100. $e^t i - \cos t\, j + \frac{1}{2}\ln(2t-1)k + C$

$$\int \left(e^t + \sin t + \frac{1}{2t-1}\right)dt = e^t - \cos t + \frac{1}{2}\ln(2t-1) + C$$

101. $\frac{3}{4} i + \ln(2)\, j + (1 - \frac{1}{e})k$

$$\int_0^1 \sqrt[3]{t}\, dt = \frac{3}{4}t^{\frac{4}{3}}\Big]_0^1 = \frac{3}{4} \quad \int_0^1 \frac{1}{t+1}dt = \ln(t+1)\Big]_0^1 = \ln 2 \quad \int_0^1 e^{-t}dt = -e^{-t}\Big]_0^1 = \frac{-1}{e} + 1$$

3.3

104. $\sqrt{29}\,\pi$

$f'(t) = 2\cos t \quad g'(t) = 5 \quad h'(t) = -2\sin t$

$s = \int_0^{\pi} \sqrt{4\cos^2 t + 25 + 4\sin^2 t}\; dt = \int_0^{\pi} \sqrt{4(\cos^2 t + \sin^2 t) + 25}\, dt$

$= \int_0^{\pi} \sqrt{4+25}\; dt = \sqrt{29}\,t\Big]_0^{\pi} = \sqrt{29}\,\pi$

132. $\frac{1}{5}$; No.

$r'(t) = -5\sin t\, i + 5\cos t\, j \quad T(t) = \frac{\langle -5\sin t, 5\cos t\rangle}{\sqrt{25(\sin^2 t + \cos^2 t)}} = \langle -\sin t, \cos t\rangle$

$T'(t) = \langle -\cos t, -\sin t\rangle \quad k = \frac{\|\langle -\cos t, -\sin t\rangle\|}{\|\langle -5\sin t, 5\cos t\rangle\|} = \frac{\sqrt{\cos^2 t + \sin^2 t}}{\sqrt{25(\sin^2 t + \cos^2 t)}} = \frac{1}{5}$

133. $\frac{1}{2}$

$y' = 1 - \frac{1}{2}x \quad y'' = -\frac{1}{2} \quad k = \frac{\frac{1}{2}}{(1 + (1 - \frac{1}{2}x)^2)^{\frac{3}{2}}} = \frac{1}{2(1 + (\frac{1}{4}x^2 - x + 1))^{\frac{3}{2}}} = \frac{1}{\frac{1}{2}x^2 - 2x + 4}$

$k(2) = \frac{1}{2 - 4 + 4} = \frac{1}{2}$

134. $\frac{1}{\sqrt{2}}$

$y' = x^2 \quad y'' = 2x \quad k = \frac{|2x|}{(1 + x^4)^{\frac{3}{2}}} \quad k(1) = \frac{2}{2^{\frac{3}{2}}} = \frac{1}{\sqrt{2}}$

3.4

160. $v(t) = 2ti + j$, $a(t) = 2i$, $s(t) = \sqrt{4t^2+1}$

$v(t) = r'(t) = 2ti + j$ $a(t) = v'(t) = 2i$ $s(t) = \|v(t)\| = \sqrt{(2t)^2+1}$
$= \sqrt{4t^2+1}$ =

183. $r(t) = 0i + (\frac{1}{6}t^3 + \frac{9}{2}t - \frac{14}{3})j + (\frac{1}{6}t^3 - \frac{1}{2}t + \frac{1}{3})k$

$a(t) = tj + tk$ $\int a(t)\,dt = v(t) = \frac{1}{2}t^2 j + \frac{1}{2}t^2 k + C$ $v(1) = 5j$
$\frac{1}{2}j + \frac{1}{2}k + C = 5j$ $C = \frac{9}{2}j - \frac{1}{2}k$ $v(t) = \frac{1}{2}t^2 j + \frac{1}{2}t^2 k + \frac{9}{2}j - \frac{1}{2}k$
$\int v(t)\,dt = r(t) = \frac{1}{6}t^3 j + \frac{1}{6}t^3 k + \frac{9}{2}tj - \frac{1}{2}tk + C$
$\frac{1}{6}j + \frac{1}{6}k + \frac{9}{2}j - \frac{1}{2}k + C = 0i + 0j + 0k$ $C = \frac{-14}{3}j + \frac{1}{3}k$
$r(t) = 0i + (\frac{1}{6}t^3 + \frac{9}{2}t - \frac{14}{3})j + (\frac{1}{6}t^3 - \frac{1}{2}t + \frac{1}{3})k$

194. $r(t) = (\frac{1}{2}t^2 + 2)i + (e^t + t - 1)j$

$\int a(t)\,dt = v(t) = ti + e^t j + C$ $0i + j + C = 2j$ $C = j$
$v(t) = ti + e^t j + j$ $\int v(t)\,dt = r(t) = \frac{1}{2}t^2 i + e^t j + tj + C$
$0i + j + 0i + C = 2i$ $C = 2i - j$ $r(t) = (\frac{1}{2}t^2 + 2)i + (e^t + t - 1)j$

Calculus 3

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6. a. $\langle -3, 10 \rangle$ b. $\langle -3, 10 \rangle$ c. Yes d. $\langle 2, 4t+4 \rangle$

$r(t) = \langle \lim_{t\to -3} (t), \lim_{t\to -3} (t^2+1) \rangle = \langle -3, 10 \rangle \quad \lim_{t\to -3} r(t) = r(-3)$

$r(t+2) - r(t) = \langle t+2, t^2+4t+5 \rangle - \langle t, t^2+1 \rangle = \langle 2, 4t+4 \rangle$

10. $\langle 0, \frac{2}{3}, \frac{\pi}{2} \rangle$

$\lim_{t\to\infty} e^{-2t} = 0 \quad \lim_{t\to\infty} \frac{2t+3}{3t-1} = \frac{2}{3} \quad \lim_{t\to\infty} (\arctan 2t) = \frac{\pi}{2}$

13. Does not exist

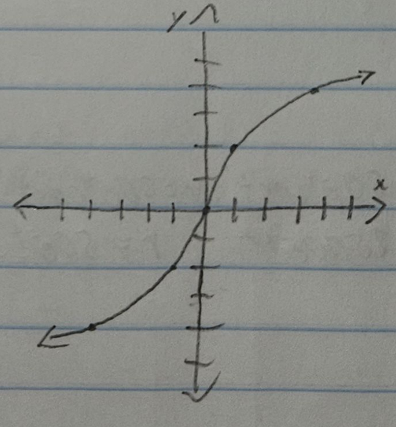
$\lim_{t\to\infty} 2e^{-t} = 0 \quad \lim_{t\to\infty} e^{-t} = 0 \quad \lim_{t\to\infty} \ln(t-1) = DNE$

14. A straight lines starting at $(1,2,-1)$ extending in the direction of vector $\langle 1,5,6 \rangle$

$P = (1,2,-1) \quad V = \langle 1,5,6 \rangle$

23. $y = 2\sqrt[3]{x}$

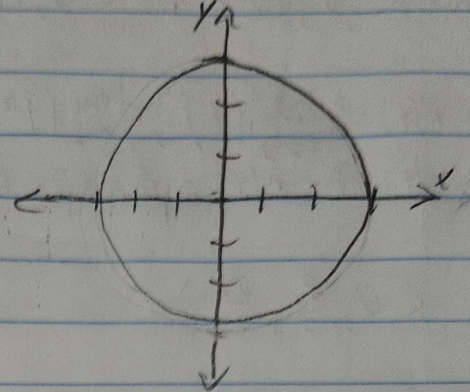
$x = t^3 \quad y = 2t \quad t = \sqrt[3]{x} \quad y = 2\sqrt[3]{x}$



25. $x^2+y^2=9$

$x=3\cos t \quad y=3\sin t$

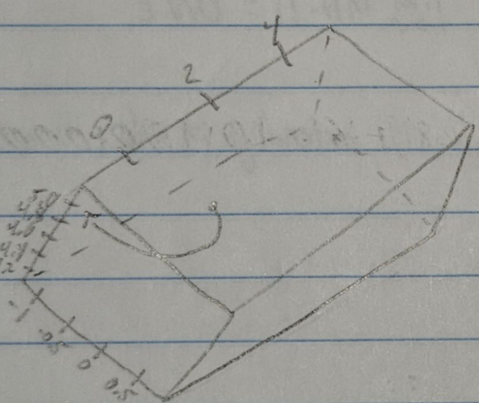
$x^2+y^2=9\cos^2 t+9\sin^2 t=9(1)=9$



34. $\lim_{t\to\infty} r(t)=5k$

$\lim_{t\to\infty} 50e^{-t}\cos t=0 \quad \lim_{t\to\infty} 50e^{-t}\sin t=0 \quad \lim_{t\to\infty} 5-5e^{-t}=5$

35.



36. $x=50e^{-t}\cos t \quad y=50e^{-t}\cos t \quad r^2=x^2+y^2=2500e^{-2t}\cos^2 t+2500e^{-2t}\sin^2 t$

$=2500e^{-2t} \quad r=50e^{-t} \quad 5-\frac{r}{10}=5-\frac{50e^{-t}}{10}=5-5e^{-t}=z$

3.2

52. $9i + 4j - k$

$$r'(t) = 9t^2 i + 4t j - \frac{1}{t^2}k \quad r'(1) = 9i + 4j - k$$

56. $T(t) = \frac{-\sin t}{\sqrt{1+\cos^2 t}} i + \frac{\cos t}{\sqrt{1+\cos^2 t}} j + \frac{\cos t}{\sqrt{1+\cos^2 t}} k$

$$r'(t) = -\sin(t) i + \cos(t) j + \cos(t) k \quad \|r'(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2 + (\cos t)^2}$$
$$= \sqrt{\sin^2 t + \cos^2 t + \cos^2 t} = \sqrt{1 + \cos^2 t}$$

63. $900t^7 + 16t$

$$r'(t) = -15t^4 i + 5j + 4t k \quad r''(t) = -60t^3 i + 4k$$
$$r'(t) \cdot r''(t) = 900t^7 + 16t$$

94. $-2\cos t(t - \frac{1}{t}) j + 2\sin t(t - \frac{1}{t}) k$

$$\begin{vmatrix} i & j & k \\ t & 2\sin t & 2\cos t \\ \frac{1}{t} & 2\sin t & 2\cos t \end{vmatrix} = (4\sin t\cos t - 4\sin t\cos t) i - (2t\cos t - \frac{2\cos t}{t}) j + (2t\sin t - \frac{2\sin t}{t}) k = -2\cos t(t - \frac{1}{t}) j + 2\sin t(t - \frac{1}{t}) k$$

95. $\langle 0, 2\sin t(t - \frac{1}{t}) - 2\cos t(1 + \frac{1}{t^2}), 2\sin t(1 + \frac{1}{t^2}) + 2\cos t(t - \frac{2}{t}) \rangle$

$$r'(t) = i + 2\cos t j - 2\sin t k \quad u'(t) = \frac{-1}{t^2} i + 2\cos t j - 2\sin t k$$

$$r'(t) \times u(t) = \begin{vmatrix} i & j & k \\ 1 & 2\cos t & -2\sin t \\ \frac{1}{t} & 2\sin t & 2\cos t \end{vmatrix} = (4\cos^2 t + 4\sin^2 t) i - (2\cos t + \frac{2\sin t}{t}) j + (2\sin t - \frac{2\cos t}{t}) k = 4i - 2(\cos t + \frac{\sin t}{t}) j + 2(\sin t - \frac{\cos t}{t}) k$$

$$r(t) \times u'(t) = \begin{vmatrix} i & j & k \\ t & 2\sin t & 2\cos t \\ \frac{-1}{t^2} & 2\cos t & -2\sin t \end{vmatrix} = (-4\sin^2 t - 4\cos^2 t) i - (-2t\sin t + \frac{2\cos t}{t^2}) j + (2t\cos t + \frac{2\sin t}{t^2}) k = -4i + 2(t\sin t - \frac{\cos t}{t^2}) j + 2(t\cos t + \frac{\sin t}{t^2}) k$$

$$r'(t) \times u(t) + r(t) \times u'(t) = 0i + (2\sin t(t - \frac{1}{t}) - 2\cos t(1 + \frac{1}{t^2})) j + (2\sin t(1 + \frac{1}{t^2}) + 2\cos t(t - \frac{2}{t})) k$$