

Algebra Survival Sheet

Basic Properties

$$\begin{aligned} a + b &= b + a, \quad ab = ba \\ (a + b) + c &= a + (b + c), \quad (ab)c = a(bc) \\ a(b + c) &= ab + ac \end{aligned}$$

Special Products

$$\begin{aligned} (a + b)^2 &= a^2 + 2ab + b^2, \quad (a - b)^2 = a^2 - 2ab + b^2 \\ (a + b)(a - b) &= a^2 - b^2 \end{aligned}$$

Exponent Rules

$$\begin{aligned} a^m a^n &= a^{m+n}, \quad \frac{a^m}{a^n} = a^{m-n} \\ (a^m)^n &= a^{mn}, \quad (ab)^n = a^n b^n \\ a^0 &= 1, \quad a^{-n} = \frac{1}{a^n} \end{aligned}$$

Radical Rules

$$\begin{aligned} \sqrt[n]{a^m} &= a^{m/n} \\ \sqrt{ab} &= \sqrt{a}\sqrt{b} \quad (a \geq 0, b \geq 0) \\ \sqrt{\frac{a}{b}} &= \frac{\sqrt{a}}{\sqrt{b}} \quad (a \geq 0, b > 0) \end{aligned}$$

Logarithm Rules

$$\begin{aligned} \log_b(xy) &= \log_b x + \log_b y \\ \log_b\left(\frac{x}{y}\right) &= \log_b x - \log_b y \\ \log_b(x^r) &= r \log_b x \\ \log_b x &= \frac{\ln x}{\ln b} \end{aligned}$$

Absolute Value

$$|a| = \begin{cases} a & a \geq 0 \\ -a & a < 0 \end{cases} \quad |a| = \sqrt{a^2}$$

Rational Expressions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \quad \frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$$

Factoring

$$\begin{aligned}x^2 + bx + c &= (x + m)(x + n) \\a^2 - b^2 &= (a - b)(a + b) \\a^3 \pm b^3 &= (a \pm b)(a^2 \mp ab + b^2)\end{aligned}$$

Zero Product Property

$$ab = 0 \Rightarrow a = 0 \text{ or } b = 0$$

Functions

Composition

$$(f \circ g)(x) = f(g(x))$$

Inverse Functions

$$f(f^{-1}(x)) = x$$

Find inverse: swap x, y and solve

Even / Odd

$$f(-x) = f(x) \text{ (even)}, \quad f(-x) = -f(x) \text{ (odd)}$$

Domain Restrictions

Denominator $\neq 0$

Even root: inside ≥ 0

$\log(x) : x > 0$

Linear Equations

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Forms of a Line

$$y - y_1 = m(x - x_1)$$

$$y = mx + b$$

$$Ax + By = C$$

Systems

Solve via substitution or elimination

Quadratics

Forms

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = a(x - r_1)(x - r_2)$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant

$$b^2 - 4ac$$

Geometry & Coordinates

Distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Inequalities & Intervals

Inequalities

Multiply/divide by negative \Rightarrow flip sign

Domain vs. Range

Domain: All possible *x-values* you are allowed to plug into a function **Range:** All possible *y-values* the function can output

Domain = inputs (horizontal / x-axis), Range = outputs (vertical / y-axis)

How to Find Domain:

- Denominator cannot be zero
- Even roots require inside ≥ 0
- Logarithms require inside > 0

How to Think About Range:

- Look at the graph: what y-values are hit?
- Consider max/min values (especially for quadratics)
- Think about transformations (shifts, stretches)

Example:

$$f(x) = x^2$$

Domain: $(-\infty, \infty)$, Range: $[0, \infty)$

Interval Notation

$$(a, b), [a, b], (-\infty, a), (a, \infty)$$

Rates of Change

$$\frac{f(b) - f(a)}{b - a}$$