MGRECON - Class 2 (after class)

Summary of Class 1

1) $WTP = f(\overset{+}{P_S},\ \overset{-}{P_C},\ \overset{+}{I},\ \overset{+}{U})$;

2) Consumers maximize CS = WTP − P;

3) MWTP is decreasing;

4) if we read the MWTP graph *horizontally*,
we have the demand curve $Q_D = f(P; P_S, P_C, I, U)$;

5) Movements along the Demand $\neq$ Demand shifts

6) Consumer Surplus = area below the demand & above the price line

7) Elasticity $E_p = \frac{\Delta Q/Q}{\Delta P/P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$

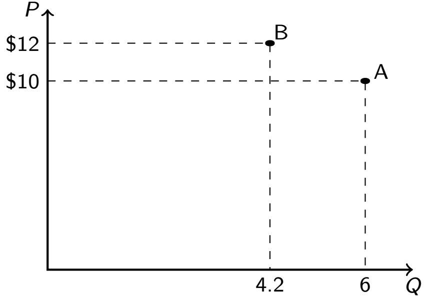
- if you have a linear (inverse) demand equation $P = a - b \cdot Q$,
  use the formula $E = -\frac{1}{b} \cdot \frac{P}{Q}$,
- if we only have 2 points,
  use the two-point formula

Summary of Class 1

Exercise 1 A recent article on titanium reports that the price has increased from \$10 to \$12 per kilogram, while quantity has decreased from 6 to 4.2 thousand metric tons. This information leads you to conclude that

a. demand is inelastic
b. demand is elastic
c. supply is elastic
d. the cross-price demand elasticity with aluminum is negative

Answer to Exercise 1
We do not have the demand equation. We only have two points on the demand. So we use the two-point elasticity formula.



mid-point values:

$$P_m = \frac{P_A + P_B}{2} = \frac{12+10}{2} = 11$$

$$Q_m = \frac{Q_A + Q_B}{2} = \frac{4.2+6}{2} = 5.1$$

$$E = \frac{\frac{Q_B - Q_A}{Q_m}}{\frac{P_B - P_A}{P_m}} = \frac{\frac{4.2-6}{5.1}}{\frac{12-10}{11}} \simeq -1.9412$$

Market Power

Market Power = ability to charge above cost

advertising, perceived high quality

1) strong need/want for the good

2) no good substitutes

$\leftrightarrow$ high market power (high WTP)

little competition
(patents/licenses/franchises, economies of scale)

Requirement 1 by itself is not sufficient, e.g. water.

Requirement 2 by itself is not sufficient, e.g. the autograph of a non-celebrity.

Examples:

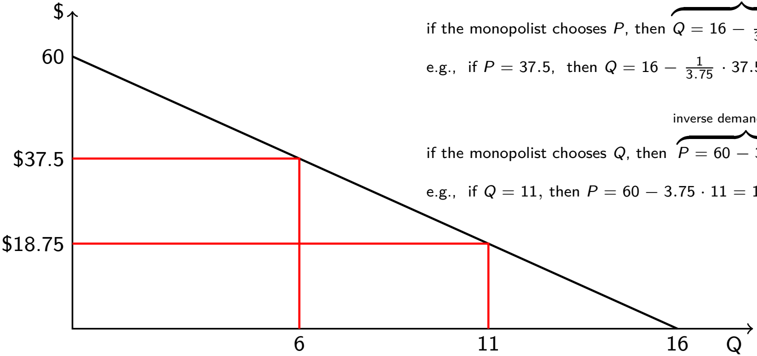
food/drinks inside stadiums, theaters, museums,

pharmaceutical companies, De Beers, Microsoft, Monsanto, Luxottica

Revenue Maximization

A monopolist can choose any point **on the demand curve**.

Example: $Q = 16 - \frac{1}{3.75} \cdot P \quad \Leftrightarrow \quad P = 60 - 3.75 \cdot Q$



if the monopolist chooses P, then $\overbrace{Q = 16 - \frac{1}{3.75} \cdot P}^{\text{demand equation}}$

e.g., if $P = 37.5$, then $Q = 16 - \frac{1}{3.75} \cdot 37.5 = 6$

if the monopolist chooses Q, then $\overbrace{P = 60 - 3.75 \cdot Q}^{\text{inverse demand equation}}$

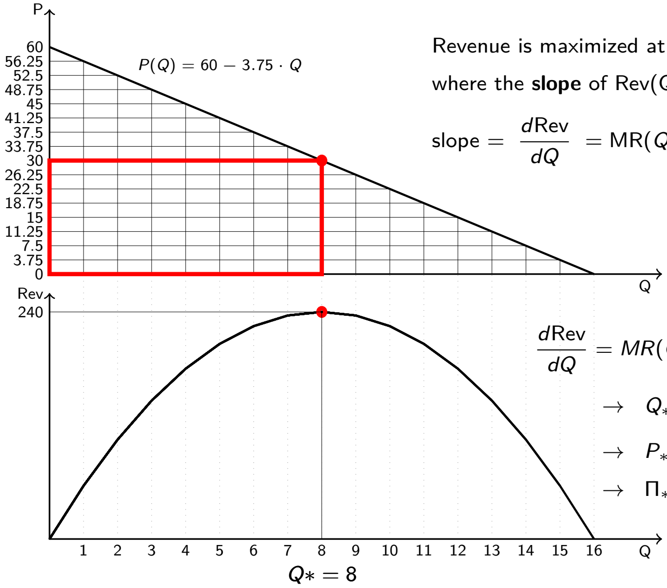
e.g., if $Q = 11$, then $P = 60 - 3.75 \cdot 11 = 18.75$

Suppose **there are no costs**, hence profit maximization is just revenue maximization

$$\text{Rev}(Q) = P(Q) \cdot Q = (60 - 3.75 \cdot Q) \cdot Q = 60 \cdot Q - 3.75 \cdot Q^2$$

How do we find the point on the demand that maximizes total revenue?

Revenue Maximization: cont'd



Revenue is maximized at the point Q_*

where the **slope** of Rev(Q) is equal to zero

$$\text{slope} = \frac{d\text{Rev}}{dQ} = \text{MR}(Q) = 60 - 7.5 \cdot Q$$

$$\frac{d\text{Rev}}{dQ} = MR(Q) = 60 - 7.5 \cdot Q = 0$$

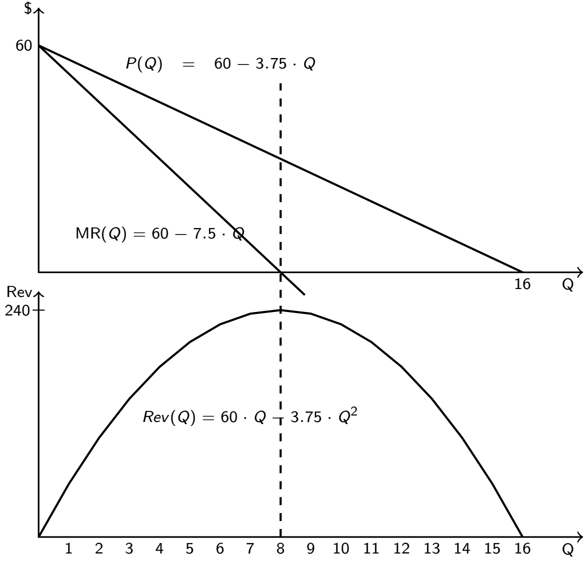
$\rightarrow \quad Q_* = 8$

$\rightarrow \quad P_* = 60 - 3.75 \cdot 8 = 30$

$\rightarrow \quad \Pi_* = 30 \cdot 8 = 240$

Revenue Maximization: cont'd

To maximize revenue, solve the equation $\text{MR}(Q) = 0$



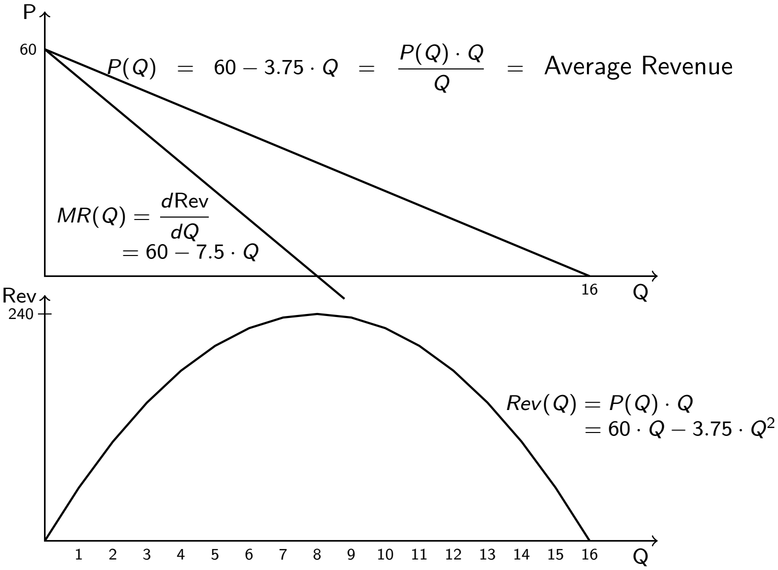
$$\overbrace{60 - 7.5 \cdot Q}^{\text{MR}(Q)=0} = 0 \quad \rightarrow \quad Q_* = 8$$

Why $\text{MR}(Q_*) = 0$?

if $\text{MR}(Q) > 0$,
increasing Q increases Revenue

if $\text{MR}(Q) < 0$,
decreasing Q increases Revenue

Marginal Revenue vs. Average Revenue



Revenue Maximization Exercise

Given the demand equation $Q = 240 - 2 \cdot P$,
find the revenue-maximizing quantity Q_* and price P_*, and
the resulting revenue $P_* \cdot Q_*$

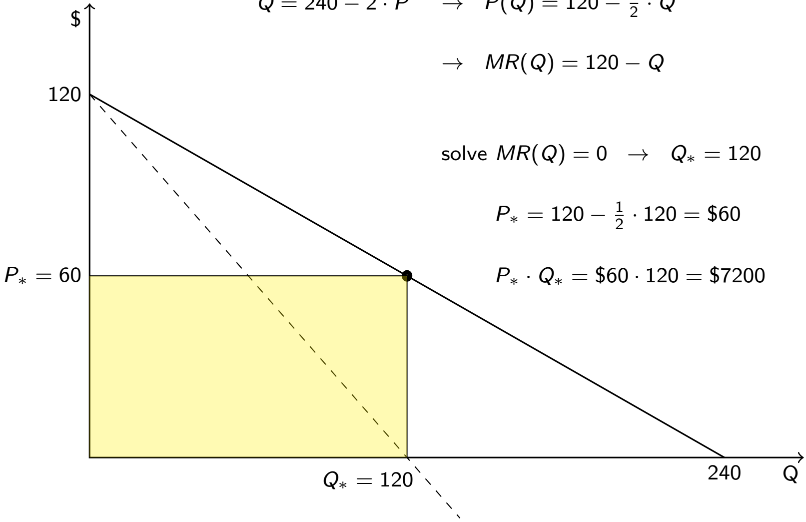
Answer

1. Inverse demand P(Q) ____________
2. $Rev(Q) = P(Q) \cdot Q$ ____________
3. $MR(Q) = \dfrac{dRev(Q)}{dQ}$ ____________
4. Solve $MR(Q) = 0$ ____________
5. $P_* = P(Q_*)$ ____________
6. $P_* \cdot Q_* =$ ____________

Revenue Maximization Exercise: Answers

1. Inverse demand P(Q): $Q = 240 - 2 \cdot P \;\rightarrow\; P = 120 - \frac{1}{2} \cdot Q$
2. $Rev(Q) = P(Q) \cdot Q \;=\; (120 - \frac{1}{2}Q) \cdot Q \;=\; 120 \cdot Q - \frac{1}{2} \cdot Q^2$
3. $MR(Q) = \dfrac{dRev(Q)}{dQ} \;=\; 120 - Q$
4. Solve $MR(Q) = 0$: $120 - Q = 0 \;\rightarrow\; Q_* = 120$
5. $P_* = P(Q_*) = 120 - \frac{1}{2}Q_* = 120 - \frac{1}{2} \cdot 120 = \$60$
6. $P_* \cdot Q_* \;=\; \$60 \cdot 120 = \$7200$

Revenue Maximization Exercise: Figure



$$Q = 240 - 2 \cdot P \quad \rightarrow \quad P(Q) = 120 - \tfrac{1}{2} \cdot Q$$

$$\rightarrow \quad MR(Q) = 120 - Q$$

solve $MR(Q) = 0 \quad \rightarrow \quad Q_* = 120$

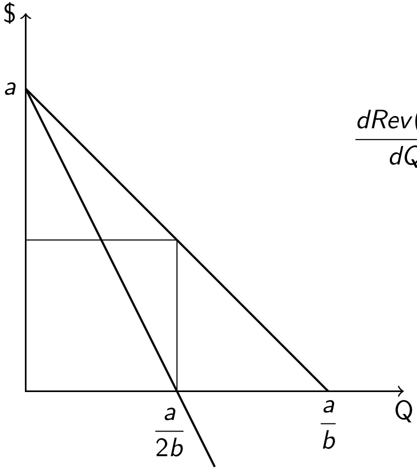
$$P_* = 120 - \tfrac{1}{2} \cdot 120 = \$60$$

$$P_* \cdot Q_* = \$60 \cdot 120 = \$7200$$

Marginal Revenue of a Linear Demand

If $P(Q)$ is **linear**, then $MR(Q)$ is **also linear**, with **twice the slope**:

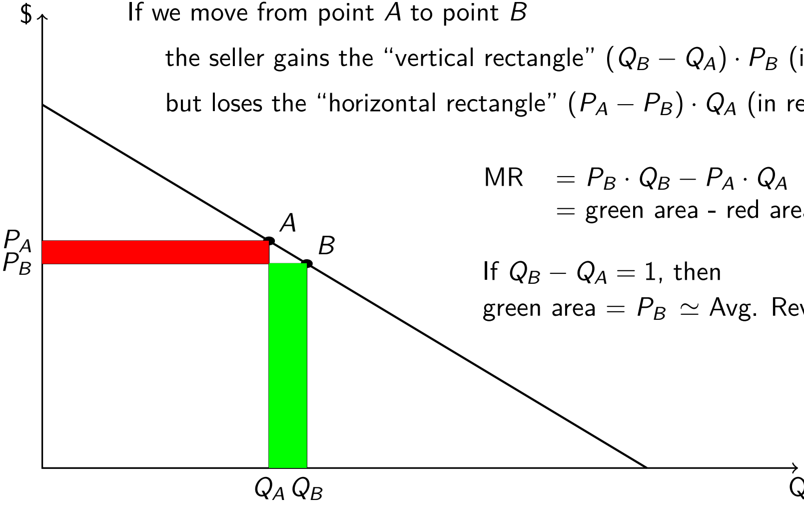
$$\boxed{P = a - b \cdot Q \quad \rightarrow \quad MR(Q) = a - 2b \cdot Q}$$



$$\begin{aligned} \frac{dRev(Q)}{dQ} &= \frac{d}{dQ}(\overbrace{(a - b \cdot Q)}^{P(Q)} \cdot Q) \\ &= \frac{d}{dQ}(a \cdot Q - b \cdot Q^2) \\ &= a - 2b \cdot Q \end{aligned}$$

Why $MR(Q) < P(Q)$?

Since demand is downward sloping, we have $MR(Q) < P(Q)$, at any $Q > 0$.



If we move from point A to point B

the seller gains the "vertical rectangle" $(Q_B - Q_A) \cdot P_B$ (in green),

but loses the "horizontal rectangle" $(P_A - P_B) \cdot Q_A$ (in red)

$$\begin{aligned} \text{MR} &= P_B \cdot Q_B - P_A \cdot Q_A \\ &= \text{green area - red area} \end{aligned}$$

If $Q_B - Q_A = 1$, then
green area $= P_B \simeq$ Avg. Rev.

Marginal Revenue and Elasticity

Marginal Revenue and Elasticity are related:

$$\boxed{MR(Q) = P(Q) \cdot \left(1 - \frac{1}{|E(Q)|}\right)}$$

$$|E(Q)| < 1 \quad \Leftrightarrow \quad \frac{1}{|E(Q)|} > 1 \quad \Leftrightarrow \quad 1 - \frac{1}{|E(Q)|} < 0 \quad \Leftrightarrow \quad \mathrm{MR}(Q) < 0$$

$$|E(Q)| > 1 \quad \Leftrightarrow \quad \frac{1}{|E(Q)|} < 1 \quad \Leftrightarrow \quad 1 - \frac{1}{|E(Q)|} > 0 \quad \Leftrightarrow \quad MR(Q) > 0$$

At any point where the demand is **inelastic** ($|E(Q)| < 1$), revenue "follows P"

if $P \downarrow$ by 1%, then $Q \uparrow$ by <u>less</u> than 1%, and thus Rev $= (P \cdot Q) \downarrow$

if $P \uparrow$ by 1%, then $Q \downarrow$ by <u>less</u> than 1%, and thus Rev $= (P \cdot Q) \uparrow$

If demand is inelastic, Rev and Q move in opposite directions, hence MR $= \frac{\Delta \text{Rev}}{\Delta Q} < 0$

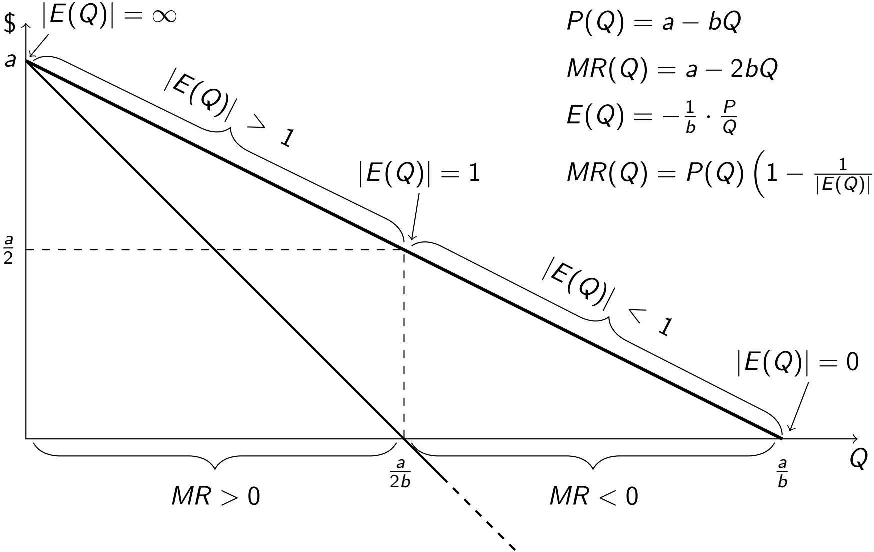
At any point where the demand is **elastic** ($|E(Q)| > 1$), revenue "follows Q"

if $P \uparrow$ by 1%, then $Q \downarrow$ by <u>more</u> than 1%, and thus Rev $= (P \cdot Q) \downarrow$

if $P \downarrow$ by 1%, then $Q \uparrow$ by <u>more</u> than 1%, and thus Rev $= (P \cdot Q) \uparrow$

If demand is elastic, Rev and Q move in the same direction, hence MR $= \frac{\Delta \text{Rev}}{\Delta Q} > 0$

Marginal Revenue and Elasticity when Demand is Linear



$$P(Q) = a - bQ$$

$$MR(Q) = a - 2bQ$$

$$E(Q) = -\frac{1}{b} \cdot \frac{P}{Q}$$

$$MR(Q) = P(Q)\left(1 - \frac{1}{|E(Q)|}\right)$$

Profit Maximization

$$\Pi(Q) = \text{Rev}(Q) - \text{Cost}(Q)$$

Suppose that:

1) demand is linear, and
2) total cost increases at a **nondecreasing rate**, i.e. MC is nondecreasing

Then

$$\max\ \Pi \quad \Rightarrow \quad \frac{d\,\Pi}{dQ} = \frac{d\,\text{Rev}}{dQ} - \frac{d\,\text{Cost}}{dQ} = 0$$

$$\max\ \Pi \quad \Rightarrow \quad \boxed{\text{MR}(Q) = \text{MC}(Q)}$$

Why $\text{MR}(Q) = \text{MC}(Q)$?

If $\text{MR}(Q) > \text{MC}(Q)$, increasing Q increases profit (Rev grows more than Cost)

If $\text{MR}(Q) < \text{MC}(Q)$, decreasing Q increases profit (Rev decreases less than Cost)

Exercise: Profit Maximization with constant MC

A monopolist operates in a market with demand

$$Q = 16 - \frac{1}{3.75}P \quad \Leftrightarrow \quad P = 60 - 3.75 \cdot Q$$

therefore

$$MR(Q) = 60 - 7.5 \cdot Q$$

Total cost of production is

$$TC(Q) = 15 \cdot Q$$

therefore

$$MC(Q) = 15$$

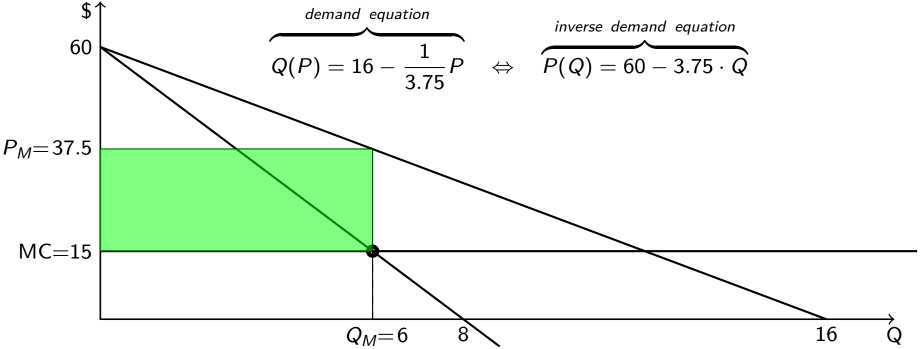
Find the profit maximizing quantity and price, and the resulting profit:

$Q_M =$ ______________

$P_M =$ ______________

$\Pi_M =$ ______________

Profit Maximization with constant MC: Answers



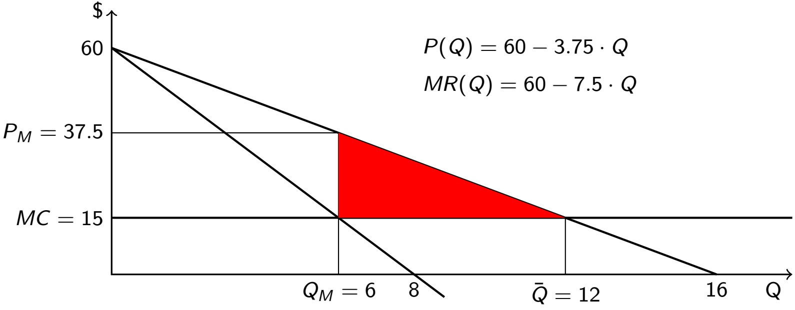
$$\overbrace{Q(P) = 16 - \frac{1}{3.75}P}^{demand\ \ equation} \quad \Leftrightarrow \quad \overbrace{P(Q) = 60 - 3.75 \cdot Q}^{inverse\ \ demand\ \ equation}$$

$$\overbrace{60 - 7.5 \cdot Q}^{MR} = \overbrace{15}^{MC} \rightarrow Q_M = 6$$

$$P_M = 60 - 3.75 \cdot Q_M = 60 - 3.75 \cdot 6 = 37.5$$

$$\Pi_M = \overbrace{37.5 \cdot 6}^{\text{revenue}} - \overbrace{15 \cdot 6}^{\text{cost}} = 135$$

The Social Cost of Monopoly: the "Deadweight Loss"



For each unit up to $\bar{Q} = 12$, there are *gains from trade*, i.e. MWTP $>$ MC

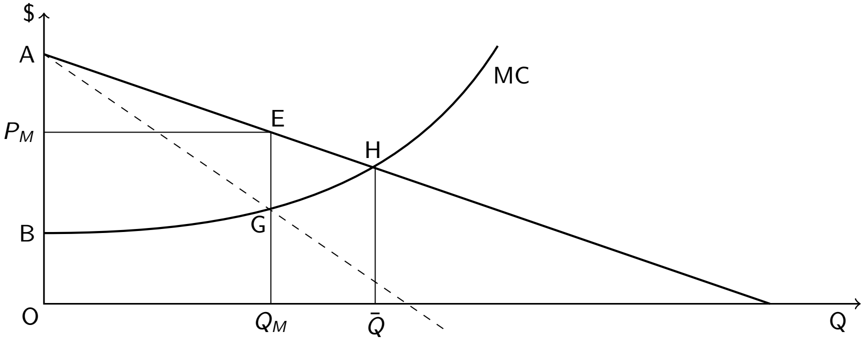
All units up to $\bar{Q} = 12$ should be sold, but the monopolist only sells $Q_M = 6$ units.

The area between $\bar{Q}$ and Q_M, below the demand and above the MC curve is the "Deadweight Loss"
generated by the monopoly

$$DWL = \frac{1}{2}(37.5 - 15)(12 - 6) = 67.5$$

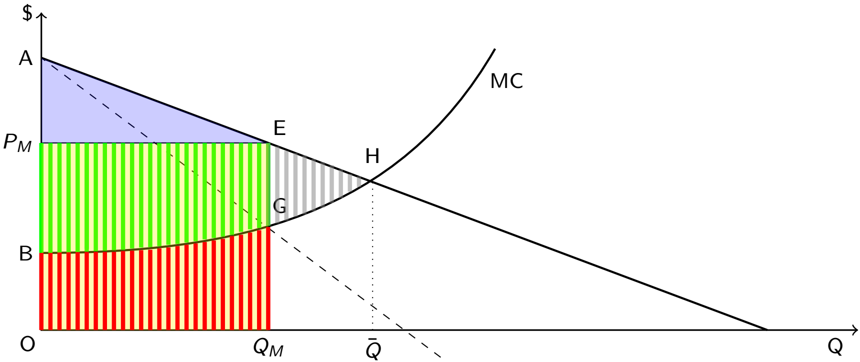
The DWL is the monetary value of the *unrealized gains from trade*.

Classic Monopoly Picture



$$
\begin{aligned}
\text{Consumer Surplus} &= \underline{\hspace{3cm}} \\
\text{Revenue} &= \underline{\hspace{3cm}} \\
\text{Total Variable Cost} &= \underline{\hspace{3cm}} \\
\text{Profit} &= \underline{\hspace{3cm}} \\
\text{Deadweight Loss} &= \underline{\hspace{3cm}}
\end{aligned}
$$

Classic Monopoly Picture: Answers



$$\begin{aligned}
\text{Consumer Surplus} &= AP_ME \\
\text{Revenue} &= OP_MEQ_M \\
\text{Total Variable Cost} &= OBGQ_M \\
\text{Profit} &= BP_MEG \\
\text{Deadweight Loss} &= GEH
\end{aligned}$$

The Lerner Index

Recall the link between MR and Elasticity:

$$\mathrm{MR}\,(Q) = P\,(Q) \cdot \left(1 - \frac{1}{|E(Q)|}\right)$$

A profit maximizing seller chooses the output level Q_M where

$$\mathrm{MR}(Q_M) = \mathrm{MC}(Q_M)$$

Combining these two equalities and rearranging yields

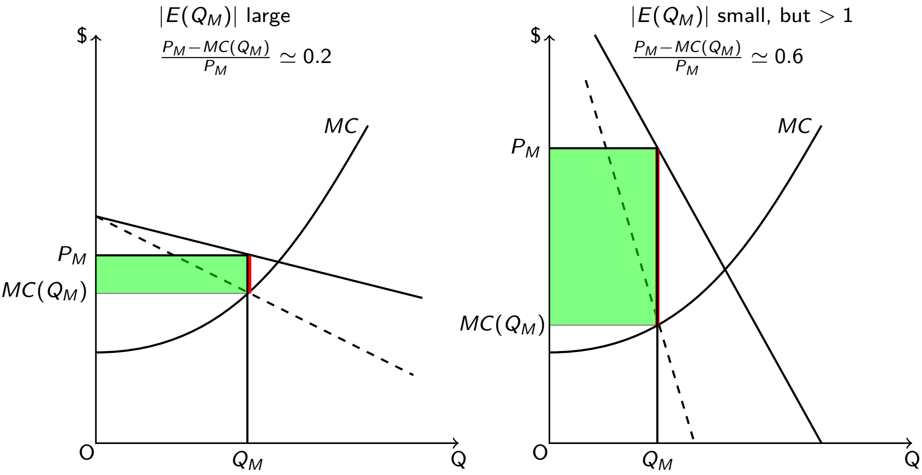
$$\overbrace{\frac{P_M - \mathrm{MC}(Q_M)}{P_M}}^{\text{Lerner Index}} = \frac{1}{|E(Q_M)|}$$

The Lerner Index is a **measure of market power**:

if $|E(Q_M)|$ is large, the Lerner index is small

if $|E(Q_M)|$ is small (close to 1), the Lerner index is large

The Lerner Index in pictures



Exercises

1. A good is sold at price $P_M = \$24$.
You know that, at that point on the demand curve, $E_P = -2$.
What is MC?

Answer: $\dfrac{\$24 - \text{MC}}{\$24} = \dfrac{1}{2} \;\; \rightarrow \;\; \text{MC} = \$12$

2. Working as a consultant for a company, you discover that the price has been set at a point where $E_P = -0.82$. You see an opportunity to make a recommendation immediately. What recommendation?
(Note: you have not learned *anything* yet about the cost structure of this company)

Answer: If $|E| < 1$, increasing P raises revenue and cannot increase costs.

Swan's Pies

We have data on:

Q = our sales

P = our prices

A = our advertising expenditures

P_X = competitors' prices

Y = Income of our buyers

Pop = Population near our stores

We want to:

- Identify significant demand drivers (P and shifters)
- Predict the direction of most effects; e.g. if P goes up Q goes down
- Choose a model (linear, log, polynomial ...)
- Perform the statistical estimation
- Use the estimated coefficients to assist managerial decisions

Swan's Gourmet Frozen Fruit Pie, European Market Demand Data, 2016-1 to 2017-4

City	Year-Quarter	Row	Q	P (€)	A (€)	Px (€)	Y (€)	Pop	T
Copenhagen	2016-1	1	82887	4.79	11870	5.19	19785	1764754	1
	2016-2	2	72901	5.41	15614	3.63	19816	1775081	2
	2016-3	3	106199	4.31	10546	3.96	19940	1808791	3
	2016-4	4	78568	5.06	12729	4.63	20058	1811837	4
	2017-1	5	120952	4.77	14111	4.77	20106	1834492	5
	2017-2	6	87201	4.49	10632	3.66	20288	1869017	6
	2017-3	7	71789	4.98	12772	3.92	21182	1870754	7
	2017-4	8	155061	3.98	10842	4.35	21335	1877867	8
Barcelona	2016-1	9	214919	4.56	20387	3.74	9625	4912883	1
	2016-2	10	220539	4.6	20679	4.6	9851	4941102	2
	2016-3	11	172218	5.43	25775	4.37	9877	4949927	3
	2016-4	12	238150	4.56	20171	4.54	9936	4959864	4
	2017-1	13	256591	4.8	22904	4.03	9974	4974649	5
	2017-2	14	245715	4.5	20009	3.65	10028	4986292	6
	2017-3	15	237510	4.47	19955	3.97	10270	4986914	7
	2017-4	16	201141	5.41	24570	3.67	10412	5000497	8
Stuttgart	2016-1	17	110145	4.91	14910	5.23	14310	2285176	1
	2016-2	18	116912	4.58	14363	4.69	14554	2302945	2
	2016-3	19	142545	4.22	11807	4.52	14763	2324662	3
	2016-4	20	135300	4.55	13134	3.81	15120	2334683	4
	2017-1	21	134443	4.37	12738	3.97	15420	2334686	5
	2017-2	22	160029	4.04	11773	4.64	15505	2341437	6
	2017-3	23	65749	5.56	16542	5.21	15535	2343293	7
	2017-4	24	138459	4.64	13095	4.79	16100	2349031	8
Paris	2016-1	25	527428	4.31	32193	4.16	18785	11695450	1
	2016-2	26	504765	4.96	37724	5.05	18794	11811599	2
	2016-3	27	499079	3.77	28023	3.78	18821	11813719	3
	2016-4	28	448951	5.07	37951	3.46	18952	11818503	4
	2017-1	29	451161	5.28	40171	4.39	19221	11842401	5
	2017-2	30	464889	5.03	36683	4.22	19638	11909047	6
	2017-3	31	546251	4.91	37066	5.21	19934	11934859	7
	2017-4	32	529288	4.3	33045	4.78	19989	11935574	8
The Hague	2016-1	33	133740	4.01	10422	4.34	16010	1362486	1
	2016-2	34	70139	4.75	20571	3.55	16171	1363325	2
	2016-3	35	263328	4.46	23723	5.03	16450	1370405	3
	2016-4	36	34559	5.25	29382	3.44	16515	1384396	4
	2017-1	37	270526	3.89	25153	3.88	16625	1385026	5
	2017-2	38	132062	5.27	40232	3.86	16773	1386713	6
	2017-3	39	126794	5.27	45213	4.1	16857	1394751	7
	2017-4	40	239474	3.92	40196	4.55	16893	1404000	8
Hamburg	2016-1	41	167495	4.35	16969	4.92	14502	3224922	1
	2016-2	42	184777	3.99	17022	4.79	14572	3226820	2
	2016-3	43	142506	4.76	19106	3.4	14732	3248190	3
	2016-4	44	191225	4.46	19300	4.73	14748	3266896	4
	2017-1	45	149219	4.51	17832	4.68	14831	3268014	5
	2017-2	46	134449	5.27	20658	3.65	15313	3284702	6
	2017-3	47	178994	4.28	17889	3.65	15463	3285469	7
	2017-4	48	123281	5.52	22865	3.99	15656	3297523	8
Average			210006.31	4.67875	21902.44	4.27	16042.4	4267821.33	
max			546251	5.56	45213	5.23	21335	11935574	
min			34559	3.77	10422	3.4	9625	1362486	

Q = units sold

P = price

A = advertising

Px = Competitors' Average Price

I = Income

Pop = Population

T = Time

Swan's Pies: a Linear Demand Model

The regression equation is:

$$Q = a + b \cdot P + c \cdot A + d \cdot P_X + e \cdot Y + f \cdot Pop + g \cdot T + z$$

The "noise" term z includes measurement errors and omitted variables.

We expect:

$b < 0$	demand is downward sloping!
$c > 0$	hopefully, money spent on advertising has increased sales!
$d > 0$	our competitors' products should be substitute goods
$e > 0$	hopefully, our product is not an inferior good
$f > 0$	more people around $\rightarrow$ more potential buyers
g	no expectation

Swan's Pies: Discussion Questions

1. How would you calculate price elasticities?

2. Do you expect Paris and Stuttgart to have the similar demands for Swan's Pies?

3. Can you say anything about whether Swan's has set prices optimally, *without any information about their costs*?

Swan's Pies: Demand Estimate

Estimation yields

$$Q = 282\,031 - 81\,266 \cdot P + 3.6636 \cdot A + 19\,970 \cdot P_X + 0.0524 \cdot Y + 0.0312 \cdot Pop + 1\,918 \cdot T + z$$

Let's look at Paris in 2016-3: $Q_0 = 499,079$ and $P_0 = 3.77$

Was the price set optimally at 3.77?

Had we charged $P_1 = 4$, our sales would have been (on average)

$$\begin{aligned} Q_1 &= Q_0 + b \cdot (P_1 - P_0) \\ &= 499,079 - 81,266 \cdot (4 - 3.77) \\ &= 480,390 \end{aligned} \qquad \left[\begin{array}{c} \frac{\Delta Q}{\Delta P} = b \\ \Delta Q = b \cdot \Delta P \\ Q_1 - Q_0 = b \cdot (P_1 - P_0) \end{array} \right]$$

Revenue would have been

$$P_1 \cdot Q_1 = 4 \cdot 480,390 = 1,921,560 \; > \; P_0 \cdot Q_0 = 3.77 \cdot 499,079 = 1,881,528$$

Had we charged a bit more in Paris 2016-3, our revenue would have been higher.

What about profit?

We know nothing about costs, but ... *lowering Q cannot increase costs!*

Swan's Pies: Elasticity in Paris 2016-3

If we measure elasticity, we get

$$|E_P| = b \cdot \frac{P}{Q} = 81,266 \cdot \frac{3.77}{499079} = 0.61 < 1$$

Indeed, $|E_P| < 1$, and thus revenue follows P.

Swan's Pies: pricing decisions in Paris

Were all pricing decisions wrong in Paris?

Date	Q	P	$\lvert E_P \rvert$
2016-1	527,428	4.31	0.66
2016-2	504,765	4.96	0.80
2016-3	499,079	3.77	0.61
2016-4	448,951	5.07	0.92
2017-1	451,161	5.28	0.95
2017-2	464,889	5.03	0.88
2017-3	546,251	4.91	0.73
2017-4	529,288	4.30	0.66

It is never optimal to operate on the inelastic part of the demand curve