Discrete Structures

11/16/23

HW 9

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5.1.57 $\sum_{j=0}^{n-2} \frac{j+1}{(n-j-1)^2}$

$i = j+1 \quad j = n-1 \quad j = 0 \quad \sum_{j=0}^{n-3} \frac{j+1}{(n-j-1)^2}$

5.2.12 Show $P(1)$ is true: $\frac{1}{1\cdot 2} = \frac{1}{1+1} = \frac{1}{2}$
Show that for $k \geq 1$, if $P(k)$ is true then $P(k+1)$ is true: If
$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{1}{k(k+1)} = \frac{k}{k+1}$ is true, $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$ is true.
$P(k+1) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{k+2}{k(k+1)(k+2)} + \frac{k}{k(k+1)(k+2)}$
$= \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{2(k+1)}{k(k+1)(k+2)} = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{2}{k(k+2)} = 1 + \frac{2}{k(k+2)} = \frac{k(k+2)}{k(k+2)} + \frac{2}{k(k+2)}$
$= \frac{k+1}{k+2}$

5.2.27 $5^{k+1} - \frac{125}{4}$

$\frac{5^{k+1}-1}{5-1} - (5^2+5+1) = \frac{5^{k+1}-1}{4} - 31 = \frac{1}{4}(5^{k+1}-1) - 31 = 5^{k+1} - \frac{125}{4}$

5.3.5 a. $4 < 3!$; This is true because $4 < 6$. b. $2^k < (k+1)!$
c. $2^{k+1} < (k+2)!$ d. Must show: If k is an integer such that $k \geq 0$ and $2^k < (k+1)!$, then $2^{k+1} < (k+2)!$.

5.3.9 $P(0)$ is "$7^0 - 1$ is divisible by 6", which is true because $7^0 - 1 = 6$ which is divisible by 6. Must show "$7^{k+1} - 1$ is divisible by 6",
$7^{k+1} - 1 = 7^k \cdot 7 - 1 = 7^k \cdot (6+1) - 1 = 7^k \cdot 6 + (7^k - 1) \quad 7^k - 1 = 6r$
$7^{k+1} - 1 = 7^k \cdot 6 + 6r = 6(7^k + 6r) \therefore 7^{k+1} - 1$ is divisible by 6.