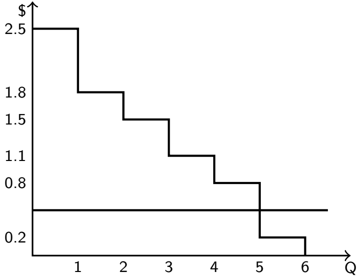
MGRECON - Class 4 (after class)

Summary of Class 3: Perfect Price Discrimination

A seller with ***perfect knowledge*** of a buyer's WTP for any unit, can "extract" all gains from trade

Q	MWTP
1	\$2500
2	\$1800
3	\$1500
4	\$1100
5	\$800
6	\$200
7	\$0



$$\Pi_* = \overbrace{\$2500 + \$1800 + \$1500 + \$1100 + \$800}^{\text{Total Revenue} = \$7700} - \overbrace{5 \cdot \$500}^{\text{Total Cost} = \$2500} = \$5200$$

$$\boxed{CS_* = 0, \quad DWL_* = 0, \quad \Pi_* = \text{all gains from trade}}$$

Summary of Class 3: Multi-Market Price Discrimination

Question *A firm selling in two (separate) markets, 1 and 2, can produce up to K units, at total cost $TC(Q_1 + Q_2)$. Given the inverse demand equation in each market, find the profit maximizing quantities (Q_1^* and Q_2^*), and prices (P_1^* and P_2^*), and compute the resulting profit.*

Answer

Step 1: Get the MR_1 and MR_2, by doubling the slope of each inverse demand;

Step 2: Ignore the capacity K and solve the system of equations

$$\text{MR}_1(Q_1) = \text{MR}_2(Q_2) = \text{MC}(Q_1 + Q_2) \quad \rightarrow \quad (Q_1^*, Q_2^*)$$

if $Q_1^* + Q_2^* \leq K$, you are done; otherwise go to Step 3.

Step 3: Solve the system of equations

$$\left[\begin{array}{c} MR_1(Q_1) = MR_2(Q_2) \\ \\ Q_1 + Q_2 = K \end{array} \right] \rightarrow (Q_1^{**}, Q_2^{**})$$

Plug the optimal quantities into their respective inverse demands to get the optimal prices, and then compute $\Pi_* = P_1^* \cdot Q_1^* + P_2^* \cdot Q_2^* - TC(Q_1^* + Q_2^*)$

Summary of Class 3: Multi-Market Price Discrimination

A **profit-maximizing** firm that sells in two (separate) markets:

1. charges *less* in the market where demand is *more elastic*
2. may sell *below its average cost*, if it has *high fixed costs*

Menu Pricing

The Problem
The seller:

has several "types" of customers (with different WTP),
cannot observe directly any customer's type.

The Pricing Strategy
Design a "menu" e.g.

# of units	price
1	\$30
2	\$45
5	\$72
...	...

*There is **no** point in offering **more** options **than the number of** customers' **types***

*Each customer will select the option that **maximizes its consumer surplus***

What is the profit-maximizing menu?

Acqualand

The amusement park "AcquaLand" has 3 types of customers:

Total WTP

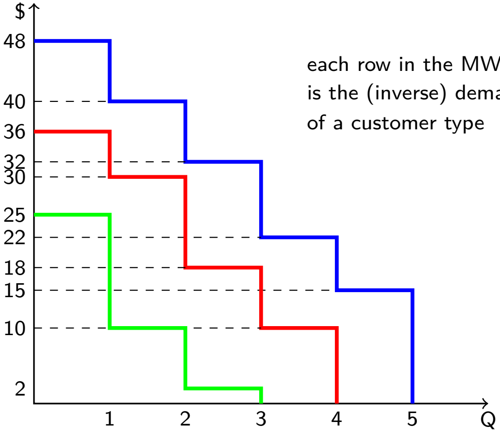
Units	1	2	3	4	5	Group Size
type A	\$48	\$88	\$120	\$142	\$157	1000
type B	\$36	\$66	\$84	\$94	\$94	1000
type C	\$25	\$35	\$37	\$37	\$37	1000

Marginal WTP

Units	1	2	3	4	5	Group Size
type A	\$48	\$40	\$32	\$22	\$15	1000
type B	\$36	\$30	\$18	\$10	\$0	1000
type C	\$25	\$10	\$2	\$0	\$0	1000

Assume zero costs.

Units	*Marginal* WTP 1	2	3	4	5	Group Size
type A	$48	$40	$32	$22	$15	1000
type B	$36	$30	$18	$10	$0	1000
type C	$25	$10	$2	$0	$0	1000



each row in the MWTP table is the (inverse) demand of a customer type

Acqualand: Perfect Price Discrimination

If all customers' type were ***observable***,

Total WTP

Units	1	2	3	4	5	Group Size
type A	\$48	\$88	\$120	\$142	\$157	1000
type B	\$36	\$66	\$84	\$94	\$94	1000
type C	\$25	\$35	\$37	\$37	\$37	1000

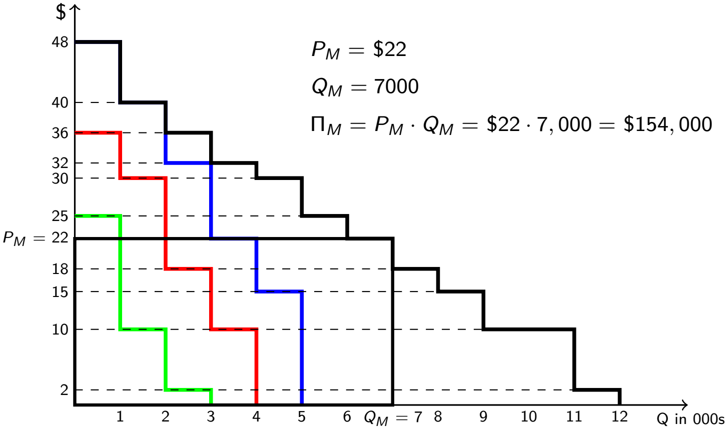
AcquaLand would offer:

- a bundle of 5 units for \$157 to the A types $(MWTP_A(5) > MC = 0)$
- a bundle of 4 units for \$94 to the B types $(MWTP_B(4) > MC = 0)$
- a bundle of 3 units for \$37 to the C types $(MWTP_C(3) > MC = 0)$

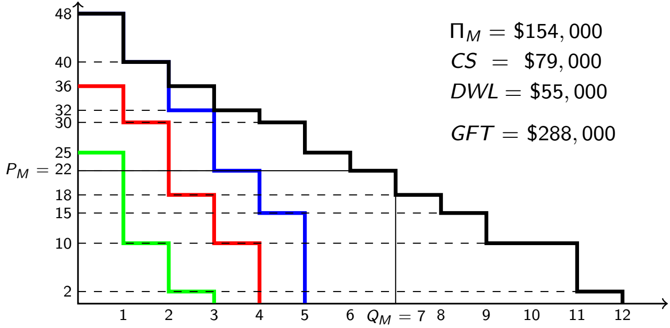
$$\boxed{\begin{aligned} \Pi &= \$157 \cdot 1,000 + \$94 \cdot 1,000 + \$37 \cdot 1,000 = \$288,000 \\ CS &= \$0 \\ DWL &= \$0 \\ GFT &= \$288,000 \end{aligned}}$$

Acqualand: Linear Pricing

If Acqualand ***could not price-discriminate,***
it would choose the profit-maximizing point on the *market* demand



Acqualand: Linear Pricing



$\Pi_M = \$154,000$

$CS = \$79,000$

$DWL = \$55,000$

$GFT = \$288,000$

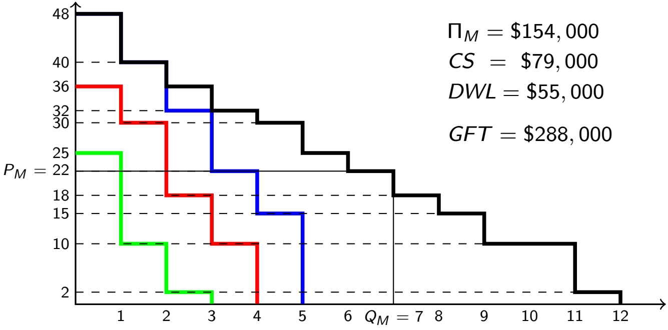
each A customer buys ____ units $\Rightarrow$ $CS_A =$ ________ $DWL_A =$ ________

each B customer buys ____ units $\Rightarrow$ $CS_B =$ ________ $DWL_B =$ ________

each C customer buys ____ units $\Rightarrow$ $CS_C =$ ________ $DWL_C =$ ________

CS = ________ DWL = ________

Acqualand: Linear Pricing



$\Pi_M = \$154,000$
$CS = \$79,000$
$DWL = \$55,000$

$GFT = \$288,000$

each A customer buys 4 units $\Rightarrow$ $CS_A = \overset{(48\text{-}22)}{26} + \overset{(40\text{-}22)}{18} + \overset{(32\text{-}22)}{10} + \overset{(22\text{-}22)}{0} = 54$ $\quad DWL_A = 15$

each B customer buys 2 units $\Rightarrow$ $CS_B = 14 + 8 = 22$ $\quad DWL_B = 18 + 10 = 28$

each C customer buys 1 unit $\Rightarrow$ $CS_C = 3$ $\quad DWL_C = 10 + 2 = 12$

CS $= 79$ $\quad$ DWL $= 55$

Acqualand: the Menu-Pricing Algorithm

The *menu* of 3 choices (we have 3 customer types) that maximizes total profit can be designed in two steps:

Step 1: Find the best price for each unit (based on the MWTP table)

Step 2: Bundle the units that each customer type buys

Marginal WTP

Units	1	2	3	4	5	Group Size
type A	\$48	\$40	\$32	\$22	\$15	1000
type B	\$36	\$30	\$18	\$10	\$0	1000
type C	\$25	\$10	\$2	\$0	\$0	1000
Unit Price						
Unit Profit						

Acqualand: Pricing Algorithm - Step 1

Find the best price for each unit

Marginal WTP

Units	1	2	3	4	5	Group Size
type A	\$48	\$40	\$32	\$22	\$15	1000
type B	\$36	\$30	\$18	\$10	\$0	1000
type C	\$25	\$10	\$2	\$0	\$0	1000
Unit Price	\$25	\$30	\$18	\$22	\$15	
Unit Profit	\$75,000	\$60,000	\$36,000	\$22,000	\$15,000	\$208,000

$48 \cdot 1 = 48$	$40 \cdot 1 = 40$	$32 \cdot 1 = 32$	$22 \cdot 1 = 22$	$15 \cdot 1 = 15$
$36 \cdot 2 = 72$	$30 \cdot 2 = 60$	$18 \cdot 2 = 36$	$10 \cdot 2 = 20$	$0 \cdot 2 = 0$
$25 \cdot 3 = 75$	$10 \cdot 3 = 30$	$2 \cdot 3 = 6$	$0 \cdot 3 = 0$	$0 \cdot 3 = 0$

Acqualand: Pricing Algorithm - Step 2

Package individual units into “bundles”, one for each customer type

	Marginal WTP					
Units	1	2	3	4	5	Group Size
type A	\$48	\$40	\$32	$\boxed{\$22}$	$\boxed{\$15}$	1000
type B	\$36	$\boxed{\$30}$	$\boxed{\$18}$	\$10	\$0	1000
type C	$\boxed{\$25}$	\$10	\$2	\$0	\$0	1000
Unit Price	\$25	\$30	\$18	\$22	\$15	
Unit Profit	\$75,000	\$60,000	\$36,000	\$22,000	\$15,000	\$208,000

At the optimal unit prices:
type A buys 5 units,
and pays \$25 + \$30 + \$18 + \$22 + \$15 = \$110
type B buys 3 units,
and pays \$25 + \$30 + \$18 = \$73
type C buys 1 units,
and pays \$25

Profit-maximizing Menu
5 units for \$110
3 units for \$73
1 units for \$25

Incentive Compatibility of the Optimal Menu

Menu Pricing: Consumer Surplus

	Option A 5 units for \$110	Option B 3 units for \$73	Option C 1 unit for \$25
type A	$WTP_A(5) - P_A =$ \$157 - \$110 = $\boxed{\$47}$	$WTP_A(3) - P_B =$ \$120 - \$73 = \$47	$WTP_A(1) - P_C =$ \$48 - \$25 = \$23
type B	$WTP_B(5) - P_A =$ \$94 - \$110 = - \$16	$WTP_B(3) - P_B =$ \$84 - \$73 = $\boxed{\$11}$	$WTP_B(1) - P_C =$ \$36 - \$25 = \$11
type C	$WTP_C(5) - P_A =$ \$37 - \$110 = -\$73	$WTP_C(3) - P_B =$ \$37 - \$73 = -\$36	$WTP_C(1) - P_C =$ \$25 - \$25 = $\boxed{\$0}$

Acqualand: Final Comparison

Linear Pricing	
$\Pi =$	\$154
$CS_A =$	\$54
$CS_B =$	\$22
$CS_C =$	\$3
$DWL =$	\$55
$GFT =$	\$288

Menu Pricing	
$\Pi =$	\$208
$CS_A =$	\$47
$CS_B =$	\$11
$CS_C =$	\$0
$DWL =$	\$22
$GFT =$	\$288

Perfect PD	
$\Pi =$	\$288
$CS_A =$	\$0
$CS_B =$	\$0
$CS_C =$	\$0
$DWL =$	\$0
$GFT =$	\$288

Acqualand: many A types

How does the optimal menu change when there are more A types?

	Marginal WTP					
Units	1	2	3	4	5	Group Size
type A	$\boxed{\$48}$	$\boxed{\$40}$	$\boxed{\$32}$	$\boxed{\$22}$	$\boxed{\$15}$	**5000**
type B	\$36	\$30	\$18	\$10	\$0	1000
type C	\$25	\$10	\$2	\$0	\$0	1000
Unit Price	\$48	\$40	\$32	\$22	\$15	
Unit Profit	\$240,000	\$200,000	\$160,000	\$110,000	\$75,000	\$785,000

$48 \cdot 5 = 240$	$40 \cdot 5 = 200$	$32 \cdot 5 = 160$	$22 \cdot 5 = 110$	$15 \cdot 5 = 75$
$36 \cdot 6 = 216$	$30 \cdot 6 = 180$	$18 \cdot 6 = 108$	$10 \cdot 6 = 60$	$0 \cdot 6 = 0$
$25 \cdot 7 = 175$	$10 \cdot 7 = 70$	$2 \cdot 7 = 14$	$0 \cdot 7 = 0$	$0 \cdot 7 = 0$

offer only 1 option: 5 units for \$157 (\$157= \$48 + \$40 + \$32 + \$22 + \$15)

Acqualand: many C types

What if there are many C types, willing to pay a positive amount for every unit?

	Marginal WTP					
Units	1	2	3	4	5	Group Size
type A	\$48	\$40	\$32	\$22	\$15	1000
type B	\$36	\$30	\$18	\$10	\$7	1000
type C	$\boxed{\$25}$	$\boxed{\$20}$	$\boxed{\$17}$	$\boxed{\$8}$	$\boxed{\$5}$	**4000**
Unit Price	\$25	\$20	\$17	\$8	\$5	
Unit Profit	\$150,000	\$120,000	\$102,000	\$48,000	\$30,000	\$450,000

$48 \cdot 1 = 48$	$40 \cdot 1 = 40$	$32 \cdot 1 = 32$	$22 \cdot 1 = 22$	$15 \cdot 1 = 15$
$36 \cdot 2 = 72$	$30 \cdot 2 = 60$	$18 \cdot 2 = 36$	$10 \cdot 2 = 20$	$7 \cdot 2 = 14$
$25 \cdot 6 = 150$	$20 \cdot 6 = 120$	$17 \cdot 6 = 102$	$8 \cdot 6 = 48$	$5 \cdot 6 = 30$

offer only 1 option: 5 units for \$75 $(\$75 = \$25 + \$20 + \$17 + \$8 + \$5)$

Air-One

Air-One has two types of customers:
400 Business travelers (B types)
600 Leisure travelers (L types);

Air-One can offer two types of tickets:
- Restricted (no refunds or upgrades, no extra luggage ...) = low quality
- Unrestricted = high quality

AirOne knows the WTP of each customer type, for each ticket:

	WTP		group size
B types	\$500	\$800	400
L types	\$350	\$450	600
	R-ticket	U-ticket	

but cannot observe whether any customer is a B-type or an L-type.

Assume $MC = 0$, and the plane has more than 1000 seats.

What are the prices P_R and P_U that maximize Air-One's profit (revenue)?

Air-One: selling quality upgrades

Air-One sells *quality*:

R ticket = 1 unit of quality

U ticket = 2 units of quality

	MWTP for quality 1	2	group size
B-type	\$500	\$300	400
L-type	\$350	\$100	600
Unit Price			
Unit Profit			

Air-One: Pricing Algorithm, Step 1

Find the profit maximizing price for each unit of quality,

based on the marginal WTP for quality of each type:

	Marginal WTP		
Units	1	2	Group Size
B type	\$500	$\boxed{\$300}$	400
L type	$\boxed{\$350}$	\$100	600
Unit Price	\$350	\$300	
Unit Profit	\$350,000	\$120,000	\$470,000

$\$500 \cdot 400 = \$200{,}000$ $\quad$ $\$300 \cdot 400 = \$120{,}000$

$\$350 \cdot 1000 = \$350{,}000$ $\quad$ $\$100 \cdot 1000 = \$100{,}000$

Air-One: Pricing Algorithm, Step 2

Determine how many units each customer type buys at the optimal prices, and package the individual units into bundles

	Marginal WTP		
Units	1	2	Group Size
B type	\$500	$\boxed{\$300}$	400
L type	$\boxed{\$350}$	\$100	600
Unit Price	\$350	\$300	
Unit Profit	\$350,000	\$120,000	\$470,000

At the optimal prices:
  the L types buy 1 unit for \$350
  the B types buy 2 units for $\$350 + \$300 = \$650$

The menu that maximizes AirOne's profit is:

> the R ticket (the bundle of 1 unit of quality), at price \$350
> the U ticket (the bundle of 2 units of quality), at price \$650

Air-One Menu Pricing: Incentive Compatibility

	U-ticket 2 units of quality for \$650	R-ticket 1 unit of quality for \$350
type B	$WTP_B(2) - P_U =$ $\$800 - \$650 = \$150$	$WTP_B(1) - P_R =$ $\$500 - \$350 = \$150$
type L	$WTP_L(2) - P_U =$ $\$450 - \$650 = -\$200$	$WTP_L(1) - P_R =$ $\$350 - \$350 = 0$

Air-One: Many B types

	Marginal WTP		
Units	1	2	Group Size
B type	$\boxed{\$500}$	$\boxed{\$300}$	800
L type	\$350	\$100	200
Unit Price	\$500	\$300	
Unit Profit	\$400,000	\$240,000	\$640,000

$500 \cdot 8 = 4000$ $\quad$ $300 \cdot 8 = 2400$

$350 \cdot 10 = 3500$ $\quad$ $100 \cdot 10 = 1000$

It is optimal for AirOne to offer only one choice:

the U ticket (the bundle of 2 units of quality), at price \$800

Long Lasting Energy: Damaged Goods

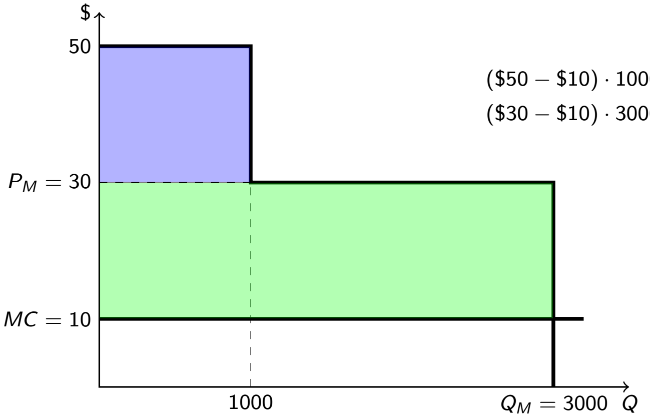
Long Lasting Energy (LLE) produces the "Wolf", a battery unit at unit cost \$10.

There are two types of buyers:
1000 are "heavy users", willing to pay \$50;
2000 are "light users", willing to pay \$30.

	WTP	group size
H	\$50	1000
L	\$30	2000
MC	\$10	

What is LLE's profit maximizing price?

Long Lasting Energy: Damaged Goods



$(\$50 - \$10) \cdot 1000 = \$40,000$

$(\$30 - \$10) \cdot 3000 = \boxed{\$60,000}$

The profit maximizing price and quantity are $P_M = 30$ and $Q_M = 3000$.

$\Pi_M =$ (\$30-\$10) $\cdot$ 3000 = \$60,000

CS = (\$50-\$30) $\cdot$ 1000 = \$20,000

DWL = \$0

Long Lasting Energy: Damaged Goods

The heavy users value the battery's long duration, but

the light users would be just as happy (almost) with a less durable battery

	duration units 1	2	group size
Heavy Users' MWTP	\$26	\$24	1000
Light Users' MWTP	\$25	\$5	2000

The less durable battery can be made from the durable one,
for an additional cost of \$3 per unit

	units of quality 1	2
TC	\$13	\$10
MC	\$13	$-\$3$

Note: $MC(2) < 0$

Long Lasting Energy: Damaged Goods

LLE introduces a lower quality battery unit: the "Cub", at unit cost \$13.

	1	2	Group Size
Heavy Users' MWTP	\$26	\$24	1000
Light Users' MWTP	\$25	\$5	2000
MC	\$13	-\$3	
Unit Price			
Unit Profit			

Long Lasting Energy: Damaged Goods

LLE introduces a lower quality battery unit: the "Cub", at unit cost \$13.

	1	2	Group Size
Heavy Users' MWTP	\$26	$\boxed{\$24}$	1000
Light Users' MWTP	$\boxed{\$25}$	\$5	2000
MC	\$13	-\$3	
Unit Price	\$25	\$24	
Unit Profit	\$36,000	\$27,000	\$63,000

$(26\text{-}13)\cdot 1 = 13$ $\quad (24+3)\cdot 1 = 27$

$(25\text{-}13)\cdot 3 = 36$ $\quad (5+3)\cdot 3 = 24$

Profit-Maximizing Menu

The Wolf (2 units of quality) for \$49 (= \$25 + \$24)

The Cub (1 units of quality) for \$25

Introducing the lower quality version increases costs, but increases profit! (from \$60,000 to \$63,000)

Deadweight Loss in Menu Pricing

Recall the AirOne example

	Marginal WTP		
Units	1	2	Group Size
B type	$500	$\boxed{\$300}$	400
L type	$\boxed{\$350}$	$100	600
Unit Price	$350	$300	
Unit Profit	$350,000	$120,000	$470,000

Profit-maximizing menu:

the R ticket (the bundle of 1 unit of quality), at price $350
the U ticket (the bundle of 2 units of quality), at price $650

Why is AirOne selling **only 1 unit to the L-types**?

If AirOne were to set $P_2 = \$100$ (i.e. sell both units to both types at $450):
the revenue from the L types would increase by $\$100 \cdot 600 =$ **$60,000**, but
the revenue from the B types would decrease by by $(\$300 - \$100) \cdot 400 =$ **$80,000**

Overall, **AirOne would lose** $80,000 - $60,000 = **$20,000**

Long Lasting Energy: Deadweight Loss

LLE's profit-maximizing menu pricing generates some deadweight loss:

The gains from trade are:

$$(\$50 - \$10) \cdot 1,000 + (\$30 - \$10) \cdot 2,000 = \$80,000$$

The social surplus generated by LLE's menu pricing (i.e. the *realized* gains from trade) is

$$\overbrace{\$63,000}^{\Pi} + \overbrace{\$1,000}^{CS} = \$64,000$$

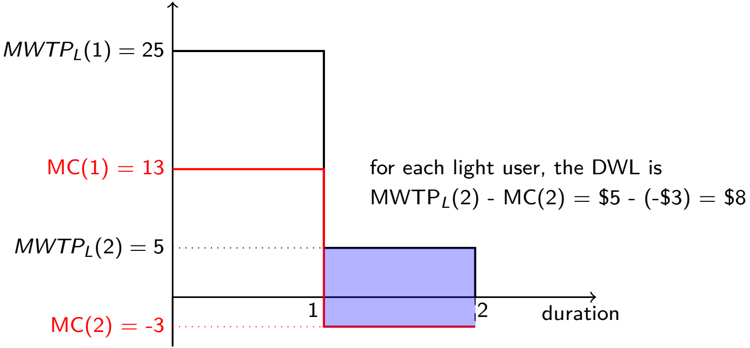
The Deadweight loss is the difference

$$GFT - \Pi - CS = \$80,000 - \$64,000 = \$16,000$$

Long Lasting Energy: Deadweight Loss

The 2000 light users do not buy the *second* duration unit.

Since $MWTP_L(2) > MC(2)$, there are unrealized gains from trade.



The total DWL is

$$(MWTP_L(2) - MC(2)) \cdot 2,000 = (\$5 - (-\$3)) \cdot 2,000 = \$16,000$$

Examples of Versioning/Damaged Goods

1. GM's Fairfax Assembly plant, in Kansas City produces
the Buick Lacrosse (high quality), and
the Chevy Malibu (low quality).

The Buick Lacrosse is made first.
Then some Lacrosse units, with additional work, become Malibus.

This is common in the auto industry (Infinity and Nissan, Lexus and Toyota ...)

2. In the late 1980s, IBM's LaserPrinter, retailing at $2,395, printed ten pages per minute.
In 1990 IBM launched the LaserPrinter E. for $1,495, printing at half the speed.
The E printer was just the original printer with **microchips added to slow it down**.

Dupuit's quote

"It is not because of the few thousand francs which would have to be spent to put a roof over the ***third class*** *carriages, or to upholster the third class seats that some company or other has open carriages with wooden benches.*

What the company is trying to do is ***prevent the passengers who can pay the second class fare from traveling third class;***
hits the poor not because it wants to hurt them, but to frighten the rich*.*

And it is again for the same reason that the companies, having proved almost cruel to second class ones, become lavish in dealing with first class passengers.

Having refused the poor what is necessary, they give the rich what is superfluous."

J.Dupuit (1849) "On Tolls and Transport Charges"