Hello everyone,

Thank you all for your replies to my previous email! I was glad to hear about what you all enjoyed learning about and how fond everyone’s university organic chemistry experience was.

My courses are coming to a close and I’m (kind of) done with my exams now - since one has now been rescheduled to January. Again, feel free to check out the hyperlinks attached in this email and respond :)

Here it goes…

1. BMEG 371: Transport Phenomena

This course has taught me all about mathematical modeling in biology/biomedical systems. We’ve followed the diffusion of drugs from the skin to the blood, tracked the trip of a catheter through an artery, and optimized single cell RNA sequencing with mathematical modeling. However, I will share the details of my final project for this course: modeling the transport of water and sugar in trees.

As many of you know, I’ve been really interested in ecology lately as I’ve been greatly inspired by the books *The Hidden Life of Trees* by Peter Wohlleben and *Entangled Life* by Merlin Sheldrake. So I was very stoked to integrate my love of trees with some of the concepts I’ve learned in class.

Aside: featuring my favourite tree on the UBC campus! You can see how this little tree sprouted out from one of the cut branches of an older tree:

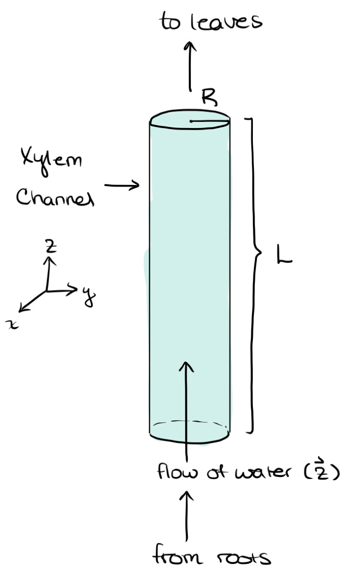
Update: December 20th 2022

Anyway, before I jump into the math, I will give a little bit of a biological background on how water and sugar is transported in trees through the hydraulic connection between the xylem and phloem. The xylem functions to move water and solutes collected from the roots to the leaves while the phloem functions to move water, food, and amino acids (produced from photosynthesis) from the leaves to the rest of the tree.

The flow inside both the xylem and phloem rely on pressure gradients (pressure differences). The xylem heavily relies on the pressure gradient of water from transpiration to move water and solutes against gravity. Transpiration is the evaporation of water into the atmosphere from the tree's leaves, thus causing there to be less water and less pressure at the leaves. In return, this allows water to be drawn up the trunk of the tree, a process known as the transpirational pull.

In the phloem, movement is inspired by the pressure gradient of sugar. Food products from photosynthesis contain a lot of sugar and the phloem's role is to distribute these sugars to the rest of the tree. Since sugar is being produced at the leaves, its concentration and pressure is highest here. As you travel down the phloem, less sugar is present and much of the sugar has already been distributed to other areas of the plant. So the phloem actively tries to move the sugar from high concentration to low concentration.

In my report, I focused on mathematically modeling transport in the xylem specifically and aimed to find an equation to describe the velocity and flow rate of the fluid. I modeled the xylem as a cylinder as shown below:



As shown by my model, I assumed that water only traveled upward (z direction), meaning that velocity only traveled in the z direction:

$$v_z \;-\; \text{velocity (m/s)}$$

I also assumed that flow throughout the xylem was continuous. Meaning that whatever liquid flowed into the xylem, must have also flowed out eventually (no build up of liquid). This assumption could be given by the equation below:

$$\frac{dp}{dt} = -\nabla(pv_z)$$

$$p \;-\; \text{density}\left(\frac{kg}{m^3}\right)$$

This equation essentially says that the change of liquid in the xylem over time is equal to the amount of liquid flowing out.

By assuming steady flow in the xylem; meaning that the viscosity of the fluid and the pressure in the xylem are independent of time (this is not entirely true in nature but makes math way simpler). And using some equations found by Navier-Stokes, we can get this equation:

$$\mu\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = \frac{-\Delta P}{L}$$

$$P - \text{pressure}\left(\frac{kg}{ms^2}\right)$$
$$L - \text{length of xylem in z direction (m)}$$
$$\mu - \text{viscocity of fluid}\left(\frac{kg}{m \cdot s}\right)$$
r - radius of the xylem (m)

This equation may look really messy right but we can integrate it and solve for the velocity as shown below. Where the ΔP in this equation is the pressure gradient, which comes from transpiration as I briefly described earlier.

$$v_z(r) = \frac{-r^2 \Delta P}{4\mu L} + C_1 \ln r + C_2$$

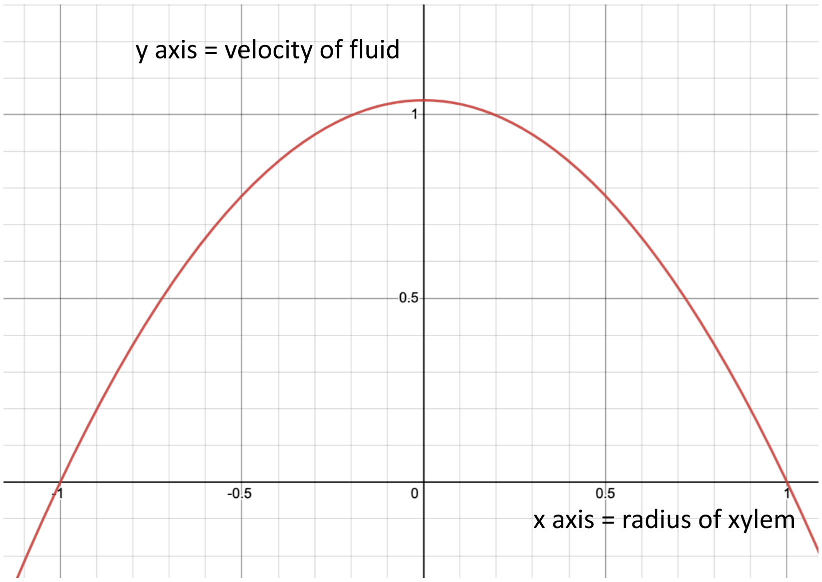
Voila our equation for fluid velocity in the xylem. But what are C1 and C2? No worries, we can solve for these unknown constants that popped up due to integration by setting some restrictions on our model to get this nice equation:

$$v_z(r) = \frac{\Delta P}{4L\mu}\left(R^2 - r^2\right)$$
$$R - \text{radius of xylem } (m)$$

Now that we have the velocity, we can find the flow rate of the liquid in the xylem, as the flow rate is dependent on just the velocity and area:

$$Q = \frac{\pi \Delta P R^4}{8L\mu}$$

To conclude this here is a plot of the velocity:



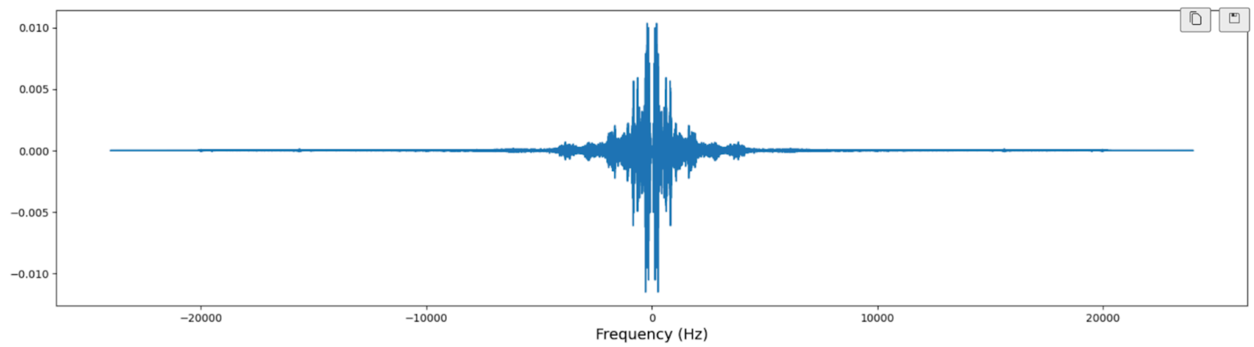
From this graph, we can see that the peak velocity is right at the center of the xylem (where the x axis = 0 is the center), and the velocity at the boundary of the xylem (the wall of the xylem) is zero (where the x axis = -1 & 1). This is the classic velocity profile for laminar flow.

I didn't realize until university how vital math is in so many industries, especially the biomedical one. Who knew how important math could be when modeling tumor metastasis or the digestion of glucose in the human body. Even though math is not my favourite part of engineering, I can appreciate its necessary role in understanding biological systems and creating safe medicine.

Now onto other things…

2. ELEC 221: Systems and Signals

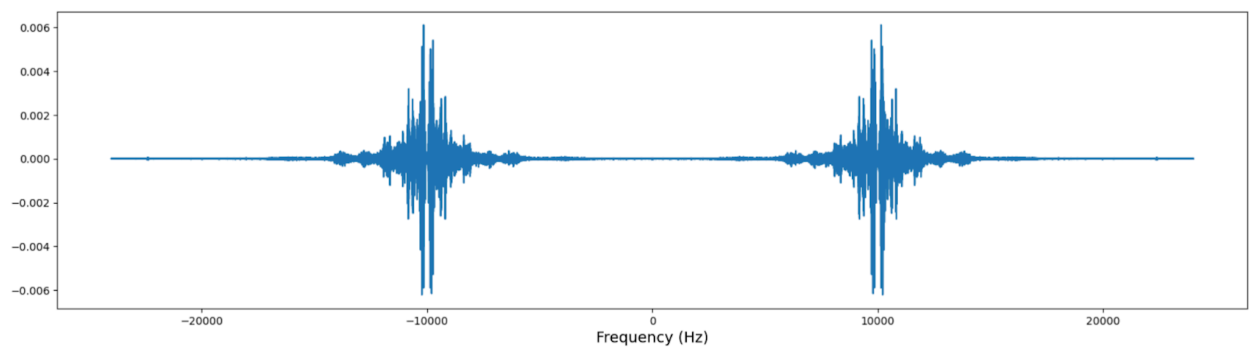
So I've talked a little bit about this course in my last email but I decided to bring it up again as I completed a pretty cool assignment for it. Last time I talked about how an audio signal could be expressed as a series of frequencies. I talked about how every note has a respective frequency, like how the middle C has a frequency of 256 Hz, and a song can be represented by what frequencies are present within it. I also shared a photo like this, to show all the frequencies present in the song Octopus Garden:



Where the x axis = frequencies present; y axis = how much of each frequency is present… bigger spike = more frequency present in song)

Now today I will talk about how I created a mock AM radio!

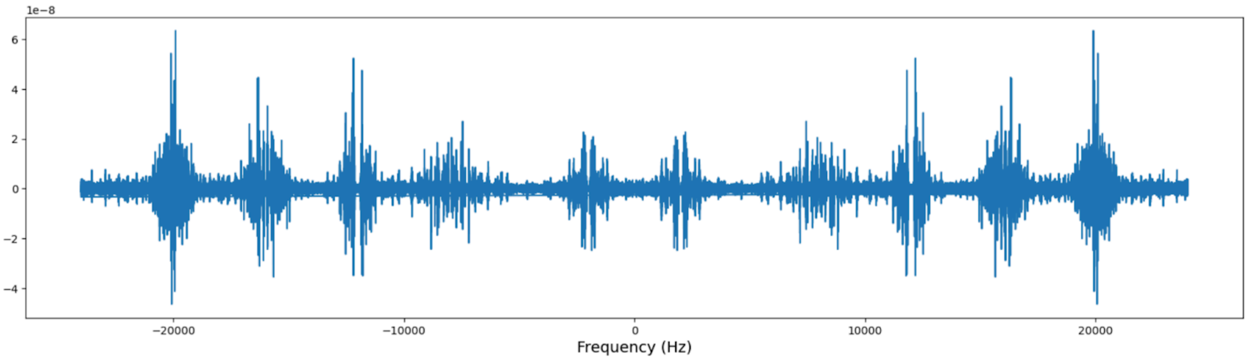
Firstly, I took a bunch of audio signals, and converted them to the frequency domain (like the previous image shown above). I then used something called amplitude modulation, which basically multiplies a signal with a sine wave so that it can move positions in the frequency domain.



For example, I can multiply the frequencies from Octopus Garden by a sine wave with a frequency of 10000 Hz. The outcome will be the image above! The shape of the signal has stayed exactly the same, but now there are two of them at +/- 10000 Hz.

It may be confusing to see negative frequencies in my plots, but this is actually completely normal. I don't quite understand what negative frequencies mean in real life, however, they appear due to the result of sine and cosine waves having complex components to them due to Euler's identity.

To build my AM radio, I basically did this with a bunch of audio signals, however, I modulated them to different frequencies! Which ended up looking like this:



Each audio signal is exactly the same as it originally was, but they are now just placed at different spots in the frequency spectrum. This is called frequency division multiplexing, where you move various separate signals into one frequency spectrum, but ensure they do not overlap so each signal can be extracted unharmed.

So for my AM radio, I can call on the song placed at say, 10000 Hz, demodulate it (multiply it by a sine wave with a frequency -10000), apply a filter to remove everything I don’t want to hear, and get the desired song played back for me!

This is essentially how an AM radio works however there are a bunch of hardware components that can ‘tune in’ to a certain frequency to extract a specific song. While the mock AM radio that I built was solely on Python. However, if you are interested in how radios work feel free to check out this informative (but slightly annoying) video.

Anyway, I will wrap it up here and I will fly out to see you all tomorrow (if my flight doesn’t get canceled).

See you soon,

Lillian