- Unit 1: Introduction to Probability
- Unit 2: Probability Distributions
- **Unit 3: Statistical Inference**
- Unit 4: Introduction to Linear Regression
- Unit 5: Regression Analysis
- Unit 6: Regression Modeling

Announcements

- Past:

  Unit 2 Individual Assignment solutions available and scores posted.

- Present:

  Unit 3 Team Assignment **due Monday, 23:59 ET** (Durham local time).

- Future:

  Unit 4 materials and individual assignment available.

- **Confidence Intervals** $PE \quad \pm \quad cv \times SE$

  PE: Point Estimate (e.g., observed sample mean)

  SE: Standard Error (e.g., observed sample std.deviation divided by the sqrt of sample size: $SE = S/\sqrt{n}$)

  cv: Critical Value is determined by a desired confidence level

  - 68% confidence interval: $PE \pm\ 1SE$
  - 95% confidence interval: $PE \pm\ 2SE$
  - 99.7% confidence interval: $PE \pm\ 3SE$

  Can compute precisely for any other confidence level

  Wider confidence interval (less informative) $\Longleftrightarrow$ Higher confidence level (more reliable)

Unit 3: Statistical Inference

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- **Estimating the (unknown) population mean**

  $PE =$ observed sample mean $\bar{x}$, $SE = S/\sqrt{n}$

- **Estimating the (unknown) population proportion**

  $PE =$ observed sample proportion $\hat{p}$, $SE = \sqrt{\hat{p}(1-\hat{p})/n}$

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- **Hypothesis Testing (supplementary)**

  1 $-$ (**p-value**): largest confidence level in a claim, based on observed sample data

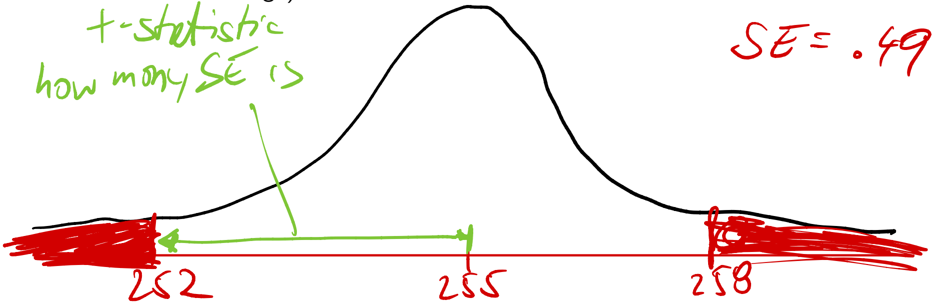
  p-value: probability that the observed sample data is "extreme" and led to making a false claim

Production process calibration (revisited)

A manufacturer of a generic drug produces caplets which contain 250mg of the medication substance. The production process includes a natural slight variability in the exact contents of each caplet, so the company set a mean weight target at 255mg per caplet. (This allowable 2% increase in the target mean ensures sufficient medication in each caplet, without risking overdosage.)

A sample of 150 caplets is selected from the output of the caplet production process. Each caplet is carefully inspected for its medication content, which is recorded in the data file `CapletWeights_Data`.

Based on the observed sample data how confident can one be that the mean weight target is not set correctly (i.e., that it is NOT set at 255mg?)



The p-value

- The p-value quantifies uncertainty about the correctness of the claim/decision/conclusion based on the observed sample data.

  The p-value can be interpreted as the probability of making a mistake due to observed sample being so extreme that the statistical analysis of sample data yield to the incorrect claim/decision/conclusion.

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- Hypothesis Testing

  A way to frame sample-based statistical analysis when determining correctness of a specific claim.

  Conclusions from any sample-based statistical analysis have inherent uncertainty (due to sampling)

  The p-value is the probability that hypothesis testing lead to an incorrect claim.

  1 - (p-value) is the confidence level that the claim is correct.

Efficacy of Moderna mRNA-1273 SARS-CoV-2 Vaccine

	mRNA-1273 Vaccine N=14,550	Placebo N=14,598
Symptomatic Covid-19	11	185
Severe Covid-19	0	30

Vaccine efficacy of 94.1% (95% CI, 89.3–96.8%; P<0.001)

"the trial was designed for the null hypothesis that the efficacy of the mRNA-1273 vaccine is 30% or less"

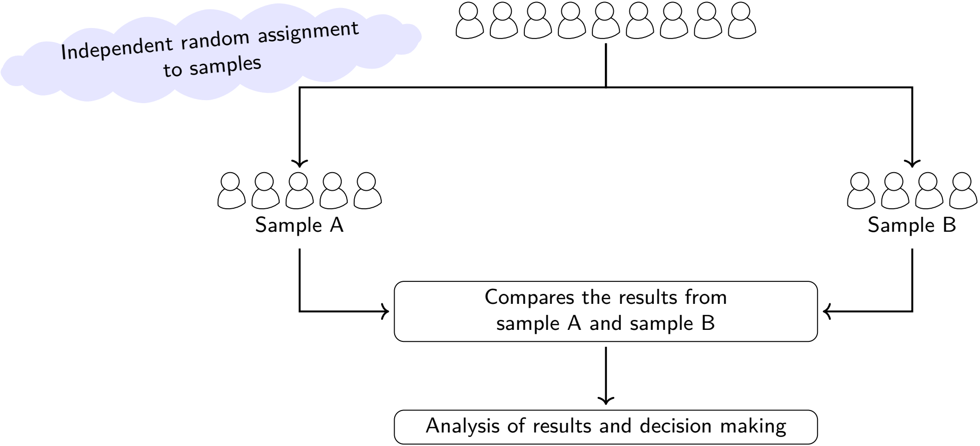
(FDA requirement: *"point estimate..at least 50%"* and *"lower bound of..confidence interval..is >30%"*)

- Want to claim that the vaccine has efficacy larger than 30% in the population (i.e., vaccination reduces the incidence of symptomatic disease by more than 30%)
- Want to reject the null hypothesis

▶ Having observed 94.1% efficacy in the trial, what is the probability that the efficacy is 30% or less?

From L.R. Baden et al., NEJM, 2020

A/B testing: comparing independent samples



Note. The two samples might also be referred to as the *control* and *treatment* (or experimental) samples (or groups)

A/B testing formally

Question: Is $p_A \neq p_B$? In other words, is difference $d = p_A - p_B$ different from zero?

Sample A

- Sample A size: n_A
- Sample A proportion: $\widehat{p}_A$
- Standard error for $\widehat{p}_A$: $\sqrt{\frac{\widehat{p_A}(1-\widehat{p_A})}{n_A}}$

Sample B

- Sample B size: n_B
- Sample B proportion: $\widehat{p}_B$
- Standard error for $\widehat{p}_B$: $\sqrt{\frac{\widehat{p_B}(1-\widehat{p_B})}{n_B}}$

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Sample B

- Sample B size: n_B
- Sample B proportion: $\widehat{p}_B$
- Standard error for $\widehat{p}_B$: $\sqrt{\frac{\widehat{p_B}(1-\widehat{p_B})}{n_B}}$

- Compute the point estimate for the difference of two sample proportions $\widehat{d} = \widehat{p}_A - \widehat{p}_B$
- Compute the standard error for the difference of two sample proportions $SE = \sqrt{\frac{\widehat{p}_A(1-\widehat{p}_A)}{n_A} + \frac{\widehat{p}_B(1-\widehat{p}_B)}{n_B}}$
- Compute t-statistic $= \widehat{d}/SE$, and determine the p-value using the two-sided t-test

Unit 3 Team Assignment

- Due Monday, 23:59 ET (Durham local time).
- Five questions with different point allocations per question
- Need to provide support for your answers. Communicating effectively is important.
- One submission per team: two-page pdf document (no other documents)

Assignment questions:

- Questions 1-2: confidence intervals
- Question 3-5: analytics supported business decisions