MGRECON - Class 3 (after class)

Summary of Class 2: Revenue Maximization 1/5

Question *A monopolist faces the following demand curve*

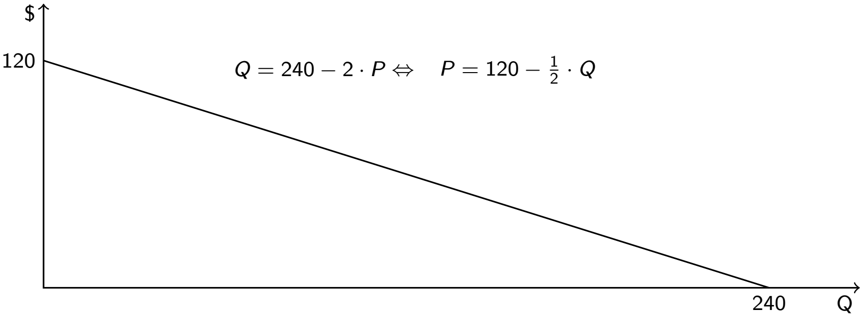
$$Q = 240 - 2P$$

What are the quantity Q_* and the price P_* that maximize the monopolist's revenue? What is the resulting revenue?

Answer - **Step 1**: Get the inverse demand

$$P = 120 - 0.5 \cdot Q$$

Answer - **Step 1b**: Plot the demand



Summary of Class 2: Revenue Maximization 2/5

Question *A monopolist faces the following demand curve*

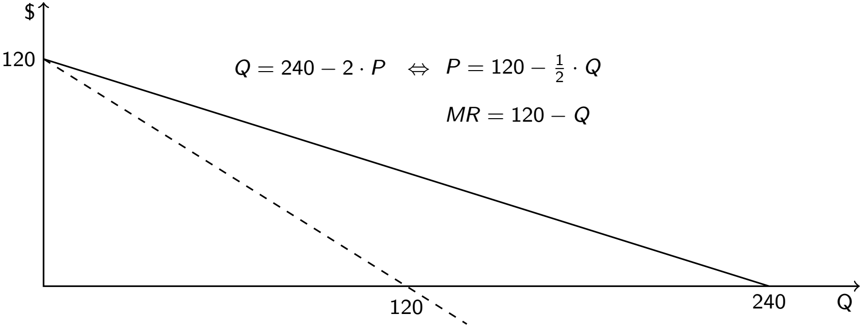
$$Q = 240 - 2P$$

What are the quantity Q_* and the price P_* that maximize the monopolist's revenue? What is the resulting revenue?

Answer - **Step 2**: Double the slope of the inverse demand to get MR

$$MR = 120 - Q$$

Answer - **Step 2b**: Add the MR curve to the graph



Summary of Class 2: Revenue Maximization 3/5

Question *A monopolist faces the following demand curve*

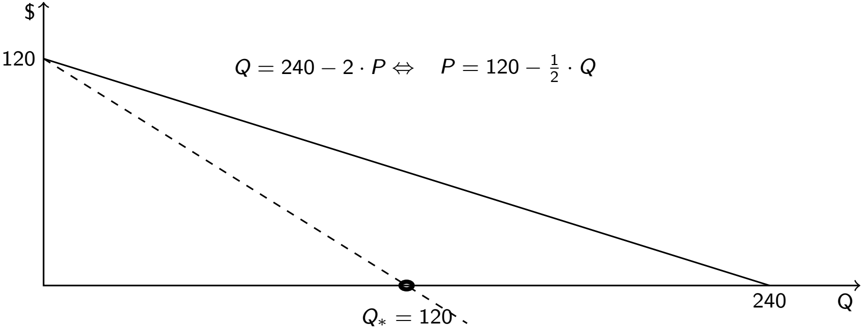
$$Q = 240 - 2P$$

What are the quantity Q_* and the price P_* that maximize the monopolist's revenue? What is the resulting revenue?

Answer - Step 3: Solve the equation MR = 0, to find the optimal quantity Q_*

$$120 - Q = 0 \rightarrow Q_* = 120$$

Answer - Step 3b: Mark Q_* in the graph



Summary of Class 2: Revenue Maximization 4/5

Question *A monopolist faces the following demand curve*

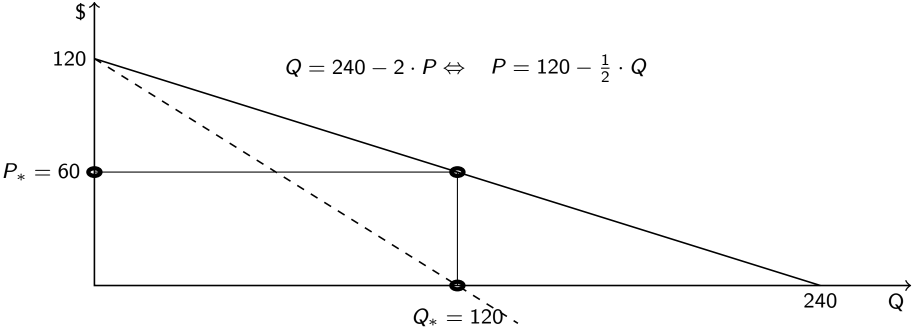
$$Q = 240 - 2P$$

What are the quantity Q_* and the price P_* that maximize the monopolist's revenue? What is the resulting revenue?

Answer - Step 4: Plug Q_* into the inverse demand to find the optimal price P_*

$$P_* = 120 - \frac{1}{2}Q_* = 60$$

Answer - Step 4b: Mark P_* in the graph



Summary of Class 2: Revenue Maximization 5/5

Question *A monopolist faces the following demand curve*

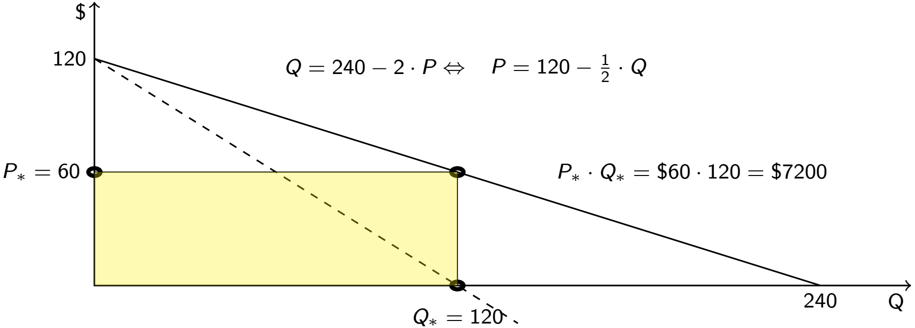
$$Q = 240 - 2P$$

What are the quantity Q_* and the price P_* that maximize the monopolist's revenue? What is the resulting revenue?

Answer - Step 5: Multiply P_* by Q_* to find the resulting revenue

$$P_* \cdot Q_* = 60 \cdot 120 = 7200$$

Answer - Step 5b: Highlight the revenue rectangle in the graph



Summary of Class 2: Profit Maximization 1/5

Question *A monopolist faces the following demand curve*

$$Q = 240 - 2P$$

and its total and marginal cost functions are given by

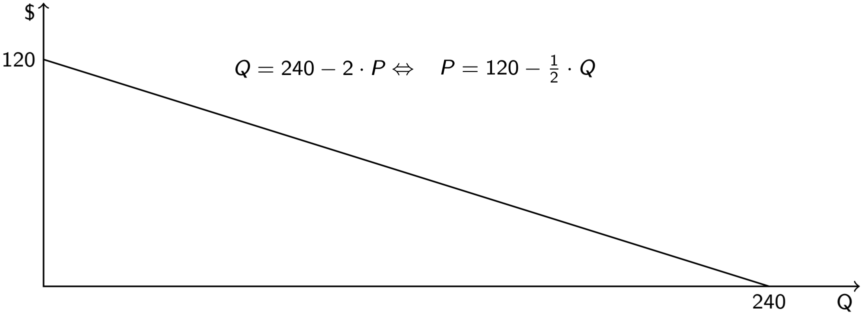
$$TC(Q) = 500 + 10 \cdot Q \quad \rightarrow \quad MC(Q) = 10$$

What are the quantity Q_M and the price P_M that maximize the monopolist's profit?
What is the resulting profit ?

Answer - **Step 1**: Get the inverse demand

$$P = 120 - 0.5 \cdot Q$$

Answer - **Step 1b**: Plot the demand



Summary of Class 2: Profit Maximization 2/5

Question *A monopolist faces the following demand curve*

$$Q = 240 - 2P$$

and its total and marginal cost functions are given by

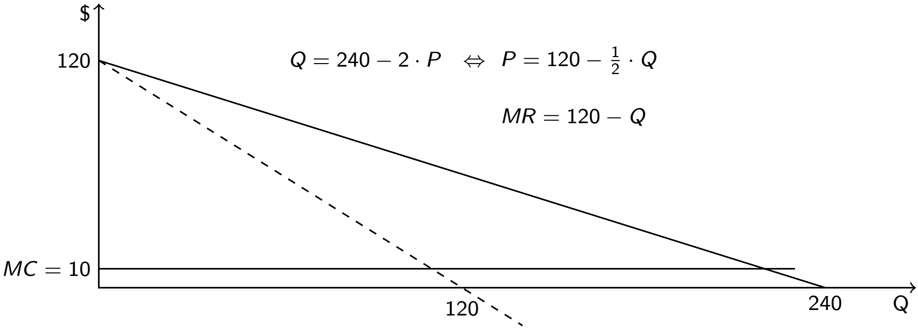
$$TC(Q) = 500 + 10 \cdot Q \quad \rightarrow \quad MC(Q) = 10$$

What are the quantity Q_M and the price P_M that maximize the monopolist's profit?
What is the resulting profit ?

Answer - **Step 2**: Double the slope of the inverse demand to get MR

$$MR = 120 - Q$$

Answer - **Step 2b**: Add the MR curve and the MC curve to the graph



Summary of Class 2: Profit Maximization 3/5

Question *A monopolist faces the following demand curve*

$$Q = 240 - 2P$$

and its total and marginal cost functions are given by

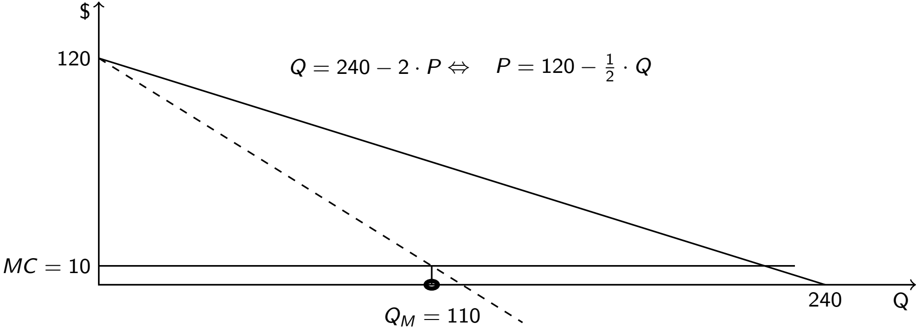
$$TC(Q) = 500 + 10 \cdot Q \quad \rightarrow \quad MC(Q) = 10$$

What are the quantity Q_M and the price P_M that maximize the monopolist's profit? What is the resulting profit ?

Answer - **Step 3**: Solve the equation MR = MC, to find the optimal quantity Q_M

$$120 - Q = 10 \rightarrow Q_M = 110$$

Answer - **Step 3b**: Mark Q_M in the graph



Summary of Class 2: Profit Maximization 4/5

Question *A monopolist faces the following demand curve*

$$Q = 240 - 2P$$

and its total and marginal cost functions are given by

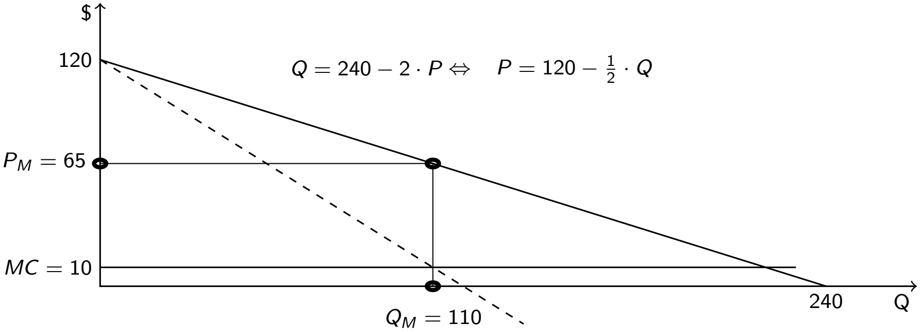
$$TC(Q) = 500 + 10 \cdot Q \quad \rightarrow \quad MC(Q) = 10$$

What are the quantity Q_M and the price P_M that maximize the monopolist's profit?
What is the resulting profit ?

Answer - **Step 4**: Plug Q_M into the inverse demand to find the optimal price P_M

$$P_M = 120 - \tfrac{1}{2} Q_M = 65$$

Answer - **Step 4b**: Mark P_M in the graph



Summary of Class 2: Profit Maximization 5/5

Question *A monopolist faces the following demand curve*

$$Q = 240 - 2P$$

and its total and marginal cost functions are given by

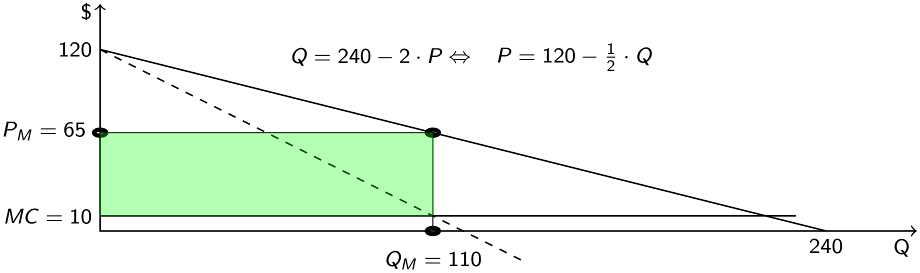
$$TC(Q) = 500 + 10 \cdot Q \quad \rightarrow \quad MC(Q) = 10$$

What are the quantity Q_M and the price P_M that maximize the monopolist's profit?
What is the resulting profit ?

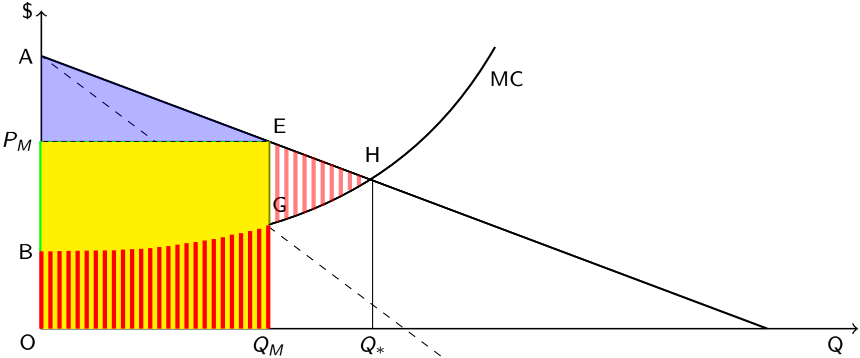
Answer - Step 5: Compute the resulting profit

$$P_M \cdot Q_M - TC(Q_M) = \overbrace{\$65 \cdot 110}^{\text{Revenue}} - \overbrace{\$10 \cdot 110}^{TVC} - \overbrace{\$500}^{FC} = \overbrace{\$6050}^{\text{green area}} - \overbrace{\$500}^{FC} = \$5550$$

Answer - Step 5b: Highlight the profit area in the graph

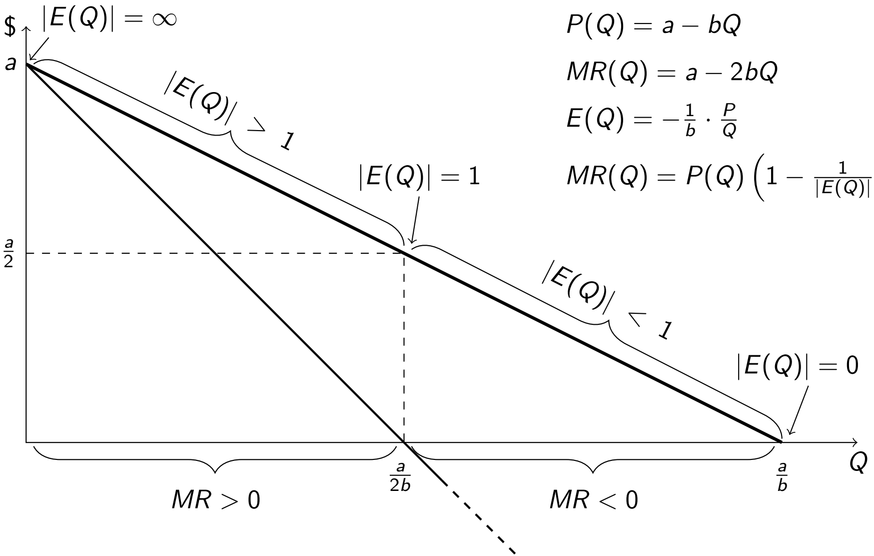


Classic Monopoly Picture



Consumer Surplus	$=$	AP_ME
Profit	$=$	BP_MEG
Deadweight Loss	$=$	GEH
Revenue	$=$	OP_MEQ_M
Total Variable Cost	$=$	$OBGQ_M$

Marginal Revenue and Elasticity when Demand is Linear



$$P(Q) = a - bQ$$

$$MR(Q) = a - 2bQ$$

$$E(Q) = -\frac{1}{b} \cdot \frac{P}{Q}$$

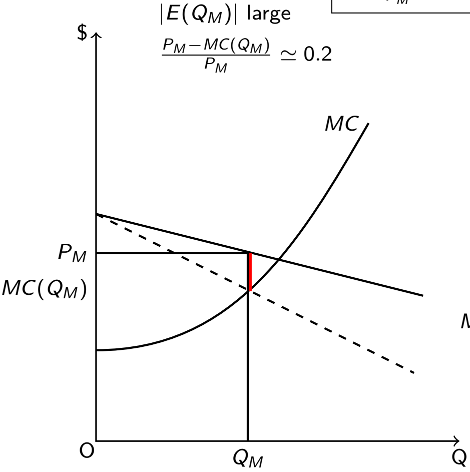
$$MR(Q) = P(Q)\left(1 - \frac{1}{|E(Q)|}\right)$$

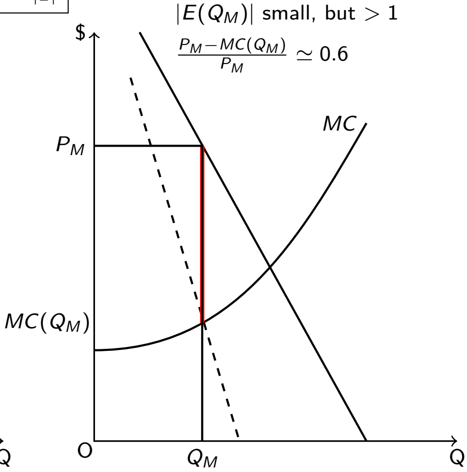
The Lerner Index

$$\frac{P_M - MC(Q_M)}{P_M} = \frac{1}{|E|}$$

The Lerner Index in Pictures

$$\boxed{\frac{P_M - MC(Q_M)}{P_M} = \frac{1}{|E|}}$$





Two More Things to Remember from Class 2

1. Suppose that:
   - we see the price charged by a seller, and
   - we have an estimate of the elasticity (at that point).

   If we assume that the seller is *maximizing profit*, can we infer MC (at that point)?

   Yes!

   $$\frac{P - MC}{P} = \frac{1}{|E|} \quad \rightarrow \text{solve for MC}$$

2. Any point on the demand where $|E| < 1$ cannot be profit maximizing. We can say this, without knowing anything about costs (beyond $MC \geq 0$).

Profit Maximization with a Flat Demand

Question A seller with costs

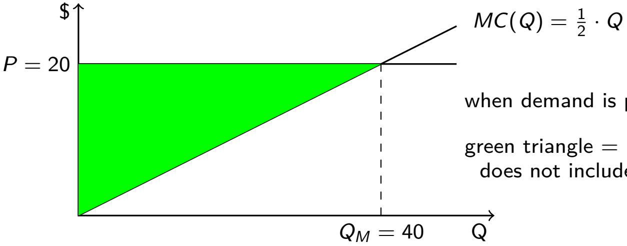
$$TC(Q) = 100 + \frac{1}{4} \cdot Q^2 \quad \rightarrow \quad MC(Q) = \frac{1}{2} \cdot Q,$$

faces a perfectly elastic (horizontal) demand $P = 20$.

What is the quantity that maximizes its profit?

Answer

$$\overbrace{20}^{\text{MR}} \;=\; \overbrace{\frac{1}{2} \cdot Q}^{\text{MC}} \;\rightarrow\; Q = 40$$



when demand is perfectly elastic, MR = P

green triangle $= P_M \cdot Q_M - TVC(Q)$
does not include the \$100 Fixed Cost

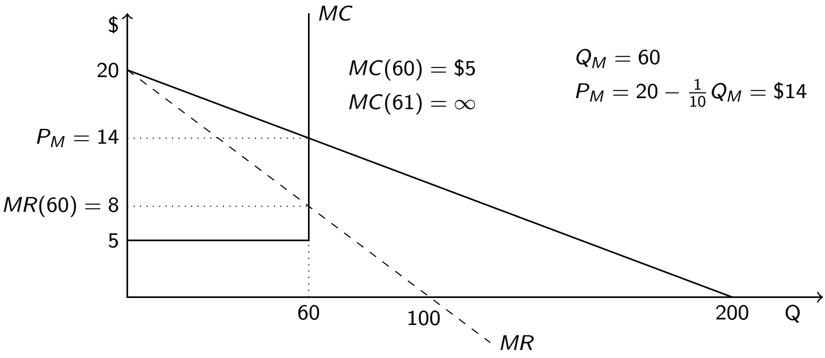
Profit Maximization with a binding Capacity Constraint

Question A seller can produce ***up to 60 units*** at constant marginal cost, MC $= 5$.

Its (inverse) demand is given by $P = 20 - \frac{1}{10}Q$.

What are the quantity Q_M and the price P_M that maximize its profit?

Answer The capacity constraint is *binding*, meaning $MR(60) = 20 - \frac{2}{10} \cdot 60 = 8 > MC$. Therefore the optimal quantity is $Q_M = 60$.



Profit Max'n with a non-binding Capacity Constraint

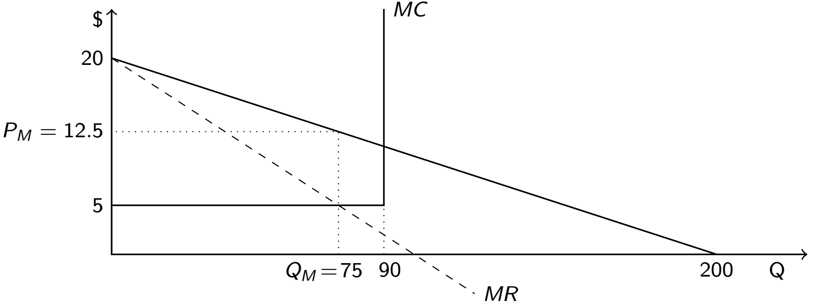
Question A seller can produce ***up to 90 units*** at constant marginal cost, MC $= 5$.

Its (inverse) demand is given by $P = 20 - \frac{1}{10}Q$.

What are the quantity Q_M and the price P_M that maximize its profit?

Answer The capacity constraint is *not binding*, meaning $MR(90) = 20 - \frac{2}{10} \cdot 90 = 2 < MC$. Therefore the optimal quantity is given by

$$\overbrace{20 - \tfrac{2}{10} \cdot Q}^{\text{MR}(Q)} = \overbrace{5}^{MC} \quad \rightarrow Q_M = 75 \quad \rightarrow P_M = 20 - \frac{1}{10}75 = \$12.5$$



Price Discrimination Classification

Price Discrimination = selling identical goods at different prices

There are 3 kinds of price discrimination:

1. **Perfect Price Discrimination**
   prices are tailored to each customer;
   this is a theoretical benchmark — it *requires perfect knowledge of the buyers' WTP*

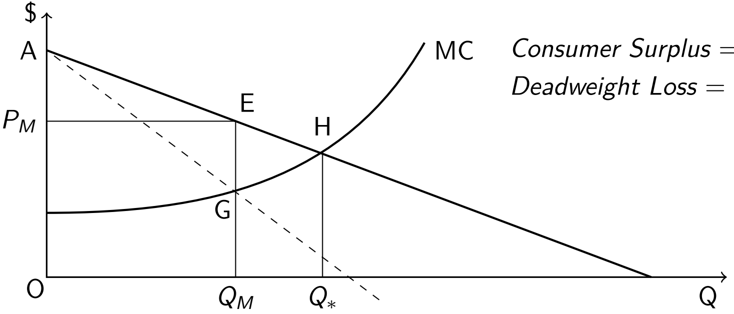
2. **Multi-market Price Discrimination**
   prices are based on *observable characteristics* of the buyers;
   e.g. age, membership, geographical location, ...

3. **Menu Pricing**
   prices are based on the *amount purchased*;
   e.g. quantity discounts

Why Price-Discriminate?

Selling all units at the same unit price "leaves money on the table."

A monopolist that cannot price discriminate,
sets its price at P_M and sells Q_M units.



Consumer Surplus = area AP_ME

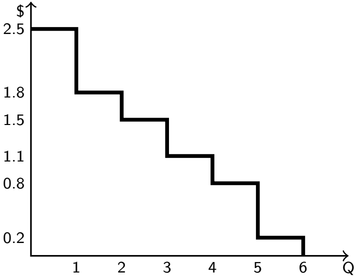
Deadweight Loss = area GEH

Price discrimination aims at converting (some of) the CS and DWL into profit

Perfect Price Discrimination

A rental car (R) company buys maintenance services from a local mechanic (M).

Inspections per Month	Marginal WTP
1	\$2500
2	\$1800
3	\$1500
4	\$1100
5	\$800
6	\$200
7	\$0



If M knows R's demand, then it can charge:

\$2499 for the 1^{st} inspection
\$1799 for the 2^{nd} inspection
\$1499 for the 3^{rd} inspection
...

thus converting the entire WTP into revenue

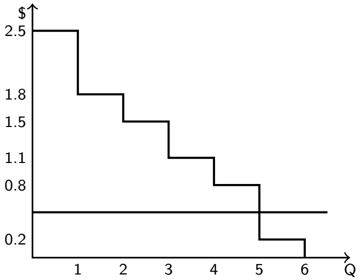
Note: in this case

$\boxed{MR = MWTP}$

Perfect Price Discrimination

If $MC = \$500$, then

Q	MWTP
1	\$2500
2	\$1800
3	\$1500
4	\$1100
5	\$800
6	\$200
7	\$0



it is optimal for M to sell $Q_* = 5$ inspections per month, because

$$MWTP(1) > ... > MWTP(5) = \$800 > MC = \$500 > MWTP(6) = \$200$$

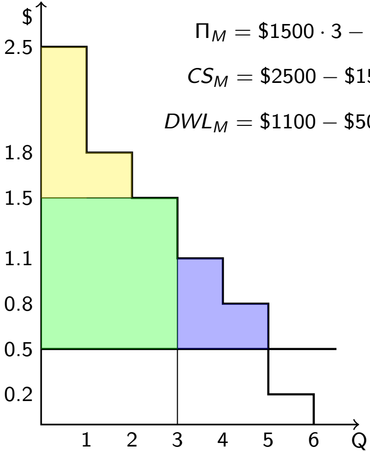
$$\Pi_* = \overbrace{\$2500 + \$1800 + \$1500 + \$1100 + \$800}^{\text{Total Revenue}} - \overbrace{5 \cdot \$500}^{\text{Total Cost}} = \$5200$$

$$\boxed{\text{Note: } CS_* = 0, \quad DWL_* = 0, \quad \Pi_* = \text{all gains from trade}}$$

Perfect Price Discrimination vs. Linear Pricing

With perfect price discrimination: $\Pi_* = \$5200$

With linear pricing $Q_M = 3, \qquad P_M = \$1500$



$$\Pi_M = \$1500 \cdot 3 - \$500 \cdot 3 = \$3000$$

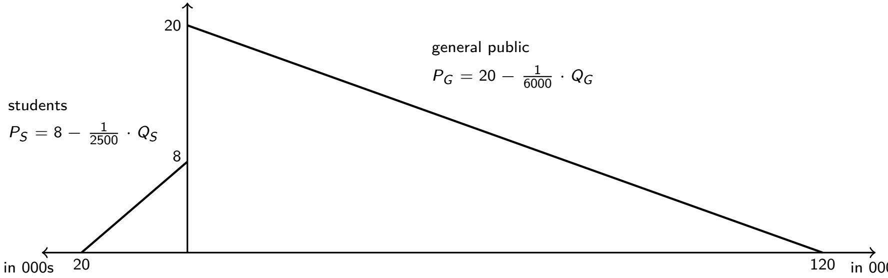
$$CS_M = \$2500 - \$1500 + \$1800 - \$1500 = \$1300$$

$$DWL_M = \$1100 - \$500 + \$800 - \$500 = \$900$$

$$\begin{aligned} \Pi_M \ + \ \mathrm{CS}_M \ + \mathrm{DWL}_M &= \Pi_* \\ \$3000 + \$1300 + \$900 \quad &= \$5200 \end{aligned}$$

Multi-market Price Discrimination: Problem 1

The demand for a college football game consists of two segments



The stadium has a maximum capacity of 75,000 seats.

All costs are zero.

Resale of tickets is impossible.

The seller can choose:

- a price P_S for students
- a price P_G for members of the general public

At what levels should P_S and P_G be set, in order to maximize profit (i.e. revenue)?

Multi-market Price Discrimination: Problem 1 - Answer

Maximize revenue **independently** in each market:

$P_S = 8 - \frac{1}{2500} \cdot Q_S$

$MR_S(Q_S) = 8 - \frac{1}{1250} \cdot Q_s = 0$

$\rightarrow Q_s^* = 10{,}000$

$\rightarrow P_s^* = 8 - \frac{1}{2500} \cdot 10{,}000 = \$4$

$P_S^* \cdot Q_S^* = \$4 \cdot 10,000 = \$40{,}000$

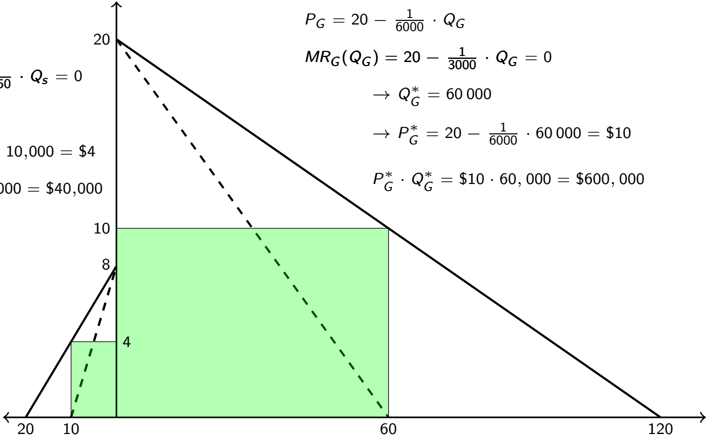
$P_G = 20 - \frac{1}{6000} \cdot Q_G$

$MR_G(Q_G) = 20 - \frac{1}{3000} \cdot Q_G = 0$

$\rightarrow Q_G^* = 60\,000$

$\rightarrow P_G^* = 20 - \frac{1}{6000} \cdot 60\,000 = \$10$

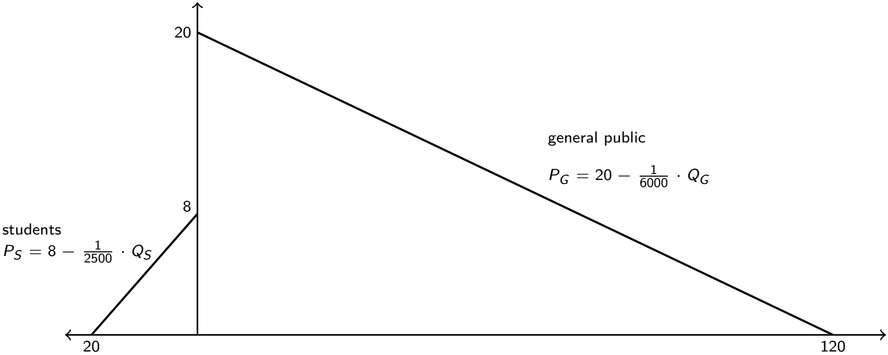
$P_G^* \cdot Q_G^* = \$10 \cdot 60,000 = \$600,000$



Since the stadium has 75 000 seats, this is feasible.

Multi-market Price Discrimination: Problem 2

The demand for a NCAA football game consists of two segments



The arena has a maximum capacity of **54 000** seats.

All costs are zero.

Resale of tickets is impossible, so we can set

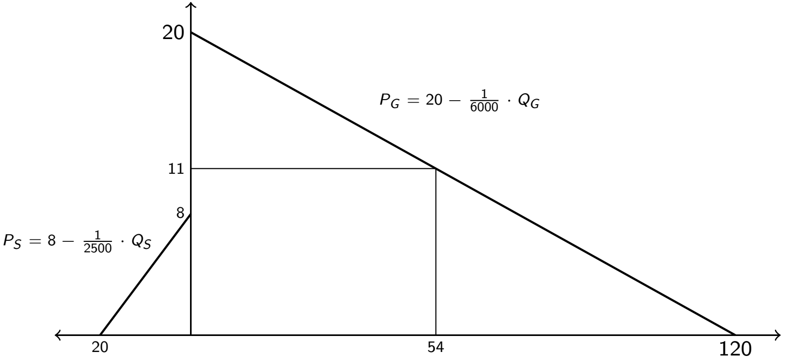
- a price P_S for students
- a price P_G for members of the general public

At what levels should we set P_S and P_G in order to maximize profit (i.e. revenue)?

AJ's proposal

Our friend AJ proposes:

> *"We should sell all 54,000 seats to the general public for \$11 each.*
>
> *Indeed,* $P_G\,(54{,}000) = 20 - \frac{1}{6000} \cdot 54{,}000 = \$11$*.*
>
> *Why bother with the students? No student pays more than \$8!"*



Is AJ correct?

Is AJ correct?

What happens if we move a few tickets, say 3,000 tickets, from non-students to the students? We would sell:

- 51,000 tickets to the general public,
  at $P_G(51000) = 20 - \frac{1}{6000} \cdot 51000 = \$11.50$ each
- 3,000 tickets to students
  at $P_S(3000) = 8 - \frac{1}{2500} \cdot 3000 = \$6.8$ each

Our total revenue would be

$$\begin{array}{lcll} \text{from non-students} & & \text{from students} & \\ \$11.50 \cdot 51{,}000 & + & \$6.8 \cdot 3,000 & \\ \$586{,}500 & + & \$20{,}400 & = \$606{,}900 \end{array}$$

The revenue generated by AJ's strategy is \$594,000.

AJ is wrong! What is he missing?

AJ's strategy:

$\Pi = \$11 \cdot 54\,000$
$\quad = \$594\,000$

A better strategy:

$\Pi = \$11.5 \cdot 51\,000 + \$6.8 \cdot 3\,000$
$\quad = \$606\,900$

AJ forgets that

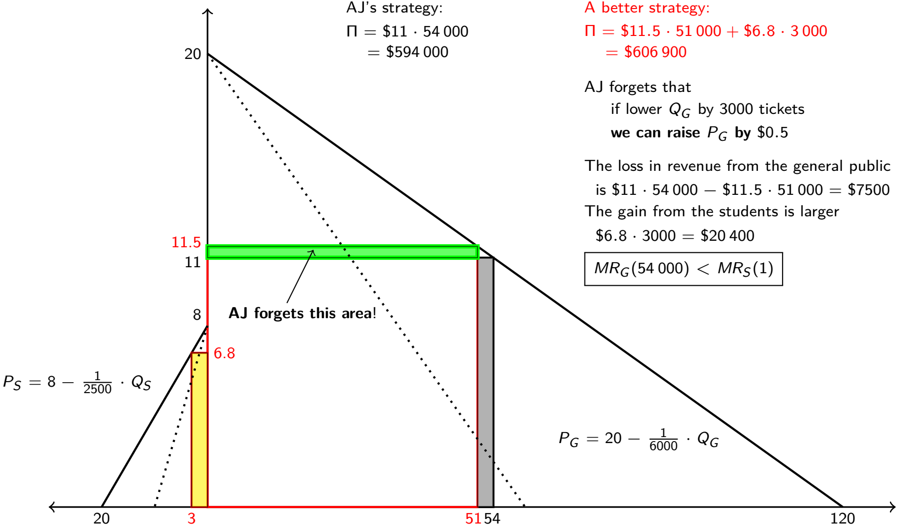
if lower Q_G by 3000 tickets

we can raise P_G by \$0.5

The loss in revenue from the general public is $\$11 \cdot 54\,000 - \$11.5 \cdot 51\,000 = \$7500$

The gain from the students is larger $\$6.8 \cdot 3000 = \$20\,400$

$\boxed{MR_G(54\,000) < MR_S(1)}$



The optimal way to allocate tickets across the two segments **must satisfy**

$$\boxed{MR_S(Q_S) = MR_G(Q_G)}$$

If $MR_S(Q_S) > MR_G(Q_G)$, we can increase total revenue by moving a few tickets from the G group to the S group, i.e. by raising P_G and lowering P_S slightly

If $MR_S(Q_S) < MR_G(Q_G)$, we can increase total revenue by moving a few tickets from the S group to the G group, i.e. by raising P_S and lowering P_G slightly

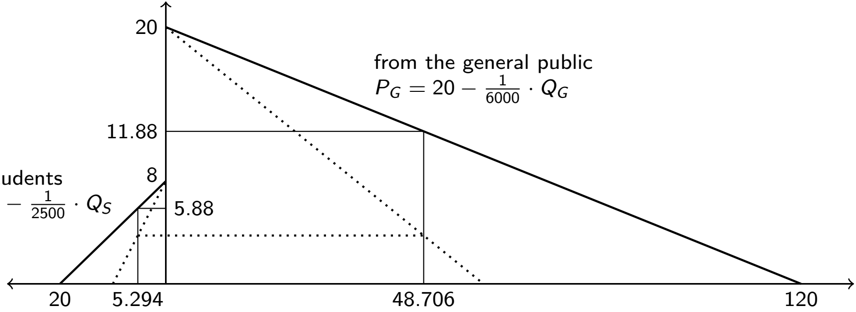
To find the profit (revenue) maximizing quantities we solve the two equations:

$$\begin{bmatrix} 8 - \frac{1}{1250} Q_S & = 20 - \frac{1}{3000} Q_G \\ \\ Q_S + Q_G & = 54{,}000 \end{bmatrix} \rightarrow \begin{bmatrix} Q_S^* = 5{,}294 \\ \\ Q_G^* = 48{,}706 \end{bmatrix}$$

Once we have the optimal quantities, we find the optimal prices:

$$\begin{bmatrix} P_S = 8 - \frac{1}{2500} \cdot 5294 = \$5.88 \\ \\ P_G = 20 - \frac{1}{6000} \cdot 48706 = \$11.88 \end{bmatrix}$$

Total revenue $= \$5.88 \cdot 5294 + \$11.88 \cdot 48706 = \$609,756$



Multi-market Price Discrimination and Elasticities

Recall that MR and elasticity are related

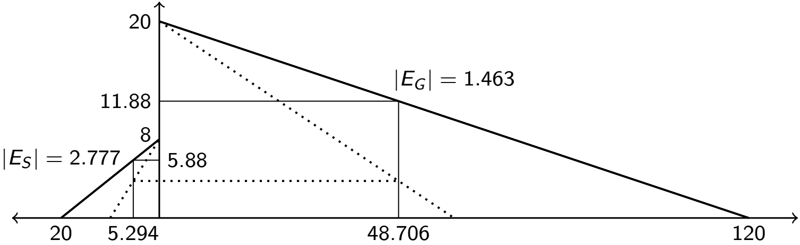
$$MR_1 = P_1\left(1 - \frac{1}{|E_1|}\right) \quad \text{and} \quad MR_2 = P_2\left(1 - \frac{1}{|E_2|}\right)$$

Since profit maximization requires $MR_1 = MR_2$, we have

$$P_1\left(1 - \frac{1}{|E_1|}\right) = P_2\left(1 - \frac{1}{|E_2|}\right)$$

which implies

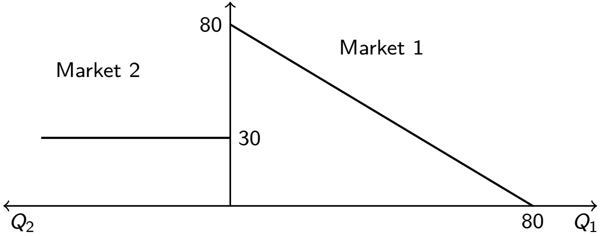
$$|E_1| < |E_2| \quad \Leftrightarrow \quad P_1 > P_2$$



Multi-market Price Discrimination and Dumping

A firm is a monopolist in its domestic market (market 1), where demand is $P_1 = 80 - Q_1$

The firms also sells in a foreign market (market 2), where it faces a flat demand curve $P_2 = 30$



The firm's cost structure is

$$TC\left(Q_1 + Q_2\right) = \overbrace{1200}^{\text{fixed cost}} + \overbrace{\tfrac{1}{4}\left(Q_1 + Q_2\right)^2}^{\text{variable cost}}$$

and thus

$$MC(Q_1 + Q_2) = \tfrac{1}{2}(Q_1 + Q_2)$$

What quantity Q_1, price P_1 and quantity Q_2 should this firm choose,
in order to maximize its profit? ($P_2 = \$30$ is given and cannot be changed)

The profit maximizing quantities Q_1^* and Q_2^* must satisfy

$$\boxed{MR_1(Q_1) = MR_2(Q_2) = MC(Q_1 + Q_2)}$$

Equality among MRs $\rightarrow$ how to allocate a given total output *across markets*:
if $MR_1(Q_1) > MR_2(Q_2)$, lowering Q_2 and raising Q_1 by the same small amount increases total revenue without changing the total output $Q_1 + Q_2$ and the total cost;
if $MR_1(Q_1) < MR_2(Q_2)$, raising Q_2 and lowering Q_1 by the same small amount increases total revenue without changing the total output $Q_1 + Q_2$ and the total cost;

Equality between MR and MC $\rightarrow$ how to choose the *total output* $Q_1 + Q_2$:
if $MC < MR$, increasing total output increases total revenue more than it increases total cost;
if $MC > MR$, reducing total output decreases total revenue less than it decreases total cost.

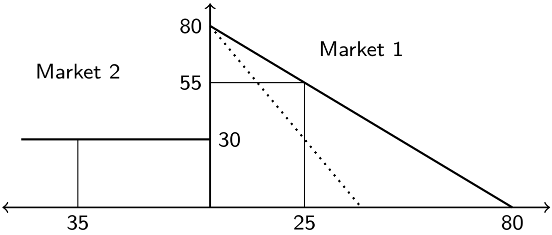
Solving the two equations simultaneously we have

$$\left[\begin{array}{c} MR_1 = MR_2 \\ \\ MC = MR_2 \end{array}\right] \Leftrightarrow \left[\begin{array}{c} 80 - 2Q_1 = 30 \\ \\ \frac{1}{2}(Q_1 + Q_2) = 30 \end{array}\right] \rightarrow \left[\begin{array}{l} Q_1^* = 25 \rightarrow P_1^* = 80 - 25 = 55, \\ \\ Q_2^* = 35 \end{array}\right]$$

The firm's profit is

$$\begin{aligned} \Pi_* &= P_1^* \cdot Q_1^* + P_2 \cdot Q_2^* - TC\left(Q_1^* + Q_2^*\right) \\ &= \$55 \cdot 25 + \$30 \cdot 35 - \$\left(1200 + \tfrac{1}{4}\,60^2\right) \\ &= \$1375 + \$1050 - \$2100 = \$325 \end{aligned}$$

Multi-market Price Discrimination and Dumping



What is the average cost at the profit maximizing level of output?

$$ATC(60) = \frac{TC(60)}{60} = \frac{1200 + \frac{1}{4}\,60^2}{60} = 35$$

Note: $ATC(60) = 35 > P_2 = 30!$

Is this dumping?

If the firm cannot sell in market 2,

$$P_1 = 80 - Q_1$$

$$TC(Q_1 + Q_2) = \overbrace{1200}^{\text{fixed cost}} + \overbrace{\tfrac{1}{4}Q_1^2}^{\text{variable cost}}$$

and thus

$$MC(Q_1) = \tfrac{1}{2}Q_1$$

$$\overbrace{80 - 2Q_1}^{MR_1} = \overbrace{\tfrac{1}{2}Q_1}^{MC} \rightarrow Q_1^0 = 32 \rightarrow P_1^0 = 80 - 32 = \$48$$

$$\Pi_1^0 = 48 \cdot 32 - 1200 - \tfrac{1}{4}(32)^2 = \boxed{\$80}$$

$$AC(32) = \frac{1200 + \frac{1}{4}(32)^2}{32} = \$45.5 < P_1^0 \quad \text{(not dumping now)}$$

If the firm can sell in both markets

$$\begin{aligned}
\Pi_* &= P_1^* \cdot Q_1^* + P_2 \cdot Q_2^* - TC(Q_1^* + Q_2^*) \\
&= \$55 \cdot 25 + \$30 \cdot 35 - \$\left(1200 + \tfrac{1}{4}60^2\right) \\
&= \$1375 + \$1050 - \$2100 = \boxed{\$325}
\end{aligned}$$