4.5

215. $y\cos z + 2xt\cos z - \frac{xy\sin z}{\sqrt{1-t^2}}$

$\frac{dw}{dx} = y\cos z \quad \frac{dw}{dy} = x\cos z \quad \frac{dw}{dz} = xy\sin z \quad \frac{dx}{dt} = 1 \quad \frac{dy}{dt} = 2t \quad \frac{dz}{dt} = \frac{1}{\sqrt{1-t^2}}$

$\frac{dw}{dt} = y\cos z + 2xt\cos z + \frac{-xy\sin z}{\sqrt{1-t^2}} =$

217. $-30x + 4y,\ 10x - 16y$

$\frac{dw}{dx} = 10x \quad \frac{dw}{dy} = 4y \quad \frac{dx}{ds} = -3 \quad \frac{dy}{ds} = 1 \quad \frac{dx}{dt} = 1 \quad \frac{dy}{dt} = -4$

$\frac{dw}{ds} = -30x + 4y \quad \frac{dw}{dt} = 10x - 16y$

222. $t\left(\frac{2t^2+1}{\sqrt{t^2+t^4}}\right.$

$\frac{df}{dx} = \frac{x}{\sqrt{x^2+y^2}} \quad \frac{df}{dy} = \frac{y}{\sqrt{x^2+y^2}} \quad \frac{dx}{dt} = 1 \quad \frac{dy}{dt} = 2t$

$\frac{df}{dt} = \frac{x+2yt}{\sqrt{x^2+y^2}} = \frac{t+2t^3}{\sqrt{t^2+t^4}} = t\left(\frac{2t^2+1}{\sqrt{t^2+t^4}}\right)$

225. 1

$\frac{df}{dx} = \frac{1}{x+y} \quad \frac{df}{dy} = \frac{1}{x+y} \quad \frac{dx}{dt} = e^t \quad \frac{dy}{dt} = e^t$

$\frac{df}{dt} = \frac{2e^t}{x+y} = \frac{2e^t}{2e^t} = 1$

232. $\frac{\sin(x-y) - \cos(x+y)}{\sin(x-y) + \cos(x+y)}$

$\frac{df}{dx} = \frac{d}{dx}\left(\sin(x+y) + \cos(x-y) - 4\right) = \cos(x+y) - \sin(x-y)$

$\frac{df}{dy} = \frac{d}{dy}\left(\sin(x+y) + \cos(x-y) - 4\right) = \cos(x+y) + \sin(x-y)$

$\frac{dy}{dx} = \frac{\sin(x-y) - \cos(x+y)}{\sin(x-y) + \cos(x+y)}$

234. $\frac{-e^y - ye^x + 4xy}{xe^y + e^x - 2x^2}$

$\frac{df}{dx} = e^y + ye^x - 4xy \quad \frac{df}{dy} = xe^y + e^x - 2x^2$

$\frac{dy}{dx} = \frac{-e^y - ye^x + 4xy}{xe^y + e^x - 2x^2}$

245. $\sqrt{3}e^{-\sqrt{3}}$, $(2-4\sqrt{3})e^{-\sqrt{3}}$

$z = r^2(\cos\theta)(\sin\theta)e^{\cot\theta}$ $\frac{dz}{dr} = 2r(\cos\theta)(\sin\theta)e^{\cot\theta}$

$\frac{dz}{dr}(2, \frac{\pi}{6}) = 4(\cos\frac{\pi}{6})(\sin\frac{\pi}{6})e^{\cot\frac{\pi}{6}} = \sqrt{3}e^{\sqrt{3}}$

$\frac{dz}{d\theta} = r^2 e^{\cot\theta}(\cos 2\theta - \cot\theta) = 4e^{\sqrt{3}}(\frac{1}{2} - \sqrt{3}) = (2-4\sqrt{3})e^{\sqrt{3}}$

250.

$f(tx, ty) = t^3x^2y - 2t^3y^3 = t^3(x^2y - 2y^3)$ $n=3$

$\frac{df}{dx} = 2xy$ $\frac{df}{dy} = x^2 - 6y^2$ $x\frac{df}{dx} + y\frac{df}{dx} = 2x^2y + x^2y - 6y^3 = 3x^2y - 6y^3$

$= 3(x^2y - 2y^3) = nf(x,y)$

4.6

265. $\frac{2}{\sqrt{6}}$

$h_x = yz$ $h_y = xz$ $h_z = xy$ $||v|| = \sqrt{6}$ $u = (\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}})$

$D_u = h_x(2,1,1)(\frac{2}{\sqrt{6}}) + h_y(2,1,1)(\frac{1}{\sqrt{6}}) + h_z(2,1,1)(\frac{-1}{\sqrt{6}})$

$= \frac{2}{\sqrt{6}} + \frac{2}{\sqrt{6}} - \frac{2}{\sqrt{6}} = \frac{2}{\sqrt{6}}$

272. -50

$f_x = 2xy$ $f_y = x^2$ $||v|| = 5$ $u = (\frac{3}{5}, \frac{-4}{5})$

$D_u = f_x(-5,5)(\frac{3}{5}) + f_y(-5,5)(\frac{-4}{5}) = (-50)(\frac{3}{5}) + (25)(\frac{-4}{5})$

$= -30 - 20 = -50$

284. $\frac{38}{\sqrt{41}}$

$f_x = 2x$ $f_y = 6y$ $||Q|| = \sqrt{41}$ $u_Q = (\frac{4}{\sqrt{41}}, \frac{5}{\sqrt{41}})$

$D_u = f_x(1,1)(\frac{4}{\sqrt{41}}) + f_y(1,1)(\frac{5}{\sqrt{41}}) = \frac{8}{\sqrt{41}} + \frac{30}{\sqrt{41}} = \frac{38}{\sqrt{41}}$

287. $\frac{31}{255}$

$f_x = \frac{5}{5x+4y}$ $f_y = \frac{4}{5x+4y}$ $\|u\| = 10$ $v = \left(\frac{3}{5}, \frac{4}{5}\right)$

$D_u = f_x(3,9)\left(\frac{3}{5}\right) + f_y(3,9)\left(\frac{4}{5}\right) = \frac{3}{51} + \frac{16}{255} = \frac{31}{255}$

291. $\frac{4}{3}i - 3j$

$f_x = e^y - \frac{1}{x}$ $f_y = xe^y$ $\nabla f = f_x(-3,0)i + f_y(-3,0)j = \frac{4}{3}i - 3j$

294. $\frac{-14}{-\sqrt{365}}i - \frac{13}{-\sqrt{365}}j$

$f_x = 2x+y$ $f_y = 2y+x$ $\nabla f = f_x(-5,-4)i + f_y(-5,-4)j = -14i - 13j$

$\|\nabla f\| = -\sqrt{196+169} = -\sqrt{365}$

296. $\frac{-\sqrt{2}}{2}i - \frac{\sqrt{2}}{2}j$

$f_x = \frac{-y}{x^2+y^2}$ $f_y = \frac{x}{x^2+y^2}$ $\nabla f = f_x(-9,9)i + f_y(-9,9)j = \frac{-1}{18}i - \frac{1}{18}j$

$\|\nabla f\| = \frac{\sqrt{2}}{18}$

300. $\frac{-\sqrt{17}}{6}$, $\frac{2}{3}i + \frac{1}{6}j$

$f_x = \frac{x}{\sqrt{x^2+2y}}$ $f_y = \frac{1}{\sqrt{x^2+2y}}$ $\nabla f = f_x(4,10)i + f_y(4,10)j = \frac{2}{3}i + \frac{1}{6}j$

$\|\nabla f\| = \frac{-\sqrt{17}}{6}$

Calculus 3

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171. $-36x-6y-z=-39$

$f_x=-18x \quad f_y=-6y \quad f(2,1)=-39 \quad f_x(2,1)=-36 \quad f_y(2,1)=-6$

$z=-39-36(x-2)-6(y-1)=-36x-6y+39$

174. $e^{7x^2+4y^2}(14x^2+8y^2)-z=-1$

$f_x=14xe^{7x^2+4y^2} \quad f_y=8ye^{7x^2+4y^2} \quad f(0,0)=1 \quad f_x(0,0)=14x$

$f_y(0,0)=8y \quad z=1+14x^2e^{7x^2+4y^2}+8y^2e^{7x^2+4y^2}$

180. $\frac{3x}{25}+\frac{4y}{25}-z=1-\ln 5$

$f_x=\frac{x}{x^2+y^2} \quad f_y=\frac{y}{x^2+y^2} \quad f_x(3,4)=\frac{3}{25} \quad f_y(3,4)=\frac{4}{25} \quad f(3,4)=\ln 5$

$z=\ln 5+\frac{3}{25}(x-3)+\frac{4}{25}(y-4)=\ln 5+\frac{3x}{25}+\frac{4y}{25}-1$

209. $\frac{\pi}{4}+\frac{1}{2}x+y-\frac{1}{2}$

$f_x=\frac{1}{(x+2y)^2+1} \quad f_y=\frac{2}{(x+2y)^2+1} \quad f_x(1,0)=\frac{1}{2} \quad f_y(1,0)=1 \quad f(1,0)=\frac{\pi}{4}$

$L(x,y)=\frac{\pi}{4}+\frac{1}{2}(x-1)+y=\frac{\pi}{4}+\frac{1}{2}x+y-\frac{1}{2}$

211. $\frac{3}{7}x+\frac{2}{7}y+\frac{6}{7}z$

$f_x=\frac{x}{\sqrt{x^2+y^2+z^2}} \quad f_y=\frac{y}{\sqrt{x^2+y^2+z^2}} \quad f_z=\frac{z}{\sqrt{x^2+y^2+z^2}} \quad f_x(3,2,6)=\frac{3}{7}$

$f_y(3,2,6)=\frac{2}{7} \quad f_z(3,2,6)=\frac{6}{7} \quad f(3,2,6)=7$

$L(x,y,z)=7+\frac{3}{7}(x-3)+\frac{2}{7}(y-2)+\frac{6}{7}(z-6)=7+\frac{3}{7}x+\frac{2}{7}y+\frac{6}{7}z-7$