Variability and Waiting Formulas

λ average arrival rate (customers per hour)

1/λ average time between arrivals (interarrival time); *a* = 1/λ

μ average service rate (customers per hour)

1/μ average service time per customer; *p* = 1/μ

Utilization = $\frac{\lambda}{\mu}$ = $\frac{p}{a}$ when there is a single server.

Idle time = 1 – utilization

Coefficient of variation of the arrival process:

$$CV_a = \frac{\text{Standard deviation of interarrival time}}{\text{Average interarrival time}}$$

Coefficient of variation of the service process:

$$CV_p = \frac{\text{Standard deviation of service time}}{\text{Average service time}}$$

The coefficient of variation of an exponential distribution is equal to 1.

For a single server, when the utilization is less than 100%, the approximation formula for the steady-state Expected Waiting Time in Queue is

$$W_q = \text{Service time} \times \left(\frac{\text{Utilization}}{1 - \text{Utilization}}\right) \times \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$

M/M/1 Formulas (require that λ < μ):

Utilization = $\frac{\lambda}{\mu}$

In steady state:

Expected waiting time in queue: $W_q = \frac{\lambda}{\mu(u-\lambda)}$

Expected waiting time in the system: $W = W_q + \frac{1}{\mu} = \frac{1}{\mu-\lambda}$

Expected number of customers in queue: $L_q = \frac{\lambda^2}{u(\mu-\lambda)}$

Expected number of customers in the system: $L = \frac{\lambda}{\mu-\lambda}$

Little's Law: $L = \lambda W$

Determining important characteristics of queueing models if we know W_q

Expected number of customers in queue: $L_q = \lambda\, W_q$

Expected waiting time in the system $W = W_q + \frac{1}{\mu}$

Expected number of customers in the system: $L = \lambda\, W$

Multiple Servers

Suppose there are *m* identical servers

Single queue feeding first available server on customer first-come first-serve basis

λ is the arrival rate to the *m*-server system

Utilization = $\frac{\lambda}{m\mu}$

Approximation formula:

$$W_q = \left(\frac{\text{Service time}}{m}\right) \times \left(\frac{\text{Utilization}^{\sqrt{2(m+1)}\,-\,1}}{1-\text{Utilization}}\right) \times \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$