Calculus 3

10/4/23 — 4.1 — Josiah Schmidt

6. $x^2+y^2 \geq 4$

7. $y^2 > x$

14. $y^2-x^2=1$, hyperbola

16. $x^2+y^2=4$, circle; $x^2+y^2=9$, circle

20. $x^2-y=1$, parabola; $x^2-y=2$, parabola

25. $xy-x=-2$, $xy-x=0$, $xy-x=2$

48. $x-2y+z=4$, plane

49. $x^2+y^2+z^2=9$, sphere with radius of 3

50. $x^2+y^2-z^2=-4$, hyperboloid of two sheets

53. $4x^2+y^2=1$

$1-4x^2-y^2=C \quad 1-4(0)-1^2=C \quad 1-1=C \quad C=0 \quad 1-4x^2-y^2=0$

$1=4x^2+y^2$

4.3

120. $8x^7e^{3y}$, $3x^8e^{3y}$

$\frac{dz}{dx}(x^8e^{3y}) = 8x^7e^{3y}$ $\frac{dz}{dy}(x^8e^{3y}) = 3x^8e^{3y}$

121. $\frac{6x^5}{x^6+y^4}$, $\frac{4y^3}{x^6+y^4}$

$\frac{dz}{dx}(\ln(x^6+y^4)) = \frac{\frac{d}{dx}(x^6+y^4)}{x^6+y^4} = \frac{6x^5}{x^6+y^4}$

$\frac{dz}{dy}(\ln(x^6+y^4)) = \frac{\frac{d}{dy}(x^6+y^4)}{x^6+y^4} = \frac{4y^3}{x^6+y^4}$

124. $\frac{1}{x}$, $\frac{-1}{y}$

$\frac{dz}{dx}(\ln\frac{x}{y}) = \frac{\frac{d}{dx}(\frac{x}{y})}{\frac{x}{y}} = \frac{y(\frac{1}{y})}{x} = \frac{1}{x}$

$\frac{dz}{dy}(\ln\frac{x}{y}) = \frac{\frac{d}{dy}(\frac{x}{y})}{\frac{x}{y}} = \frac{y(\frac{-x}{y^2})}{x} = \frac{\frac{-x}{y}}{x} = \frac{-1}{y}$

126. $2\cosh(2x+3y)$, $3\cosh(2x+3y)$

$\frac{dz}{dx}(\sinh(2x+3y)) = 2\cosh(2x+3y)$ $\frac{dz}{dy}(\cosh(2x+3y)) = 3\cosh(2x+3y)$

128. $\frac{-1}{4}$, $\frac{1}{4}$

$f_x(\frac{xy}{x-y}) = \frac{f_x(xy)(x-y)-(xy)f_x(x-y)}{(x-y)^2} = \frac{y(x-y)-xy}{(x-y)^2} = \frac{y(-y)}{(x-y)^2} = \frac{-y^2}{(x-y)^2}$

$f_y(\frac{xy}{x-y}) = \frac{f_y(xy)(x-y)-(xy)f_y(x-y)}{(x-y)^2} = \frac{x(x-y)+xy}{(x-y)^2} = \frac{x(x)}{(x-y)^2} = \frac{x^2}{(x-y)^2}$

$f_x(2,-2) = \frac{-4}{(2+2)^2} = \frac{-4}{16} = \frac{-1}{4}$ $f_y(2,-2) = \frac{4}{(2+2)^2} = \frac{1}{4}$

135. $\frac{1}{(x-y)^2}$

$\frac{dz}{dx} = \frac{1}{x-y}$ $\frac{d}{dy}(\frac{1}{x-y}) = \frac{1}{(x-y)^2}$

138. $\frac{d^2z}{dxdy} = e^x \sec^2 y$; $\frac{d^2z}{dydx} = e^x \sec^2 y$

$\frac{dz}{dy} = e^x \sec^2 y$ $\frac{d^2z}{dxdy} = \frac{d}{dx}(e^x \sec^2 y) = e^x \sec^2 y$

$\frac{dz}{dx} = e^x \tan y$ $\frac{d^2z}{dydx} = \frac{d}{dy}(e^x \tan y) = e^x \sec^2 y$

143. $f_{xyz} = 6xy^2 - 18yz^2$

$f_x = 2xy^3z - 3y^2z^3 + 10xz$ $f_{xy} = 6xy^2z - 6yz^3$ $f_{xyz} = 6xy^2 - 18yz^2$

145. $(1,1)$ and $(\frac{1}{4}, \frac{1}{2})$

$f_x = 2x + 1 - 3y$ $f_y = -3x + 3y^2$ $2x + 1 - 3y = 0$

$-3x + 3y^2 = 0$

$x = \frac{3}{2}y - \frac{1}{2}$ $-3(\frac{3}{2}y - \frac{1}{2}) + 3y^2 = 0$ $3y^2 - \frac{9}{2}y + \frac{3}{2} = 0$

$3(y^2 - \frac{3}{2}y + \frac{1}{2}) = 0$ $y = 1$ and $\frac{1}{2}$ $x = \frac{3}{2} - \frac{1}{2} = 1$ $x = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

149.

$\frac{dz}{dx} = e^x \sin y$ $\frac{d^2z}{dx^2} = e^x \sin y$ $\frac{dz}{dy} = e^x \cos y$ $\frac{d^2z}{dy^2} = -e^x \sin y$

$\frac{d^2z}{dx^2} + \frac{d^2z}{dy^2} = e^x \sin y - e^x \sin y = 0$

157. $\frac{df}{dx} = e^{-2t} \cos x \sin y$ $\frac{d^2f}{dx^2} = -e^{-2t} \sin x \sin y$ $\frac{df}{dy} = e^{-2t} \sin x \cos y$

$\frac{d^2f}{dy^2} = -e^{-2t} \sin x \sin y$ $\frac{df}{dt} = -2e^{-2t} \sin x \sin y$

$-2e^{-2t} \sin x \sin y = -e^{-2t} \sin x \sin y - e^{-2t} \sin x \sin y$

158. $f_t = \cos(x+t)$ $f_{tt} = -\sin(x+t)$ $f_x = \cos(x+t)$ $f_{xx} = -\sin(x+t)$

$f_{tt}(\sin(x+t)) = f_{xx}(\sin(x+t))$ $-\sin(x+t) = -\sin(x+t)$

$f_t = -\cos(x-t)$ $f_{tt} = -\sin(x-t)$ $f_x = \cos(x-t)$ $f_{xx} = -\sin(x-t)$

$f_{tt}(\sin(x-t)) = f_{xx}(\sin(x-t))$ $-\sin(x-t) = -\sin(x+t)$