CMSI 3300 – Classwork 2

Instructions:
This worksheet will not only provide you with practice problems for your upcoming exam, but will concrete your knowledge on the theory of knowledge-based systems and inference. Specific notes:

- Provide answers to each of the following questions and write your responses in the blanks. If you are expected to show your work in arriving at a particular solution, space will be provided for you.
- Place the names of your group members below:

Group Members:

1. Cameron Scolari
2. Julian Mazzier
3. ____________________

Problem 1 – Propositional Fundamentals

Time for the warmup! Let's make sure you're comfortable with the fundamentals of propositional logic before moving onto the good stuff…

1.1. Match each concept from Propositional Logic to its intuition / definition. Place the letter of each definition from the bank below next to the concept.

A. A logical variable standing for T/F	B. An instantiation of all propositions to T/F
C. The set of all worlds for some given props.	D. A sentence that is equivalent to True
E. Propositions with some logical connectors	F. For example: $\alpha \land \neg\alpha$
G. A sentence with ONLY disjoined props.	H. Logical operator encoding if-then claims
I. When one sentence follows from another	J. A sentence with ONLY conjoined clauses
K. Inference rule that operates on clauses	L. A giant sentence composed of rules + facts

A Proposition
H Implication
F Contradiction
I Entailment

E Logical Sentence
C Truth Table
G Clause
K Resolution

B World
D Valid / Vacuous
J CNF
L Knowledgebase

One way to perform brute-force logical inference is by defining *entailment* that operates on the definition of *models* $M(\alpha) =$ the set of worlds in which the sentence α is true. Specifically, we say that sentence α *entails* sentence β iff β is true in all of the worlds that α is:

$$\alpha \vDash \beta \Leftrightarrow M(\alpha) \subseteq M(\beta)$$

1.2. In other words, if $\alpha \vDash \beta$, then knowing α to be true allows us to assume that β is too. To wit, complete the following truth table that will be used in later questions (α is done for you).

World	X	Y	Z	$\alpha = X \land Z$	$\beta = X \lor Y \lor Z$	$\gamma = \neg Y \lor \neg Z$
w_0	F	F	F	F	F	T
w_1	F	F	T	F	T	T
w_2	F	T	F	F	T	T
w_3	F	T	T	F	T	F
w_4	T	F	F	F	T	T
w_5	T	F	T	T	T	T
w_6	T	T	F	F	T	T
w_7	T	T	T	T	T	F

For example, $M(\alpha) = M(X \land Z) = \{w_5, w_7\}$ because those are the only two worlds in which this sentence is true.

1.3. From the table in 1.2., complete the following questions to learn more about entailment (some are done for you to show you the expected format).

1.3.1. [Given] List all worlds $w_i \in M(\alpha)$	$M(a) = \{w_5, w_7\}$
1.3.2. List all worlds $w_i \in M(\beta)$	$M(\beta) = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7\}$
1.3.3. List all worlds $w_i \in M(\gamma)$	$M(\gamma) = \{w_0, w_1, w_2, w_4, w_5, w_6\}$
1.3.4. True or false: $M(\alpha) \subseteq M(\beta)$?	True
1.3.5. True or false: $M(\beta) \subseteq M(\alpha)$?	False
1.3.6. True or false: $\alpha \models \beta$?	True
1.3.7. True or false: $\beta \models \alpha$?	False

1.4. Notice that sentences β and γ in table 1.2 are *clauses* (disjunctions $\vee$ of propositions), and suppose we have a CNF knowledgebase composed of them:

$$KB = \beta \wedge \gamma = (X \vee Y \vee Z) \wedge (\neg Y \vee \neg Z)$$

One of the biggest mistakes in using resolution is thinking that *multiple propositions* can cancel out at once when resolving between two clauses. For instance, if we `resolve`(β, γ) in the above, we will **NOT** be able to cancel out the negated Y **AND** Z in order to infer X, i.e., $KB \not\models X$. To prove this, answer the following questions:

1.4.1. List all worlds $w_i \in M(X)$	$M(X) = \{w_4, w_5, w_6, w_7\}$
1.4.2. List all worlds $w_i \in M(KB)$ (Hint: look at 1.3.2. and 1.3.3.)	$M(KB) = \{w_1, w_2, w_4, w_5, w_6\}$
1.4.3. True or false: $M(KB) \subseteq M(X)$?	False
1.4.4. True or false: $KB \models X$?	False
1.4.5. What is the resulting clause from the operation `resolve`(β, γ)? (Hint: from above, you should know it's NOT X).	Vacuously True

$(X \vee Y) \vee \neg Y$ OR

$(X \vee Z) \vee \neg Z$ OR

True

With the caveat from 1.4. in mind, let's try resolving a few clauses… *if* they can be resolved! Recall that the criteria for resolving two clauses is that they share at least one proposition P, and it is negated in one clause but not the other. The resulting "inferred clause" is the set-union of all remaining propositions between the clauses disjoined together.

1.5. Consider a programmatic implementation of resolution that takes two clauses as input and returns an inferred clause, `resolve(clause1, clause2) => inferred_clause`. For each of the following clause arguments, determine:

1. Whether or not the clauses are *amenable* to resolution, i.e., that it's even possible to resolve the clauses based on the criteria stated above (hint: they must disagree on *at least* 1 proposition). Place T/F in the "Amenable" column to indicate your answer.
2. If so, determine the inferred clause that would be returned by the resolve function. Leave this column blank if the clauses weren't amenable to resolution. If the resulting clause is vacuous/valid, you may reduce it to the literal "T," and if it's a contradiction, you may write ∅ for the resolved clause.
3. Lastly, determine if your inferred clause was either vacuous/valid (i.e., logically equivalent to True, [$\equiv T$]) or a contradiction [∅]. Again, leave this blank if the clauses weren't resolvable.

#	Clause 1	Clause 2	Amenable?	Inferred Clause	$\equiv T$?	∅?
1.5.1.	$X \vee Y$	$\neg Y \vee Z$	T	$X \vee Z$		
1.5.2.	X	X	F			
1.5.3.	$\neg X$	X	T	∅		✓
1.5.4.	$X \vee Y$	$Y \vee Z$	F			
1.5.5.	$X \vee Y \vee Z$	$Z \vee \neg Y$	T	$X \vee Z$		
1.5.6.	$Z \vee \neg Y$	$\neg Z \vee Y$	T	T	✓	
1.5.7.	$X \vee Y$	W	F			
1.5.8.	$X \vee Y \vee Z$	$\neg Z \vee \neg Y \vee W$	T	T	✓	

Problem 2 – Guided Inference Exercise

You didn't think you could make it through an entire classwork without me bringing up Forney Industries, did you? Here's a story I've been creating for my upcoming novel that'll help you practice some logic (although it isn't particularly logical itself). Note that although artificial, rules and axioms like those that you'll encode below can be used to create simple Q&A systems in practice, some of which you'll use on your first homework.

> Preceding the Great Forneybot Uprising of 2024, the world was in a utopic state of bliss with humankind finally served by throngs of our robotic servants. Forney Industries, sadly, had a number of fatal oversights in their robots' design, any number of whose small, environmental quirks could cause them to violently malfunction. Before the flaws were discovered, it was, of course, too late -- however, it has long been a matter of historical discourse for what event actually triggered the first mass malfunction...

The following story is written from the future perspective, with bracketed propositions [P] indicating the atoms you'll use in the sentences that follow. Some hints:

- Rules generally follow an "if-then" pattern of implication $if \Rightarrow then$ but statement of multiple facts are usually conjunction $fact_1 \wedge fact_2 \wedge \ldots$
- "If and only if" statements, or variables that follow one another hand-in-hand tend to follow the pattern of biconditionals $\alpha \Leftrightarrow \beta$

2.1. For each of the following plain-English descriptions of a knowledgebase's rules and axioms, provide the propositional logic sentence translation that uses the bracketed propositions [P]. Don't worry about converting these into CNF clauses just yet, you'll do that in the next step!

2.1.1. "Forneybots were found to malfunction $[M]$ <u>*if and only if*</u> they suffer water damage $[D]$ or overheard a logical paradox $[P]$. This one is done for you as an example."

$$M \Leftrightarrow D \vee P$$

2.1.2. "Forneybots may have been caught in a sudden rainstorm $[R]$, which, <u>*as a rule*</u>, only happen during Winter $[W]$ or Summer$[S]$, though <u>*not all*</u> Winters and Summers had rain."

$$R \Rightarrow W \vee S$$

2.1.3. "It goes without saying that rainstorms $[R]$ and water damage $[D]$ would have gone hand in hand (i.e., are found together), since the Forneybots were not aware of their water weakness."

$$R \Leftrightarrow D$$

2.1.4. "Logical paradoxes $[P]$, on the other hand, would have only been heard during election season (through TV or radio broadcast), which are held in the Fall $[F]$ (and so all Fall seasons thus came hand in hand with paradoxes—zing!).

$P \Leftrightarrow F$

2.1.5. "It also goes without saying that *if* it is one season, *then* it cannot simultaneously be the others." (Hint: there are 3 seasons in this scenario, so you'll need the rule to be repeated 3 times and then logically conjoined!).

$(F \Rightarrow \neg W \land \neg S) \land (W \Rightarrow \neg F \land \neg S) \land (S \Rightarrow \neg F \land \neg W)$

2.1.6. "The Great Forneybot Uprising occurred from a mass malfunction on Nov. 16, 2024." (Hints: there are *multiple* facts being stated here, and they are *axiomatic,* meaning it is *not* a *rule* being stated).

$F \land M$

Now that we've got our sentences in propositional logic format, we're one step from using them to answer questions about our scenario: first, they need to be converted into Conjunctive Normal Form (CNF), meaning that our KB is just one giant sentence consisting of a *conjunction of clauses.*

2.2. For each of the sentences you found in 2.1., we'll now convert them into CNF and *enumerate* the resulting *clauses* as KB1, KB2, … with the assumption that our final CNF KB will be of the format: $KB = KB1 \land KB2 \land KB3 \land \ldots$ The correct number of resulting clauses from each conversion is given in the answer box. *Show your work in performing each conversion in the space provided.* The first conversion, and expected format of answers, is done for you. Hint: you might use *logical equivalences* to reduce the number of resulting clauses for some conversions.

Sentence from 2.1.	**CNF Conversion**		
2.1.1. $M \Leftrightarrow D \lor P$	$M \Leftrightarrow D \lor P$ $\equiv \big(M \Rightarrow (D \lor P)\big) \land \big((D \lor P) \Rightarrow M\big)$ $\equiv \big(\neg M \lor (D \lor P)\big) \land (\neg(D \lor P) \lor M)$ $\equiv (\neg M \lor D \lor P) \land \big((\neg D \land \neg P) \lor M\big)$ $\equiv (\neg M \lor D \lor P) \land (\neg D \lor M) \land (\neg P \lor M)$		
	$KB1 =$ $\neg M \lor D \lor P$	$KB2 =$ $\neg D \lor M$	$KB3 =$ $\neg P \lor M$

2.1.2.	$R \rightarrow W \vee S$ $\neg R \vee (W \vee S)$
	$KB4 =$ $\neg R \vee W \vee S$
2.1.3.	$R \leftrightarrow D$ $(R \rightarrow D) \wedge (D \rightarrow R)$ $(\neg R \vee D) \wedge (\neg D \vee R)$
	$KB5 =$ $\neg R \vee D$; $KB6 =$ $\neg D \vee R$
2.1.4.	$P \leftrightarrow F$ $(P \rightarrow F) \wedge (F \rightarrow P)$ $(\neg P \vee F) \wedge (\neg F \wedge P)$
	$KB7 =$ $\neg P \vee F$; $KB8 =$ $\neg F \vee P$
2.1.5.	$(F \rightarrow \neg W \wedge \neg S) \wedge (W \rightarrow \neg F \wedge \neg S) \wedge (S \rightarrow \neg F \wedge \neg W)$ $(\neg F \vee (\neg W \wedge \neg S)) \wedge (\neg W \vee (\neg F \wedge \neg S)) \wedge (\neg S \vee (\neg F \wedge \neg W))$ $(\neg F \vee \neg W) \wedge (\neg F \vee \neg S) \wedge \cancel{(\neg W \vee \neg F)} \wedge (\neg W \vee \neg S) \wedge \cancel{(\neg S \vee \neg F)} \wedge \cancel{(\neg S \vee \neg W)}$
	$KB9 =$ $\neg F \vee \neg W$; $KB10 =$ $\neg F \vee \neg S$; $KB11 =$ $\neg W \vee \neg S$
2.1.6.	$F \wedge M$
	$KB12 =$ F ; $KB13 =$ M

2.3. Now that we have all of our KB's starting rules and axioms, it's time for inference! In the following problem, we'll use proof by contradiction to prove that it was a logical paradox $[P]$ and NOT water damage $[D]$ that caused the Great Forneybot Uprising. Specifically, we will show:

$$\alpha = \neg D \land P$$

Complete the following steps in the next table to prove that $KB \vDash \alpha$:

1. Collect all of your answers from 2.2. in the "Initial Clauses" column, clauses KB1-13
2. Add the negation of the query, $\neg\alpha$, as clause KB14 (Hint: remember DeMorgan)
3. In the "Inferred Clauses" column, perform a search for the empty clause (∅ = contradiction) by resolving pairs of clauses that are amenable to resolution. You may stop once you've found a contradiction, but must indicate which pairs were used to obtain any clauses KB15+ (e.g., if you resolved clauses KB1 and KB2, you would write [KB1, KB2] in that row of the "Resolvents" column, and then the clause they resolve to in the same row of the "Inferred Clauses" column). *<u>You may not need to use all slots for KB15-28, but won't need more than KB28 to solve the problem.</u>* Suggestion: solve the problem on scratch paper before translating your final answer here.

Initial Clauses		**Resolvents**	**Inferred Clauses**
$KB1 = \neg M \lor D \lor P$		[KB8, KB12]	$KB15 = P$
$KB2 = \neg D \lor M$		[KB14, KB15]	$KB16 = D$
$KB3 = \neg P \lor M$		[KB6, KB16]	$KB17 = R$
$KB4 = \neg R \lor W \lor S$		[KB12, KB10]	$KB18 = \neg S$
$KB5 = \neg R \lor D$		[KB12, KB9]	$KB19 = \neg W$
$KB6 = \neg D \lor R$		[KB4, KB18]	$KB20 = \neg R \lor W$
$KB7 = \neg P \lor F$		[KB19, KB20]	$KB21 = \neg R$
$KB8 = \neg F \lor P$		[KB17, KB21]	$KB22 = \emptyset$
$KB9 = \neg F \lor \neg W$			$KB23 =$
$KB10 = \neg F \lor \neg S$			$KB24 =$
$KB11 = \neg W \lor \neg S$			$KB25 =$
$KB12 = F$			$KB26 =$
$KB13 = M$			$KB27 =$
$KB14 = D \lor \neg P$			$KB28 =$