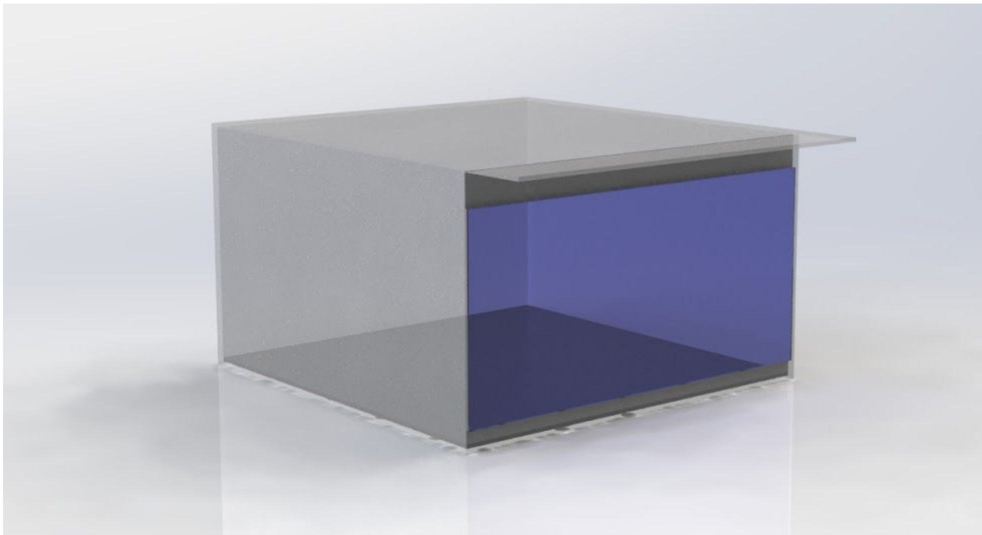
Passive Solar House Project

Lilo Heinrich & Julia Hunt



Background

What is a passive solar house, and why would you build one?

A passive solar house is designed to self-regulate its temperature by storing energy from the sun in a thermal mass during the day and using the heat stored in that thermal mass to keep the house warm at night. Considerations that factor into the decision to build and/or live in a passive solar house include the desire to reduce energy consumption, to save money on heating in the winter, and to avoid burning fossil fuels to reduce carbon footprint and minimize one's impact on climate change. Passive solar houses drastically reduce the amount of energy needed to keep the house warm in the winter.

What are its basic working principles?

How it works (north of the equator):

1) **Let in sunlight [1]:** Set up a house with some large south-facing windows which transmit a high percentage of the sun's thermal radiation to the interior of the building.
2) **Absorb the sun's thermal radiation [1]:** Cover the floor of the house in a layer of material with high thermal absorptivity, as well as decent thermal conductivity.
3) **Store the heat from the sun [1]:** Underneath the layer of thermally absorptive material, install a large thermal mass, ideally with both high specific heat and high density.
    a) This thermal mass will absorb and store heat from the sun throughout the day, and when the sun sets this stored heat will slowly transfer from your thermal mass to the air due to natural convection.

   b) This convection from the thermal mass can be increased by introducing air currents inside your home (i.e. set up a fan to blow across the floor, which you could choose to power using solar panels on your roof)
4) **Insulate the rest of the building [1]:** Line the walls, ceiling, and area underneath the thermal mass with insulation made out of a material with low thermal conductivity.
   a) The insulation will trap heat inside of your home, keeping the indoor environment warm despite the outside temperature.
5) **Control the sun:** Adjusting the temperature.
   How to make your house warmer in the winter:
   a) Uncover the windows or open the blinds all day in the winter to allow in sunlight. Cover the windows at night and enjoy the warmth emanating from your floor.
   b) It's likely that the windows will have a lower thermal resistance than the insulated walls of your home. Therefore, heat will escape more quickly through the windows so it would be prudent to cover them at night to maximize heat storage.
   c) You could also use ENERGY STAR qualified windows, which minimize heat loss by having a maximum heat flow rate, and maximize the amount of sunlight let in by requiring a minimum solar heat gain coefficient [2].

   How to keep your house cool in the summer:
   d) To keep the temperature down in your own house during the summer, we also recommend adding an overhang to the window-facing wall of your house to prevent the summer sun (which shines from a higher angle than the winter sun) from shining through your windows [1].
   e) For passive shading in the summertime, one could plant deciduous trees in front of the windows [3]. This works because the leaves provide shade in the summer and fall off in the winter.

What is your team doing to design/optimize a passive solar house for Massachusetts?

We plan to vary the thickness of the insulation and the ceramic tile in our passive solar house model to optimize heat storage during winter in Massachusetts, while maintaining the internal dimensions of the house.

Design Specifications

The design requirements are:

- Size: 100-400 sq. ft.
- Reasonably comfortable in winter in Boston climate (17-25°C air temperature indoors)
- No direct sunlight through south-facing windows at noon in summer

Model

Design choices

- Insulation thickness = the variable δ_{wall}
- Heat storage unit thickness = the variable δ_{floor}
- Insulation material: base case material, fiberglass
- Heat storage unit material: base case material, recycled ceramic tile
- Window placement: as given in project document
- House shape: as given in project document
- Window orientation: facing south
- We are planning to keep the same overhang dimensions as well

Assumptions

The assumptions we are making in our model:

- The heat capacity of the insulation and the heat capacity of the air are much smaller than the heat capacity of the heat storage unit, so they can be considered as negligible. We will only be accounting for the single lumped heat capacity of the heat storage unit.
- The heat storage unit has a uniform temperature throughout.
- There is no air convection between the outside and inside of the house, the house is assumed to be totally sealed.
- All solar radiation that reaches the window is absorbed by the heat storage unit.
- The heat storage unit has no conduction to the insulation. Heat transfers out of the heat storage unit only by convection into the inside air as if it was suspended in the air.
- Heat loss is the same through all of the walls of the house, as if the heat storage unit's proximity to the underside of the house and the underside of the house's contact with the ground doesn't matter.
- The surface area of both the inner and outer walls can be approximated as the surface area of a rectangular prism with dimensions $(L + \delta_{wall})$, $(D + \delta_{wall})$, $(H + \delta_{wall})$.

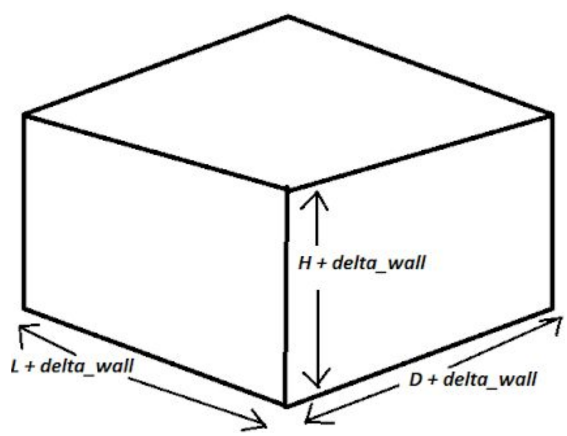


Figure 1: Diagram displaying dimensions used to calculate the surface area of the walls for computing thermal resistances.

Diagrams

In our model, R_1 is the thermal resistance between the floor and the air inside the house. R_2 is the total thermal resistance between the inside air and the outside air, which is calculated as two parallel resistances, through both the wall and the window.

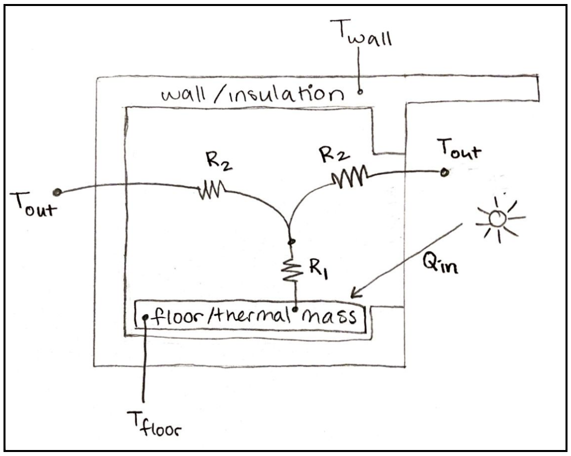


Figure 2: Diagram of heat transfer from house as a set of thermal resistances

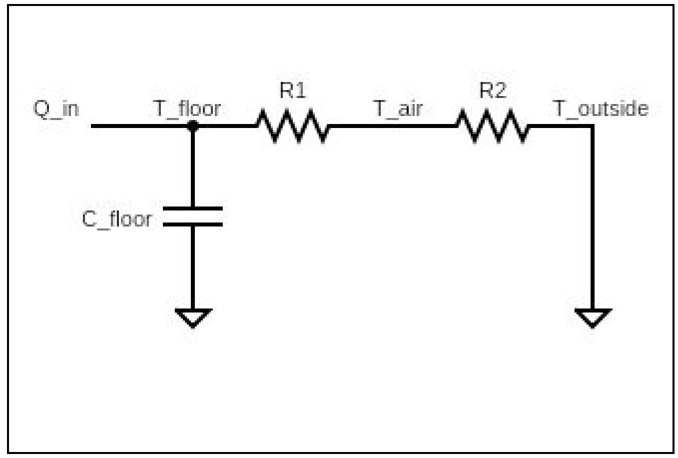


Figure 3: Overall thermal resistance & capacitance network drawn as a circuit diagram

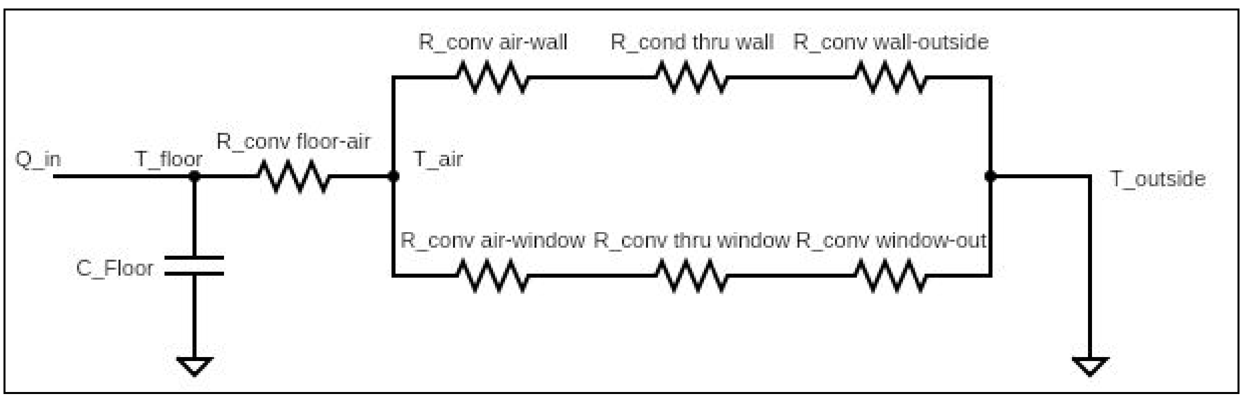


Figure 4: Expanded circuit diagram showing all of the thermal resistances

Independent Variables

$\delta_{wall} = 0.045[m]$ is thickness of wall

$\delta_{floor} = 0.35\ [m]$ is thickness of floor

Defining Constants

All of the constants below were taken from the project specification document defaults.

T_{out} = -3°C is the constant model of outside air temperature

$h_{in} = 15\ [\frac{W}{m^2 \cdot K}]$ is convective + radiative heat transfer coefficient inside house

$h_{out} = 30\ [\frac{W}{m^2 \cdot K}]$ is convective + radiative heat transfer coefficient outside house

$h_{window} = 0.7\ [\frac{W}{m^2 \cdot K}]$ is heat transfer coefficient through double-paned window
$k_{wall} = 0.04\ [\frac{W}{m \cdot K}]$ is thermal conductivity of wall material (fiberglass)
$\rho_{floor} = 3000\ [\frac{kg}{m^3}]$ is the density of the tile
$c_{floor} = 800\ [\frac{J}{kg \cdot K}]$ is the specific heat of the tile
$H = 3.0\ [m]$ is the internal height of the house
$L = 5.1\ [m]$ is the internal length of the house
$D = 5.0\ [m]$ is the internal depth of the house
$A_{floor} = L * D\ [m^2]$ is the surface area of the floor/thermal mass
$A_{window} = 2.6 * D\ [m^2]$ is the surface area of the south-facing window

Dependent Variables

The cross-sectional area of the wall:

$A_{wall} = 2*(L+\delta_{wall})*(D+\delta_{wall}) +$ the area of the floor and ceiling
$2*(L+\delta_{wall})*(H+\delta_{wall}) +$ the area of the east and west walls
$(H+\delta_{wall})*(D+\delta_{wall}) +$ the area of the north wall
$(0.4+0.2)*(D+\delta_{wall})\ [m^2]$ the area of the south facing wall

$q_{solar} = -361 cos(\pi t/(12 * 3600)) + 224\ cos(\pi t/(6 * 3600)) + 210\ [\frac{W}{m^2}]$ is the solar flux through the south-facing window.

$Q_{in} = q_{solar} A_{window}\ [W]$ is the heat absorbed by the thermal mass (ceramic tile floor)
$R_1 = R_{conv\ floor-to-air} \Rightarrow R_1 = \frac{1}{h_{in} A_{floor}}$

$$R_2 = [(\frac{1}{h_{in} A_{wall}} + \frac{\delta_{wall}}{k_{wall} A_{wall}} + \frac{1}{h_{out} A_{wall}})^{-1} + (\frac{1}{h_{in} A_{window}} + \frac{1}{h_{window} A_{window}} + \frac{1}{h_{out} A_{window}})^{-1}]^{-1}$$

$V_{floor} = A_{floor} \delta_{floor}\ [m^3]$ is the volume of tile floor
$C_{floor} = m_{floor} c_{floor} = \rho_{floor} V_{floor} c_{floor}\ [\frac{J}{K}]$ is heat capacity of tile floor

Equations

From the equation $C\frac{dT}{dt} = \frac{dU}{dt} = Q_{in} - Q_{out}$ we derived the ODE for temperature of the floor:

$$C_{floor} \frac{dT_{floor}}{dt} = Q_{in} - \frac{T_{floor} - T_{out}}{R_1 + R_2} \Rightarrow \frac{dT_{floor}}{dT} = \frac{Q_{in}}{C_{floor}} - \frac{T_{floor} - T_{out}}{C_{floor}(R_1 + R_2)}$$

To isolate T_{air} we used the voltage divider equation to set the heat flow from the floor to the air equal to the heat flow from the floor to the outside, analogous to current flow in an actual circuit.

$$\frac{T_{floor} - T_{air}}{R_1} = \frac{T_{floor} - T_{out}}{R_1 + R_2} \Rightarrow T_{air} = \frac{R_1}{R_1 + R_2}(T_{floor} - T_{out})$$

Optimization

Sweep series

We are going to sweep through a range of possible wall thickness and floor thickness combinations, solving for the maximum and minimum temperature after thirty days, to find the optimal pair of values for these variables.

To choose our parameter ranges, we started with 0.05 meter thick tile and insulation, then used trial and error. When the average temperature was too low we increased the upper bound of the insulation thickness. When the temperature fluctuated too much, we increased the upper bound of the floor thickness and vice versa.

In the final version, we set the range of values to sweep for the thickness of the wall to go from 0.005 meters up to 0.15 meters in 0.005 meter increments. For the thickness of the floor, it was from 0.05 meters up to 0.50 meters in 0.01 meter increments.

We then plotted the high and low temperatures for each combination of wall and floor thicknesses, as can be seen below in Figure 5.

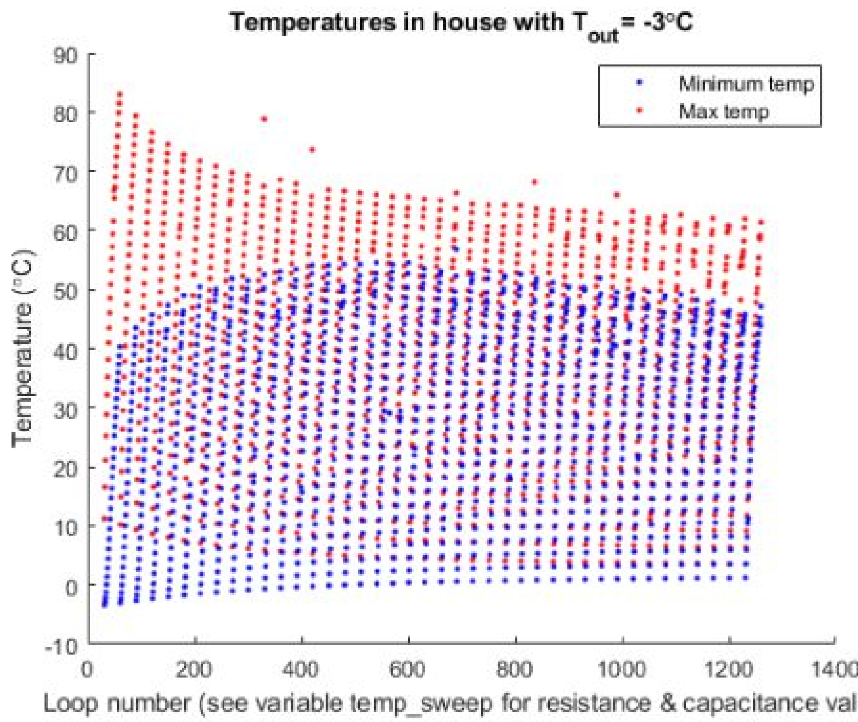


Figure 5: Plot of max & min temperature values inside house for varying capacitance & resistance values. This is the result of sweeping across both variables.

Below are snapshots of how temperature responds with the thickness of the floor and with the thickness of the wall.

As the wall/insulation unit thickness increases, the average temperature in the house increases. This is what we expected to see because the insulation provides the thermal resistance of the house. When we increase the thickness of the insulation, we are decreasing the rate at which heat leaves the house, which in turn, shifts the average temperature inside the house upwards.

As the floor/heat storage unit thickness increases, the temperature fluctuations decrease, and we observe a decrease in the temperature difference between the minimum and maximum temperatures. This is what we expected to see because the floor provides the thermal capacitance that stores heat inside the house, making the house less susceptible to temperature fluctuations.

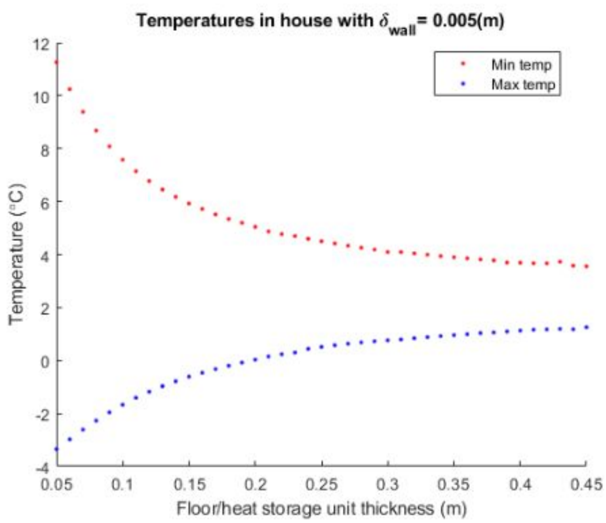


Figure 6: Max & min temps when varying floor thickness (thermal capacitance) with a fixed thermal resistance (wall thickness)

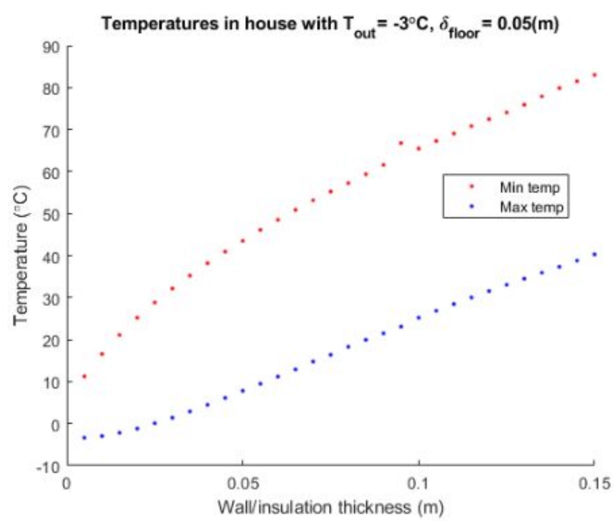


Figure 7: Max & min temps when varying wall thickness (thermal resistance) with a fixed thermal capacitance (floor thickness)

Finding Solutions

After filtering the results of our sweep to find only house designs with a minimum temperature above 17°C and a maximum temperature below 25°C, we found that there were twenty combinations of wall thickness and floor thickness that produced an acceptable result. To narrow down our options, we decided that 17°C is a bit cold for our preferences and increased the minimum temperature up to 19°C, giving ten options to choose from.

The tables below is filtered on a maximum T_{air} of 25°C and minimum T_{air} of 19°C and show the columns in this order: floor thickness in meters, wall thickness in meters, maximum temperature in Celsius, and minimum temperature in Celsius.

δ_{floor} [m]	δ_{wall} [m]	Maximum T_{air} [°C]	Minimum T_{air} [°C]
0.35	0.045	24.4936	19.0313
0.36	0.045	24.5754	18.8489
0.37	0.045	24.5411	19.1172
0.38	0.045	24.2823	19.1914
0.39	0.045	24.2024	19.2463
0.4	0.045	24.2048	19.2764
0.41	0.045	24.1706	19.1927
0.42	0.045	24.1785	19.2429
0.43	0.045	24.0867	19.3317
0.44	0.045	24.1346	19.22
0.45	0.045	23.9292	19.4133

Table 1: Table of acceptable wall & floor thicknesses with the maximum & minimum temperatures they will create

We picked the one with the smallest floor thickness to decrease cost of the materials needed to build the house, giving a final wall thickness of 0.045 meters and floor thickness of 0.35 meters.

Results

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Max air temp: 24.4936 deg C
Min equilibrium air temp: 19.0313 deg C
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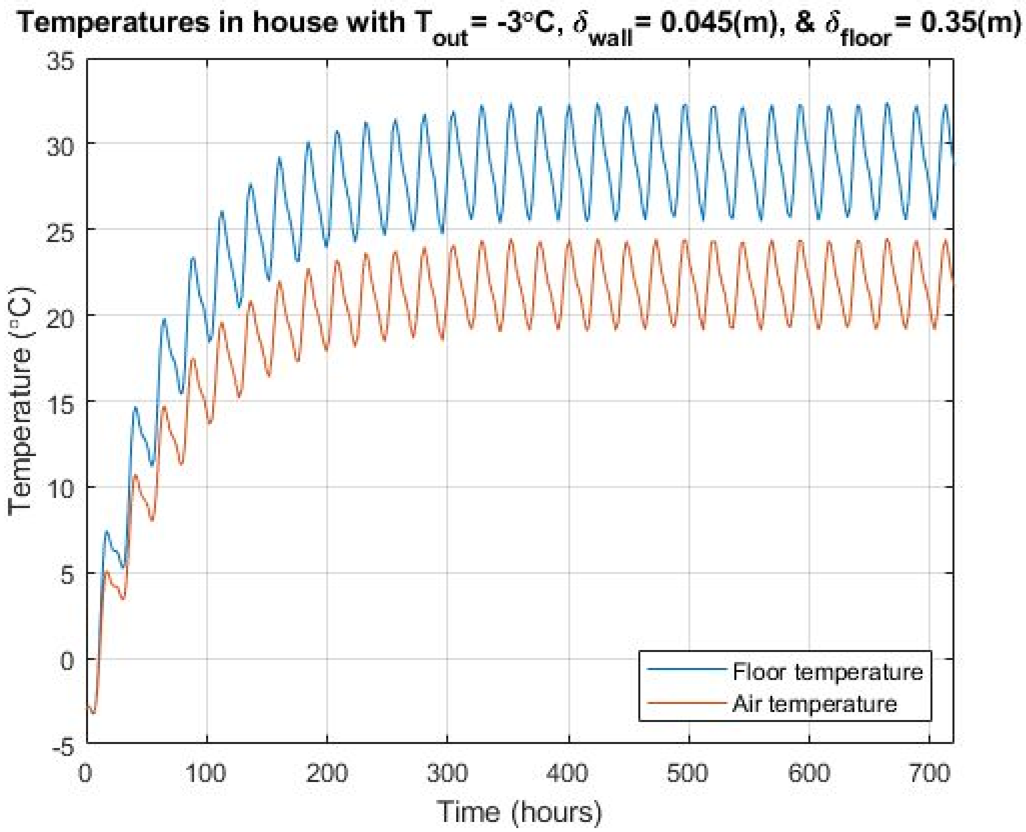

Figure 8: Plot of temperature inside the house over 720 hours (30 days).

As can be seen on the graph above, the temperature of the air in the house initially starts really cold, equal to the outdoors temperature, but warms up to reach a comfortable equilibrium temperature around 22°C within about two weeks. This looks like a successful solar house model, where the temperature remains in a comfortable range yet still fluctuates throughout the day as we expected it to.

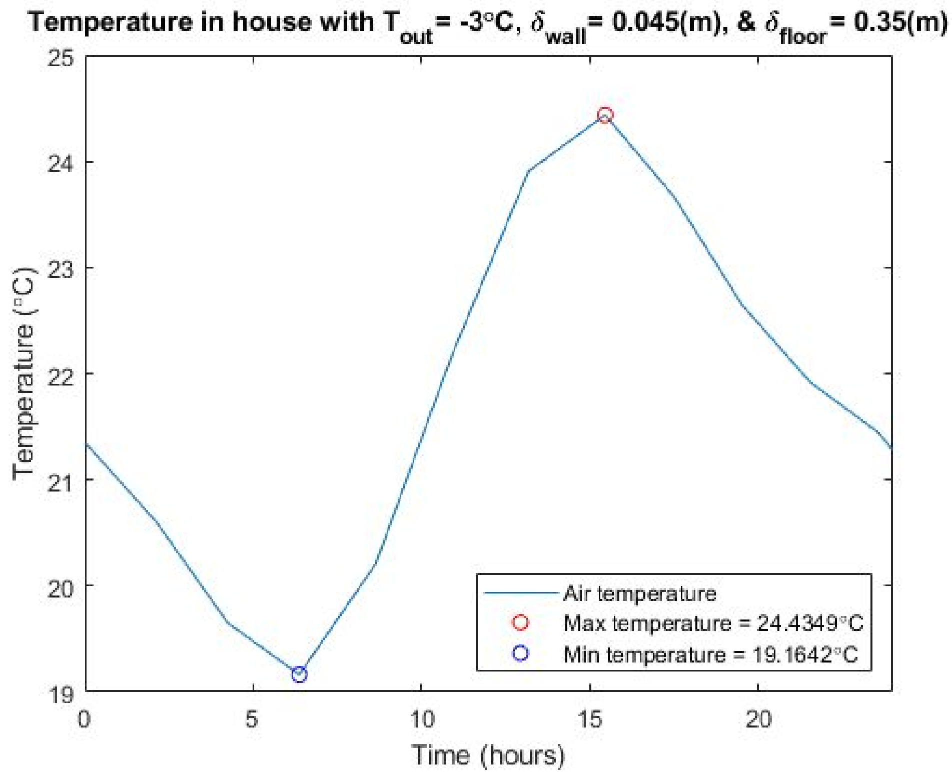

Figure 9: Plot of temperature fluctuations inside the house on Day 19.

This plot shows the temperature fluctuations inside the house over the course of a single day. The maximum temperature is ~24.4°C (76°F), and the minimum temperature is ~19.2°C (66.5°F). These results are absolutely in line with temperature fluctuations inside a normal house.

As a next step, we would like to add complexity and fidelity to the model. For example, we could model the system with a time-varying external temperature, modify our model to vary the angle of the sun as the seasons change to check that the house is comfortable in the summer, and test different building materials to optimize parameters such as price or sustainability.

Discussion

During this project, we were curious about how the temperature in our rooms compared to the temperature in our solar house, so we checked our thermostats. We both found that our room temperatures were around the mid-seventies in Fahrenheit, which translates to about 24°C, within the higher end of the temperature range inside our solar house.

Looking at the low end of the range, 19°C translates to about 66°F, which is liveable if a little bit chilly. There were no combinations of parameter values that gave us a minimum temperature as high as 20°C so this must be a limitation of our model. With the current house shape, material composition, and other modelling assumptions it isn't possible to keep this house *quite* as warm as we might like during the night. The person living in this house might wear a sweater in the evenings or have an extra blanket on their bed in the wintertime.

Other modifications we would like to add would be solar panels to the outside of the house to power any electrical devices inside the house. We'd also consider adding a front door, which increases accessibility to the interior of the structure, and potentially modeling the heat transfer through various furnishings within the house.

Bibliography [4]

[1] "Guide to Passive Solar Home Design," *U.S. Department of Energy*, Oct-2010. [Online]. Available: https://www.energy.gov/sites/prod/files/guide_to_passive_solar_home_design.pdf. [Accessed: 17-Oct-2020].

[2] "Independently Tested and Certified Energy Performance," *ENERGY STAR*, 2019. [Online]. Available: https://www.energystar.gov/products/building_products/residential_windows_doors_and_skylights/independently_tested_certified_energy_performance. [Accessed: 17-Oct-2020].

[3] C. Reardon, "Passive solar heating," *YourHome - Australia's guide to environmentally sustainable homes*, 2013. [Online]. Available: https://www.yourhome.gov.au/passive-design/passive-solar-heating. [Accessed: 17-Oct-2020].

[4] "Citation Styles: APA, MLA, Chicago, Turabian, IEEE: IEEE Style," LibGuides, 29-Sep-2020. [Online]. Available: https://pitt.libguides.com/citationhelp/ieee. [Accessed: 17-Oct-2020].