MGRECON - Class 6 (after class)

GameTheory: Road Map

- Simultaneous Games
  - Dominant Strategies
  - Dominated Strategies
  - Nash Equilibrium
  - Multiple Equilibria
- Sequential Games

Are Business Games “Zero-Sum”?

Is Business War?

Yes?
“outsmart competition”
“make a killing”
“capture market share”
“beat up suppliers”
“lock up customers”

No?
“listen to customers”
“work with suppliers”
“form strategic alliances/partnerships”

Business games almost always have **both** competition and cooperation:
  cooperation creates value
  competition distributes value across parties

The Prisoner's Dilemma Game

Rowina and Colin are caught driving a stolen car and are suspected of having committed a second crime Without further evidence, they can only be convicted of car theft

The cops put them in different rooms and tell each of them:

"You have two choices: <u>confess</u> (to the second crime), or <u>stay silent</u>

> *if you both stay silent, each of you goes to jail for 1 year (for the car theft)*
>
> *if you both confess, each gets 5 years (for the car theft and the second crime)*
>
> *if your partner confesses and you do not, you get 8 years and your partner goes free*
>
> *if you confess and your partner does not, you go free and your partner gets 8 years*

		Colin	
		Confess	Stay Silent
Rowina	Confess	-5 -5	-8 0
	Stay Silent	0 -8	-1 -1

A well-defined game must specify: i) players, ii) strategies and iii) payoffs

The Prisoner's Dilemma Game

Colin

		Confess	Stay Silent
Rowina	Confess	-5, -5	0, -8
	Stay Silent	-8, 0	-1, -1

Rowina and Colin **care only about their own jail time**: no loyalty, friendship, fairness, "doing the right" ...

Q1: What will each player do?

Q2: Does Rowina's best choice depend on her guess about Colin's choice? Does Colin's?

Q3: What is the likely outcome? Is it a "good" (i.e. efficient) outcome?

Q4: Does this game remind you of other real-life situations?

The Prisoner's Dilemma Game: Analysis

		Colin	
		Confess	Stay Silent
Rowina	Confess	-5 -5	-8 0
	Stay Silent	0 -8	-1 -1

Rowina thinks:

 if Colin confesses, I am better off confessing $(-5 > -8)$

 if Colin stays silent, I am better off confessing $(0 > -1)$

and concludes: "I will confess, no matter what Colin does"

Colin thinks:

 if Rowina confesses, I am better off confessing $(-5 > -8)$

 if Rowina stays silent, I am better off confessing $(0 > -1)$

and concludes: "I will confess, no matter what Rowina does"

For each player, "Confess" is a **dominant** strategy

"Dominant" means payoff-maximizing ***no matter what the opponent does***

Note: Selfish behavior here leads to an inefficient outcome

Iterative Elimination of Dominated Strategies (IEDS)

Player 2

		C1	C2	C3
Player 1	R1	8 0	10 0	6 6
	R2	8 0	5 5	0 10
	R3	4 4	0 8	0 8

Player 1 thinks:

my best reply to C1 is R3 ($4 > 0$)

my best reply to C2 is R3 ($8 > 5 > 0$)

my best reply to C3 is R2 ($10 > 8 > 6$)

Player 2 thinks:

my best reply to R1 is C2 ($10 > 8 > 6$)

my best reply to R2 is C1 ($8 > 5 > 0$)

my best reply to R3 is C1 ($4 > 0$)

IEDS 2

Player 2

		C1	C2	[redacted]
	[redacted]	[redacted]	[redacted]	[redacted]
Player 1	R2	8 0	5 5	[redacted]
	R3	4 4	0 8	[redacted]

C3 is never a best response
(all horizontal arrows leave C3)

C3 is a **dominated** strategy

C3 can be deleted from the game

R1 is never a best response
(all vertical arrows leave R1)

R1 is a **dominated** strategy

R1 can be deleted from the game

IEDS 3

Player 2

		C1	C2	[redacted]
	R[redacted]	[redacted]	[redacted]	[redacted]
Player 1	R[redacted]	[redacted]	[redacted]	[redacted]
	R3	4 4	[redacted]	[redacted]

In the ***residual*** (two-bytwo) game:

R2 is dominated

C2 is dominated

R2 and C2 now can be deleted

The remaining outcome (R3,C1)

is the only “rational” outcome

The IEDS procedure relies each player **believing that its opponents' are rational**

Nash Equilibrium

Colin

		C1	C2	C3
	R1	3 1	2 0	1 3
Rowina	R2	0 2	1 1	0 2
	R3	1 3	2 0	3 1

For Rowina:

R3 is the best response to C1

R2 is the best response to C2

R1 is the best response to C3

For Colin:

C1 is the best response to R1

C2 is the best response to R2

C3 is the best response to R3

The strategy profile (R2,C2) is a Nash Equilibrium

A Nash Equilibrium is a profile of strategies which are ***simultaneous best responses***

John Nash proved that any finite game has at least one Nash Equilibrium

Multiple Nash Equilibria

The Battle of the Sexes

Wife \ Husband	Football	Movie
Football	1, 3	0, 0
Movie	0, 0	3, 1

The Party Arrival Game

Guest 1 \ Guest 2	On Time	Late
On Time	2, 2	0, 1
Late	1, 0	1, 1

The Drivers' Game

Driver 2 \ Driver 1	Left	Right
Left	0, 0	-10, -10
Right	-10, -10	0, 0

The Apathy Game

P1 \ P2	C1	C2
R1	0, 0	0, 0
R2	0, 0	0, 0

A Two-Tiered Tender Offer

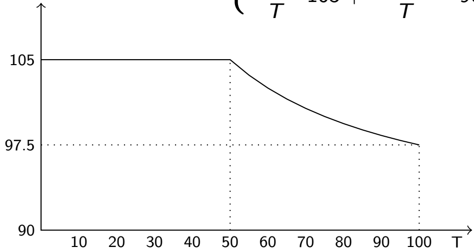
A company is owned by 100 shareholders, each owning one share.
Each share can be sold today for \$100.

A raider offers to pay

$$\begin{cases} P_I = 105 & \text{for each of the first 50 shares;} \\ P_{II} = 90 & \text{for any share beyond the } 50^{th} \end{cases}$$

All who tender get the same price: if T shareholders tender, each gets

$$P(T) = \begin{cases} 105 & \text{if } T \leq 50; \\ \dfrac{50}{T} \cdot 105 + \dfrac{T-50}{T} \cdot 90 = 90 + \dfrac{750}{T} & \text{if } T > 50. \end{cases}$$



What will each shareholder do?

Let $t = \#$ shareholders, *other than you,* who decide to tender, so $0 \leq t \leq 99$

There are 3 cases:

i) $t < 50$, the takeover fails (whether you tender or not) and your earn:

$$\begin{cases} \$105, \text{ if you tender} \\ \$100 \text{ (the current value), if you do not tender;} \end{cases}$$

ii) $t > 50$, the takeover succeeds (whether you tender or not) and you earn

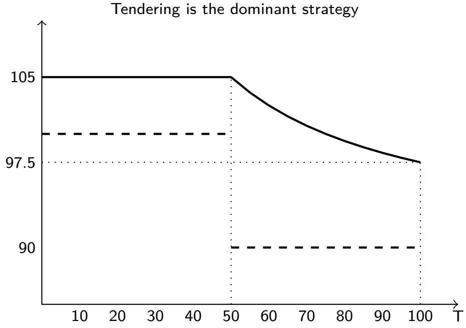
$$\begin{cases} \text{a price } P \geq P(100) = \$97.5, \text{ if you tender} \\ \$90 \text{ (by law, the raider can buy your share at \$90), if you do not tender;} \end{cases}$$

iii) $t = 50$, you are pivotal:

$$\begin{cases} \text{if you tender, the takeover succeeds, and you earn } P(51) = 104.71 \\ \text{if you do not tender, the takeover fails and you earn \$100;} \end{cases}$$

In all 3 cases you earn more by tendering; thus tendering is the ***dominant*** strategy.

The raider ends up buying all shares at $97.5 each.



Continuous line = your payoff from tendering,

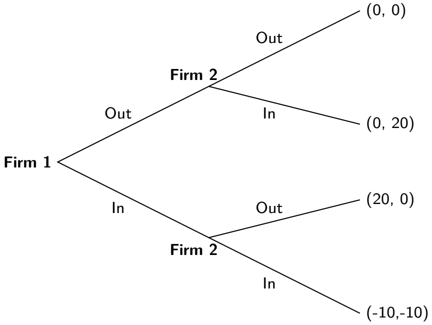
Dashed line = your payoff from not tendering

Sequential Games

In simultaneous games, players move at the same time

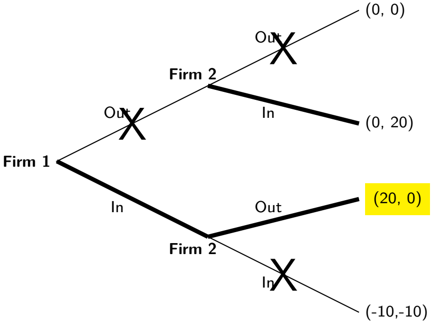
		Firm 2	
		Enter	Stay Out
Firm 1	Enter	-10, -10	20, 0
	Stay Out	0, 20	0, 0

In sequential games, some players move before others



Backward Induction

The equilibrium of a sequential game is found by backward induction



If all payoffs in the game are *different*, the equilibrium of a sequential game is **unique**.

Simultaneous vs Sequential Games

Player 2

Player 1		C1	C2	C3
	R1	37 37	40 30	37 20
	R2	30 40	32 32	25 15
	R3	20 37	15 25	0 0

In the simultaneous game

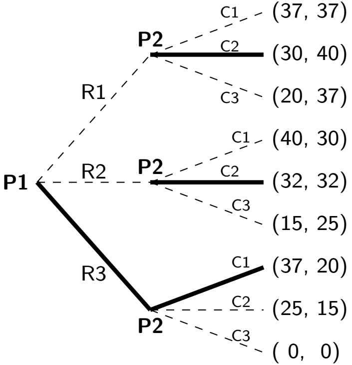
R3 and C3 are **dominated**

Delete R3 and C3, then delete R1 and C1

(R2,C2) is the unique Nash equilibrium

Simultaneous vs Sequential Games cont'd

		P2		
		C1	C2	C3
	R1	37 37	40 $\boxed{30}$	37 20
P1	R2	30 40	32 $\boxed{32}$	25 15
	R3	20 $\boxed{37}$	15 25	0 0



In the sequential game, R3 is **optimal**

In equilibrium, first Player 1 **chooses** R3, then Player 2 chooses C1