tues 7-8
||
wed 12:30

Problem set

Analysis of filters with complex impedance

Note: It's important that you complete the pre-lab before working on this problem set! You may want to reference Chapter 7 of your textbook (which is online if you forgot to bring your copy to class).

For each of the circuits in the table, figure out the complex number that represents the ratio of the output voltage divided by the input voltage using math. Write your final result in the tables. For each circuit in the table, create a plot of the magnitude of this complex number as a function of frequency. Put both the magnitude and frequency on a log scale. This plot can be in Python, MatLab, or the coding software of your choice.

For each circuit use:

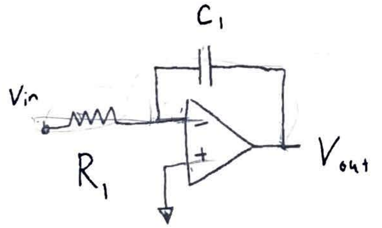
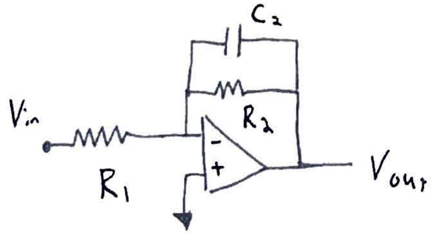
- R1 = 1 kΩ   C1 = 1 μF
- R2 = 10 kΩ   C2 = 0.1 μF

Remember, the impedance of a resistor and capacitor are

- $Z_R = R$
- $Z_C = \frac{1}{j\omega C}$

j –
ω - frequency? [Hz] ← ?
C - capacitance [F]

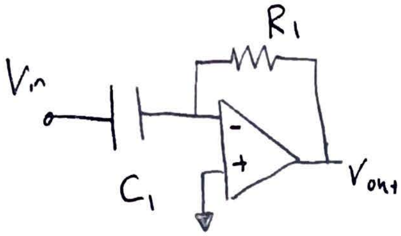
$\frac{1}{2\pi f C} = \frac{1}{\omega C}$
$\omega = 2\pi f$

	$I = \frac{V_{in}}{R} = \frac{-V_{out}}{\left(\frac{1}{j\omega C}\right)} = -V_{out}\, j\omega C$ $\frac{V_{out}}{V_{in}} = \boxed{\frac{-1}{j\omega CR}}$
	$I = \frac{V_{in}}{R_1} = \frac{-V_{out}}{R_2} + \frac{-V_{out}}{\left(\frac{1}{j\omega C_2}\right)}$ $= -V_{out}\left(\frac{1}{R_2} + j\omega C_2\right)$ $\frac{V_{out}}{V_{in}} = \frac{-1}{R_1\left(\frac{1}{R_2} + j\omega C_2\right)} = \boxed{\frac{-R_2}{R_1(1+j\omega C_2 R_2)}}$

$R_1 + \frac{1}{j\omega C_1} = \frac{R_1}{1+j\omega C_1}$

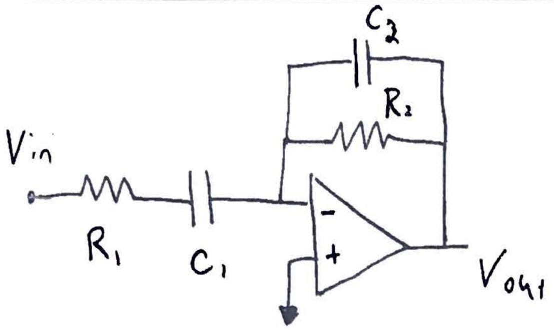
$$R_1 + \frac{1}{j\omega C_1} = \frac{R_1}{1} + \frac{R_1}{j\omega C_1 R_1} = R_1\left(1 + \frac{1}{j\omega C R_1}\right) = R_1\left(\frac{1}{1} + \frac{1}{j\omega C_1 R_1}\right) = R_1\left(\frac{j\omega C_1 R_1}{j\omega C_1 R_1} + \frac{1}{j\omega C R_1}\right)$$

$$= \cancel{R_1}\left[\frac{(j\omega C_1 R_1 + 1)}{j\omega C_1 R_1}\right]$$



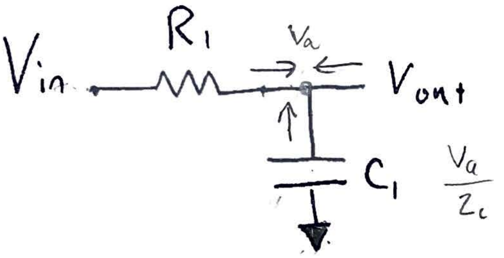
$$I = \frac{V_{in}}{\left(\frac{1}{j\omega C}\right)} = V_{in} j\omega C = \frac{-V_{out}}{R}$$

$$\frac{V_{out}}{V_{in}} = \boxed{-j\omega CR}$$



$$I = \frac{V_{in}}{\left(R_1+\frac{1}{j\omega C_1}\right)} = \frac{-V_{out}}{R_2} + \frac{-V_{out}}{\left(\frac{1}{j\omega C_2}\right)} = -V_{out}\left(\frac{1}{R_2}+j\omega C_2\right)$$

$$\frac{V_{out}}{V_{in}} = \boxed{\frac{-1}{\left(R_1+\frac{1}{j\omega C_1}\right)\left(\frac{1}{R_2}+j\omega C_2\right)}}$$

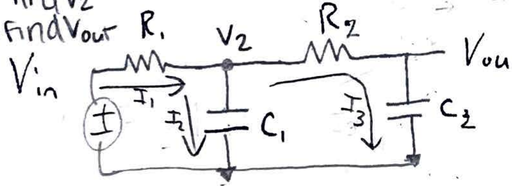


$$I = \frac{V_{in}-V_{out}}{R_1} = \frac{V_{out}}{\left(\frac{1}{j\omega C_1}\right)} = V_{out}\, j\omega C_1$$

$$V_{in} = V_{out}\, j\omega C_1 R_1 + V_{out} = V_{out}(1+j\omega C_1 R_1)$$

$$\frac{V_{out}}{V_{in}} = \boxed{\frac{1}{1+j\omega C_1 R_1}}$$

1. find V_2
2. Find V_{out}



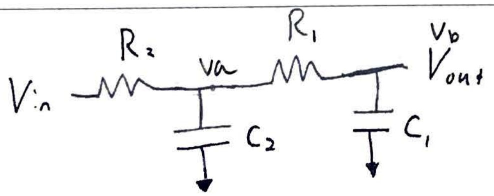
same circuit equivalent

$$Z_1 = R_1 \quad Z_2 = \frac{1}{j\omega C_1} \quad Z_3 = R_2 + \frac{1}{j\omega C_2}$$

$$\frac{V_{in}-V_2}{Z_1} = \frac{V_2}{\frac{1}{\frac{1}{Z_2}+\frac{1}{Z_3}}} = V_2\left(\frac{1}{Z_2}+\frac{1}{Z_3}\right)$$

$$\frac{V_{in}}{Z_1} = V_2\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)$$

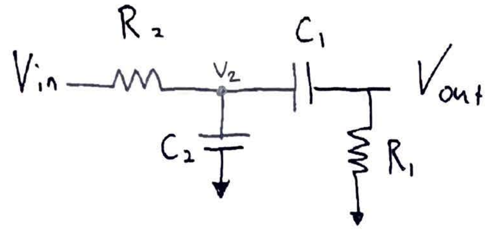
$$\frac{V_2}{V_{in}} = \frac{1}{Z_1\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)}$$



$$\frac{V_{out}}{V_{in}} = \frac{V_2}{V_{in}} \cdot \frac{R_2}{Z_3} = \frac{R_2}{Z_1 Z_3\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)}$$

$$= \frac{R_2}{R_1\left(R_2+\frac{1}{j\omega C_2}\right)\left(\frac{1}{R_1}+j\omega C_1+\frac{1}{R_2+\frac{1}{j\omega C_2}}\right)}$$

$$= \boxed{\frac{j\omega C_2 R_2}{R_1(j\omega C_2 R_2+1)\left(\frac{1}{R_1}+j\omega C_1+\frac{j\omega C_2 R_2+1}{j\omega C_2}\right)}}$$



$$I = \frac{V_{in}-V_2}{R_2} = V_2\left(\frac{1}{\frac{1}{j\omega C_2}} + \frac{1}{\frac{R_1}{j\omega C_1 R_1} + R_1}\right)$$

$$= V_2\left(j\omega C_2 + \frac{j\omega C_1 R_1 + 1}{R_1}\right)$$

$$\frac{V_{in}}{R_2} = V_2\left(j\omega C_2 + \frac{j\omega C_1 R_1 + 2}{R_1}\right)$$

$$\frac{V_2}{V_{in}} = \frac{R_2}{j\omega C_2 + \frac{j\omega C_1 R_1 + 2}{R_1}}$$

$$= \frac{R_1 R_2}{j\omega R_1 (C_1 + C_2) + 2}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{V_2}{V_{in}} \cdot \frac{1}{j\omega C_1}}{\left(\frac{1}{j\omega C_1} + R_1\right)} = \frac{R_1 R_2}{j\omega C_1\left(\frac{1}{j\omega C_1} + R_1\right)\left(j\omega R_1 (C_1 + C_2) + \right.}$$

$$= \boxed{\frac{R_1 R_2}{(1 + j\omega C_1 R_1)(j\omega R_1 (C_1 + C_2) + 2)}}$$

$$-\frac{\frac{1}{j\omega C_2 + \frac{1}{R_2}}}{R_1 + \frac{1}{j\omega C_1}} = \frac{\left(\frac{-R_2}{j\omega C_2 R_2 + 1}\right)}{\left(\frac{R_1}{(1 + j\omega C_1 R_1)}\right)} = \frac{-R_2(1 + j\omega C_1 R_1)}{R_1(1 + j\omega C_2 R_2)}$$