11/15/23 HW 10 Josiah Schmitz

103. 24

$x(t)=5\cos t \quad y(t)=5\sin t \quad x'(t)=-5\sin t \quad y'(t)=5\cos t$

$\int_0^{\tan^{-1}\frac{4}{3}} \langle -10\cos t+10\sin t, 10\cos t+10\sin t\rangle \cdot \langle -5\sin t, 5\cos t\rangle \, dt$

$=\int_0^{\tan^{-1}\frac{4}{3}} 50(-\cos t\sin t-\sin^2 t+\cos^2 t+\sin t\cos t)\,dt = \int_0^{\tan^{-1}\frac{4}{3}} 50(\cos^2 t-\sin^2 t)\,dt$

$=50\left(\frac{1}{2}(\cos t\sin t+t)+\frac{1}{4}(\sin 2t-2t)\right)\Big]_0^{\tan^{-1}\frac{4}{3}} = 50\left[\frac{1}{2}\cos t\sin t+\frac{1}{4}\sin 2t\right]_0^{\tan^{-1}\frac{4}{3}}$

$=50\left(\frac{12}{25}-0\right)=24$

107. Not conservative

$\frac{d}{dy}(-y+e^x\sin y)=e^x\cos y-1 \quad \frac{d}{dx}((x+2)e^x\cos y)=3e^x\cos y+xe^x\cos y$

$e^x\cos y-1 \neq (x+3)e^x\cos y$

109. Conservative; $3x^2+5xy+2y^2$

$\frac{d}{dy}(6x+5y)=5 \quad \frac{d}{dx}(5x+4y)=5 \quad 5=5 \quad \int(6x+5y)\,dx=3x^2+5xy+h(y)$

$\frac{d}{dy}(3x^2+5xy+h(y))=5x+h'(y) \quad 5x+h'(y)=5x+4y,\ h'(y)=4y \quad h(y)=2y^2$

108. Not conservative

$\frac{d}{dy}(e^{2x}\sin y)=e^{2x}\cos y \quad \frac{d}{dx}(e^{2x}\cos y)=2e^{2x}\cos y \quad e^{2x}\cos y\neq 2e^{2x}\cos y$

143. πa^2, 0

$\int_0^{\pi}\langle -a\sin t, a\cos t\rangle\cdot\langle -a\sin t, a\cos t\rangle\,dt=\int_0^{\pi} a^2(\sin^2 t+\cos^2 t)\,dt$

$=a^2 t\Big]_0^{\pi}=\pi a^2 \quad \Phi=\iint_D(\nabla\cdot F)\,dA=0$

144. $\frac{1}{3}\sin 3$

$dx=dt \quad dy=2t\,dt \quad \int_0^1(\cos t\cos 2t-\sin t\sin 2t)\,dt=\frac{1}{3}\sin 3t\Big]_0^1=\frac{1}{3}\sin 3$

6.4

149. $\frac{1}{12}$

$P(x,y) = (\sin x \cos y)$ $Q(x,y) = (xy + \cos x \sin y)$ $P_y = -\sin x \sin y$

$Q_x = y - \sin x \sin y$ $\int_0^1 \int_x^{\sqrt{x}} (y - \sin x \sin y + \sin y \sin x)\,dy\,dx$

$\int_x^{\sqrt{x}} y\,dy = \frac{1}{2}y^2\Big]_x^{\sqrt{x}} = \frac{1}{2}x - \frac{1}{2}x^2$ $\int_0^1 \frac{1}{2}x - \frac{1}{2}x^2\,dx = \left(\frac{1}{4}x^2 - \frac{1}{6}x^3\right)\Big]_0^1 = \frac{1}{12}$

152. $\frac{-1}{6}$

$P_y = x$ $Q_x = 0$ $\int_0^1 \int_0^y -x\,dx\,dy$ $\int_0^y -x\,dx = -\frac{1}{2}x^2\Big]_0^y = -\frac{1}{2}y^2$

$\int_0^1 -\frac{1}{2}y^2\,dy = -\frac{1}{6}y^3\Big]_0^1 = \frac{-1}{6}$

158. -1

$P_y = 2(1 + x\cos y)$ $Q_x = (2x\cos y)$ $\int_0^1 \int_0^x 2(x\cos y - 1 - x\cos y)\,dy\,dx$

$\int_0^x -2\,dy = -2y\Big]_0^x = -2x$ $\int_0^1 -2x\,dx = -x^2\Big]_0^1 = -1$

172. -2π

$x = a\cos t$ $y = a\sin t$ $\int_C F \cdot dr = -\int_{C_1} F \cdot dr$

$\int_{C_1} F \cdot dr = \int_0^{2\pi} \left\langle -\frac{\sin t}{a}, \frac{-\cos t}{a} \right\rangle \cdot \langle -a\sin t, -a\cos t \rangle\,dt = \int_0^{2\pi} \sin^2 t + \cos^2 t\,dt$

$= \int_0^{2\pi} dt = 2\pi$

173. 2π

$\int_0^{2\pi} \left\langle -\frac{\sin t}{a}, \frac{-\cos t}{a} \right\rangle \cdot \langle -a\sin t, -a\cos t \rangle\,dt = \int_0^{2\pi} \sin^2 t + \cos^2 t\,dt$

$= \int_0^{2\pi} dt = 2\pi$

184. 8π

$P_y = x^2$ $Q_x = -y^2$ $\iint_D (-y^2 - x^2)\,DA$ $x = r\cos\theta$ $y = r\sin\theta$ $0 \le r \le 2$
$0 \le \theta \le 2\pi$ $\int_0^{2\pi}\int_0^2 (-r^2\sin^2\theta - r^2\cos^2\theta)\,r\,dr\,d\theta = \int_0^{2\pi}\int_0^2 -r^3\,dr\,d\theta$
$\int_0^2 -r^3\,dr = -\frac{1}{4}r^4\Big]_0^2 = -4$ $\int_0^{2\pi} 4\,d\theta = 4\theta\Big]_0^{2\pi} = 8\pi$

201. 0

$P = -x$ $Q = y$ $P_x = -1$ $Q_y = 1$ $\iint_D 0\,dA = 0$

6.5

214. $(-y^2\cos z)i + (6xyz - e^{2z})j - 3xz^2k$

$R_y = 0$ $R_x = e^{2z}$ $Q_z = y^2\cos z$ $Q_x = 0$ $P_y = 3xz^2$ $P_z = 6xyz$
$-y^2\cos z\, i + (6xyz - e^{2z})j + (-3xz^2)k$

217. $i + j + k$

$R_y = 0$ $R_x = -1$ $Q_z = -1$ $Q_x = 0$ $P_y = -1$ $P_z = 0$

226. 3

$P_x = 1$ $Q_y = 1$ $R_z = 1$ $1 + 1 + 1 = 3$

232. Harmonic

$u_{xx} = e^{-x}(\cos y - \sin y)$ $u_{yy} = e^{-x}(\sin y - \cos y)$ $u_{zz} = 0$
$u_{xx} + u_{yy} + u_{zz} = e^{-x}(\cos y - \sin y + \sin y - \cos y) = 0$

233. Harmonic

$f_{xx} = \frac{2x^2-y^2-z^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}$ $f_{yy} = \frac{2y^2-x^2-z^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}$ $f_{zz} = \frac{2z^2-y^2-x^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}$

$f_{xx} + f_{yy} + f_{zz} = 0$

242. 0

$F = \frac{-y}{x^2+y^2} i + \frac{x}{x^2+y^2} j$ $Q_x = \frac{y^2-x^2}{(x^2+y^2)^2}$ $P_y = \frac{y^2-x^2}{(x^2+y^2)^2}$

$Q_x - P_y = 0$

246. 8

$P_x = yz$ $Q_y = 1$ $R_z = 1$ $\text{div } F = yz+1+1$

$\text{div } F(1,2,3) = (2)(3)+1+1 = 8$

252. $-3i + \frac{2}{e^6}k$

$R_y = ze^{yz}$ $R_x = 0$ $Q_z = xe^{xz}$ $Q_x = ze^{xz}$ $P_y = -xe^{-xy}$ $P_z = 0$

$(ze^{yz} - xe^{xz})i + 0j + (ze^{xz} + xe^{-xy})k$

$\text{curl } F(3,2,0) = (0-3)i + (0+2e^{-6})k = -3i + 2e^{-6}k$