Calculus 3

10/25/23 HW 7 Josiah Schmitz

318. $f(0,0)=1$ is a saddle point. $f(\frac{4}{3},\frac{4}{3})=\frac{59}{27}$ is a local max.

$f_x=-3x^2+4y$ $f_y=4x-4y$ $-3x^2+4y=0$ $4x-4y=0$
$-3x^2+4x=0$ $x=0$ and $\frac{4}{3}$ $y=0$ and $\frac{4}{3}$ $(0,0)$ $(\frac{4}{3},\frac{4}{3})$
$f_{xx}=-6x$ $f_{xy}=4$ $f_{yy}=-4$ $D=f_{xx}(0,0)\cdot f_{yy}(0,0)-(f_{xy}(0,0))^2$
$=0-16=-16$ $f(0,0)=1$ $D=f_{xx}(\frac{4}{3},\frac{4}{3})\cdot f_{yy}(\frac{4}{3},\frac{4}{3})-(f_{xy}(\frac{4}{3},\frac{4}{3}))^2$
$=(-8)(-4)-16=16$ $f(\frac{4}{3},\frac{4}{3})=\frac{59}{27}$

322. Test fails, $f(x,y)>0$ for all non-zero (x,y), so $(0,0)$ is a local min.

$f(x,y)=8x^2y+8xy^2+7$ $f_x=16xy+8y^2$ $f_y=16xy+8x^2$
$16xy+8y^2=0$ $16xy=-8y^2$ $x=-\frac{1}{2}y$ $16(-\frac{1}{2}y)y+8(-\frac{1}{2}y)^2=0$
$-8y^2+2y^2=0$ $-6y^2=0$ $y=0$ $x=0$
$f_{xx}=16y$ $f_{xy}=16x+16y$ $f_{yy}=16x$
$D=0\cdot 0-0=0$

324. $f(10,5)$ is a local min.

$f_x=3x^2-300$ $f_y=3y^2-75$ $3x^2-300=0$ $3y^2-75=0$
$x=10$ $y=5$ $f_{xx}=6x$ $f_{xy}=0$ $f_{yy}=6y$
$D=(60)(30)-0=1800$ $f_{xx}(10,5)=60$ $f(10,5)=$

333. $(-1,0)$ is a local max.

$f_x=-(2x+2)e^{-(x^2+y^2+2x)}$ $f_y=-(2y)e^{-(x^2+y^2+2x)}$
$-(2x+2)e^{-(x^2+y^2+2x)}=0$ $x=-1$ $-(2y)e^{-(x^2+y^2+2x)}=0$ $y=0$
$f_{xx}=(4x^2+8x+2)e^{-(x^2+y^2+2x)}$ $f_{xy}=(4x+4)ye^{-(x^2+y^2+2x)}$
$f_{yy}=(4y^2-2)e^{-(x^2+y^2+2x)}$ $D=(-2e)(-2e)-0=4e$ $f_{xx}(-1,0)=-2e$

339. $(\frac{1}{4}, \frac{1}{2})$ is a local max, and $(1,1)$ is a saddle point.

$f_x = 2x+1-3y$ $f_y = -3x+3y^2$ $2x+1-3y=0$ $-3x+3y^2=0$
$x = \frac{3}{2}y - \frac{1}{2}$ $-3(\frac{3}{2}y - \frac{1}{2}) + 3y^2 = 0$ $3(y^2 - \frac{3}{2}y + \frac{1}{2}) = 0$ $y = 1$ and $\frac{1}{2}$
$x = 1$ and $\frac{1}{4}$ $f_{xx} = 2$ $f_{xy} = -3$ $f_{yy} = 6y$ $D = (2)(6) - 9 = 3$
$f_{xx}(1,1) = 2$ $D = (2)(3) - 9 = -3$

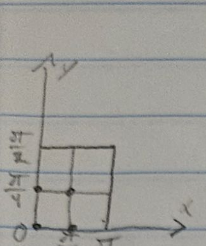
349. $(\sqrt{5}, 0, 0)$, $(-\sqrt{5}, 0, 0)$

$z = \frac{x^2-5}{y}$ $f(x,y) = x^2+y^2+(\frac{x^2-5}{y})^2$ $f(x,y) = x^2+y^2+\frac{(x^2-5)^2}{y^2}$
$f_x = 2x + \frac{4x(x^2-5)}{y^2}$ $f_y = 2y - \frac{2(x^2-5)^2}{y^3}$ $2y = \frac{2(x^2-5)^2}{y^3}$
$2y^4 = (x^2-5)^2$ $y = \sqrt{x^2-5}$ $2x + \frac{4x(x^2-5)}{x^2-5} = 0$ $x = \pm\sqrt{5}$
$y = \frac{x^2-5}{2}$ $y = 0$ $z = 0$

355. $x = 3$, $y = 6$

$f_x = -10x - 2y + 42$ $f_y = -16y - 2x + 102$
$-10x - 2y + 42 = 0$ $-16y - 2x + 102 = 0$ $-10x = 2y - 42$ $x = \frac{-1}{5}y + \frac{21}{5}$
$-16y + \frac{2}{5}y - \frac{42}{5} + 102 = 0$ $\frac{-78}{5}y + \frac{468}{5} = 0$ $y = 6$ $x = 3$

5.1

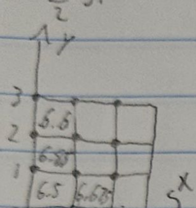


4. 6.68

$(0,0)\ (\frac{\pi}{2},0)\ (0,\frac{\pi}{4})\ (\frac{\pi}{2},\frac{\pi}{4})\quad \Delta x=\frac{\pi}{2}\quad \Delta y=\frac{\pi}{4}\quad \Delta A=\frac{\pi^2}{8}$

$V=\sum_{i=1}^{2}\sum_{j=1}^{2} f(x_{ij},y_{ij})\Delta A\quad V=\Delta A\left(f(0,0)+f(\frac{\pi}{2},0)+f(0,\frac{\pi}{4})+f(\frac{\pi}{2},\frac{\pi}{4})\right)$

$V=\frac{\pi^2}{8}\left(2+1+\frac{2+\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)=\frac{\pi^2}{8}(4+\sqrt{2})=\frac{\pi^2}{2}+\frac{\pi^2\sqrt{2}}{8}\approx 6.68$



8. a. $58.55\ ft^3$ b. $6.5\ ft$

$(0,1)(0,2)(0,3)(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)\quad \Delta x=1\quad \Delta y=1\quad \Delta A=1$

$\frac{6+6.5+6.5+7}{4}=6.5$ = depth(d) of lower left square $6.5\cdot 1=6.5$ = volume of square

V of all squares: 6.5, 6.85, 6.6, 6.675, 6.925, 6.625, 6.425, 6.175, 5.775

$V=6.5+6.85+6.6+6.675+6.925+6.625+6.425+6.175+5.775$

$V=58.55\ ft^3\qquad \frac{58.55\ ft^3}{9\ ft^2}=6.5\ ft$

24. $(1-\cos(1))(\sin(1))$

$\int_1^e \frac{\sin(\ln x)\cos(\ln y)}{xy}\,dx=\frac{-\cos(\ln x)\cos(\ln y)}{y}\Big]_1^e=\frac{-\cos(1)\cos(\ln y)}{y}-\frac{-\cos(0)\cos(\ln y)}{y}$

$=\frac{(1-\cos(1))\cos(\ln y)}{y}\qquad \int_1^e \frac{(1-\cos(1))\cos(\ln y)}{y}\,dy=(1-\cos(1))\sin(\ln y)\Big]_1^e$

$=(1-\cos(1))(\sin(1)-\sin(0))=(1-\cos(1))(\sin(1))$

26. $\frac{2e^3+1}{9}$

$\int_1^2 x^2\ln(x)\,dy=x^2y\ln(x)\Big]_1^2=2x^2\ln(x)-x^2\ln(x)=x^2\ln(x)$

$\int x^2\ln(x)\,dx=\frac{x^3}{3}\ln(x)-\int\frac{x^2}{3}\,dx=\frac{x^3}{3}\ln(x)-\frac{x^3}{9}=\frac{x^3}{3}\left(\ln(x)-\frac{1}{3}\right)+C$

$\frac{x^3}{3}\left(\ln|x|-\frac{1}{3}\right)\Big]_1^e=\frac{e^3}{3}\left(\frac{2}{3}\right)-\frac{1}{3}\left(-\frac{1}{3}\right)=\frac{2e^3}{9}+\frac{1}{9}=\frac{2e^3+1}{9}$

29. $\frac{1}{4}(e^8-e^4)$

$\int_0^1 xe^{x+4y}\,dx\quad e^{4y}\int xe^x\,dx=e^{4y}\left(xe^x-\int e^x\,dx\right)=e^{x+4y}(x-1)+C$

$e^{x+4y}(x-1)\Big]_0^1=e^{4y}\quad \int_1^2 e^{4y}\,dy=\frac{1}{4}e^{4y}\Big]_1^2=\frac{1}{4}(e^8-e^4)$

30. e^2-e

$\int_0^1 xe^{x-y}dy = -xe^{x-y}]_0^1 = -xe^{x-1}+xe^x = (e-1)xe^{x-1}$

$\int (e-1)xe^{x-1}dx = (e-1)[xe^{x-1}-\int e^{x-1}dx] = (e-1)(xe^{x-1}-e^{x-1}) = (e^x-e^{x-1})(x-1)$

$(e^x-e^{x-1})(x-1)]_1^2 = e^2-e$

33. $\tan^{-1}(2)+\ln(\frac{5}{4})-\frac{1}{2}\ln(2)-\frac{\pi}{4}$

$\int_1^2 \frac{x}{x^2+y^2}dy = \tan^{-1}(\frac{y}{x})]_1^2 = \tan^{-1}(2x^{-1})-\tan^{-1}(x^{-1})$

$\int_0^1(\tan^{-1}(2x^{-1})-\tan^{-1}(x^{-1}))dx = x\tan^{-1}(\frac{2}{x})+\ln(x^2+4)-\frac{1}{2}\ln(x^2+1)-x\cot^{-1}(x)]_0^1$

$= (\tan^{-1}(2)+\ln(5)-\frac{1}{2}\ln(2)-\cot^{-1}(1))-(\ln(4)-\frac{1}{2}\ln(1)) = \tan^{-1}(2)+\ln(\frac{5}{4})-\frac{1}{2}\ln(2)-\frac{\pi}{4}$

34. $\frac{1}{2}(5\ln(5)-2\ln(2))$

$u=x+y^2 \quad \frac{du}{dy}=2y \quad du=2ydy \quad \frac{1}{2}\int\frac{1}{u}du = \frac{1}{2}\ln(x+y^2)$

$\frac{1}{2}\ln(x+y^2)]_1^2 = \frac{1}{2}\ln(x+4)-\frac{1}{2}\ln(x+1) \quad \int\frac{1}{2}(\ln(x+4)-\ln(x+1))dx =$

$\frac{1}{2}((x+4)\ln(x+4)-x-4)-\frac{1}{2}((x+1)\ln(x+1)-x-1) = \frac{1}{2}((x+4)\ln(x+4)-(x+1)\ln(x+1)-3)$

$\frac{1}{2}((x+4)\ln(x+4)-(x+1)\ln(x+1)-3)]_0^1 = \frac{1}{2}(5\ln(5)-2\ln(2)-3)+\frac{3}{2}$

36. 38.7

$R=1 \quad \int_2^3\int_1^2(x^4+2y^3)dxdy \quad \int_1^2(x^4+2y^3)dx = (\frac{1}{5}x^5+2xy^3)]_1^2 =$

$=(\frac{1}{5}(32)+4y^3)-(\frac{1}{5}(1)+2y^3) = \frac{31}{5}+2y^3 \quad \int_2^3(2y^3+\frac{31}{5})dy = (\frac{1}{2}y^4+\frac{31}{5}y)]_2^3$

$=(\frac{1}{2}(81)+\frac{31}{5}(3))-(\frac{1}{2}(16)+\frac{31}{5}(2)) = 38.7$

38. 0.233

$R=1 \quad \int_0^1\int_0^1 \tan^{-1}(xy)dxdy \quad \int_0^1 \tan^{-1}(xy)dx = [x\tan^{-1}(xy)-\frac{\ln(x^2y^2+1)}{2y}]_0^1$

$=\tan^{-1}(y)-\frac{\ln(y^2+1)}{2y} \quad \int_0^1(\tan^{-1}(y)-\frac{\ln(y^2+1)}{2y})dy = 0.233$

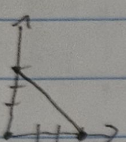
5.2

74. $\frac{19}{4}$

$\int_0^1\int_{x^3}^{x^3+1}(2x+5y)\,dy\,dx \quad \int_{x^3}^{x^3+1}(2x+5y)\,dy = (2xy+\frac{5}{2}y^2)\Big]_{x^3}^{x^3+1}$

$= 2x^4+2x+\frac{5}{2}(x^6+2x^3+1)-2x^4-\frac{5}{2}x^6 = 5x^3+2x+\frac{5}{2}$

$\int_0^1(5x^3+2x+\frac{5}{2})\,dx = (\frac{5}{4}x^4+x^2+\frac{5}{2}x)\Big]_0^1 = \frac{5}{4}+1+\frac{5}{2} = \frac{19}{4}$



78. $-\sin 3 + 3$

$\int_0^3\int_0^{-x}\sin y\,dy\,dx \quad \int_0^{-x}\sin y\,dy = -\cos y\Big]_0^{-x} = -\cos x + 1$

$\int_0^3(-\cos x+1)\,dx = (-\sin x + x)\Big]_0^3 = -\sin 3 + 3$

80. $\frac{23}{12}$

$\int_{2x}^{3x}(x+y^2)\,dy = (xy+\frac{1}{3}y^3)\Big]_{2x}^{3x} = 3x^2+9x^3-2x^2-\frac{8}{3}x^3 = \frac{19}{3}x^3+x^2$

$\int_0^1(\frac{19}{3}x^3+x^2)\,dx = (\frac{19}{12}x^4+\frac{1}{3}x^3)\Big]_0^1 = \frac{19}{12}+\frac{1}{3} = \frac{23}{12}$

82. $e^2-\frac{1}{2}e$

$\int_{\ln u}^2(v+\ln u)\,dv = (\frac{1}{2}v^2+v\ln u)\Big]_{\ln u}^2 = 2+2\ln u-\frac{1}{2}\ln^2 u-\ln^2 u = -\frac{3}{2}\ln^2 u+2\ln u+2$

$\int_e^{e^2}(-\frac{3}{2}\ln^2 u+2\ln u+2)\,du = (-\frac{1}{2}u(3\ln^2 u-10\ln u+6))\Big]_e^{e^2}$

$= (-\frac{1}{2}e^2(12-20+6))+(\frac{1}{2}e(3-10+6)) = e^2-\frac{1}{2}e$

90. $\frac{3}{2}$

$\int_0^1\int_{x^5+1}^{x^2+3}dy\,dx \quad \int_0^1(-x^5-x^2+2)\,dx = (-\frac{1}{6}x^6-\frac{1}{3}x^3+2x)\Big]_0^1 = -\frac{1}{6}-\frac{1}{3}+2 = \frac{3}{2}$

94. $-\frac{2}{9}\sin 3 + \frac{2}{3}$

found in prob. #78

$\iint_D f(x,y)\,dA = -\sin 3+3 \quad A(D) = \frac{1}{2}(3)(3) = \frac{9}{2} \quad \frac{2}{9}(-\sin 3+3) = -\frac{2}{9}\sin 3+\frac{2}{3}$

96. $1+\sin 1$

$$\int_{-1}^{\frac{\pi}{2}}\int_{0}^{x+1}\sin x\,dy\,dx = \int_{0}^{\frac{\pi}{2}+1}\int_{y-1}^{\frac{\pi}{2}}\sin x\,dx\,dy \quad \int_{y-1}^{\frac{\pi}{2}}\sin x\,dx = -\cos x\Big]_{y-1}^{\frac{\pi}{2}} = \cos(y-1)$$
$$\int_{0}^{\frac{\pi}{2}+1}\cos(y-1)\,dy = \sin(y-1)\Big]_{0}^{\frac{\pi}{2}+1} = \sin\left(\frac{\pi}{2}\right)-\sin(-1) = 1+\sin 1$$

98. $\frac{32}{105}$

$$\int_{-1}^{0}\int_{-\sqrt{y+1}}^{\sqrt{y+1}} y^2\,dx\,dy = \int_{-1}^{1}\int_{x^2-1}^{0} y^2\,dy\,dx \quad \int_{x^2-1}^{0} y^2\,dy = \frac{1}{3}y^3\Big]_{x^2-1}^{0} = -\frac{1}{3}(x^6-3x^4+3x^2-1)$$
$$\int_{-1}^{1}\left(-\frac{1}{3}(x^6-3x^4+3x^2-1)\right)dx = -\frac{1}{3}\left(\frac{1}{7}x^7-\frac{3}{5}x^5+x^3-x\right)\Big]_{-1}^{1} = \frac{2}{3}\left(-\frac{1}{7}+\frac{3}{5}-1+1\right) = \frac{-2}{21}+\frac{2}{5} = \frac{32}{105}$$

102. $\frac{355}{336}$

$$\int_{0}^{1}\int_{x}^{x^5}(2x+y^2)\,dy\,dx \quad \int_{x}^{x^5}(2x+y^2)\,dy = \left(2xy+\frac{1}{3}y^3\right)\Big]_{x}^{x^5} = 2x^6+\frac{1}{3}x^{15}+2x^2+\frac{1}{3}x^3$$
$$\int_{0}^{1}\left(\frac{1}{3}x^{15}+2x^6+\frac{1}{3}x^3+2x^2\right)dx = \left(\frac{1}{48}x^{16}+\frac{2}{7}x^7+\frac{1}{12}x^4+\frac{2}{3}x^3\right)\Big]_{0}^{1} = \frac{1}{48}+\frac{2}{7}+\frac{1}{12}+\frac{2}{3} = \frac{355}{336}$$

108. 1

$$\int_{0}^{1}\int_{0}^{-x+1} 2\,dy\,dx = \int_{0}^{-x+1} 2\,dy\,dx = 2y\Big]_{0}^{-x+1} = -2x+2 \quad \int_{0}^{1}(-2x+2)\,dx = (-x^2+2x)\Big]_{0}^{1} = 1$$