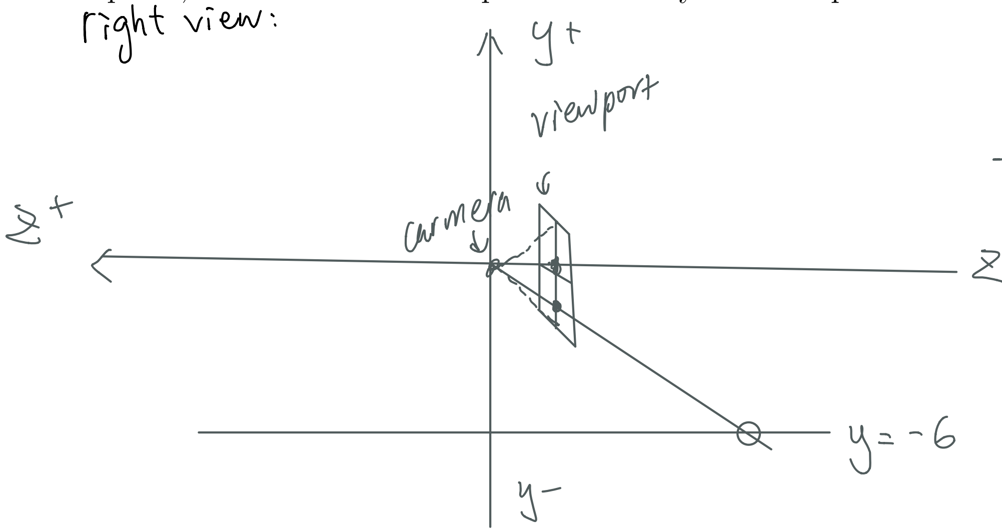
Ray-plane intersection

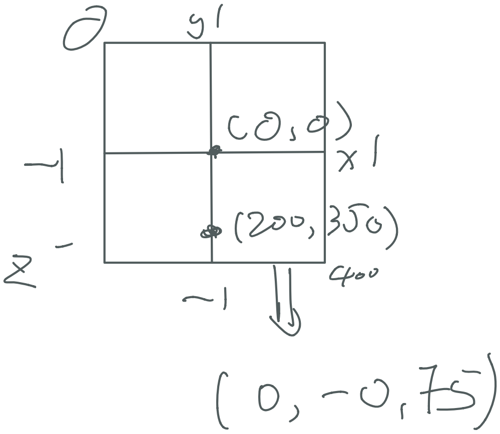
When using a ray-tracing approach to rendering, it is important to determine whether or not each ray intersects with a given object. In this example we will intersect a ray with a plane.

(a) Setup: here we will have a "ground" plane spanning the entire world at $y = -6$. We will have a camera at the origin and a screen that is 400×400 pixels. Our aspect ratio will be 1, fov=90°, and `near` = 1. This will give us a viewport at $z = -1$, with $x_{\min} = -1$, $x_{\max} = 1$, $y_{\min} = -1$, $y_{\max} = 1$, so the origin of the viewport is at the center of the screen. We will cast a ray from the camera through the pixel (200, 350).

First, draw the *right view* of this setup, labeling your axes, the viewport, the camera, the ray, the plane, and the intersection point of the ray with the plane.

right view:





(b) In the next part we will build up our parametric ray equation:

$$\vec{R}(t) = \vec{R}_0 + t\vec{R}_d$$

where $\vec{R}_0$ is the starting point of the ray, $\vec{R}_d$ is the unit vector pointing in the direction of the ray, and $t \in [0, \infty)$. Write down $\vec{R}_0$ and $\vec{R}_d$, showing all your work for $\vec{R}_d$. Put this together to form the ray equation for this specific ray.

$$\vec{R_0} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\vec{R_p} = \begin{bmatrix}0\\-0.75\\-1\end{bmatrix}$$

$$\vec{R_d} = \frac{\vec{R_p} - \vec{R_0}}{\|\vec{R_p} - \vec{R_0}\|}$$

$$\vec{R_p} - \vec{R_0} = \begin{bmatrix}0\\-0.75\\-1\end{bmatrix} - \begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\-0.75\\-1\end{bmatrix}$$

$$\|\vec{R_p} - \vec{R_0}\| = \sqrt{1^2 + 0.75^2}$$

$$= \sqrt{\frac{9}{16} + \frac{16}{16}} = 1.25$$

$$\vec{R_d} = \begin{bmatrix}0\\-0.75\\-1\end{bmatrix} / 1.25 = \begin{bmatrix}0\\-0.6\\-0.8\end{bmatrix}$$

(c) Next, find the intersection point $p = (x, y, z)$ of this ray with the plane. Show your steps clearly, including finding the value of t at the intersection point.

$$\begin{bmatrix} x \\ -6 \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ -0.6 \\ -0.8 \end{bmatrix}$$

$x = 0$

$-6 = 0 + t(-0.6)$

$t = 10$

$z = 0 + (-0.8) \times 10$

$= -8$

$t = 10$

The intersection point

$(0, -6, -8)$

(d) From your result above, what constraints should be placed on z_{far} so that some of the plane is visible on the screen?

z_{far} should be at least 8.