

### Example: Poker Test

A sequence of 1000 four-digit numbers has been generated and an analysis indicates the following combinations & frequencies:

<i>Combination Distribution (i)</i>	<i>Observed Frequency (O<sub>i</sub>)</i>
4 different digits	540
3 like digits	50
4 like digits	20
1 pair	320
2 pairs	70

Use Poker's test to determine if these random numbers are independent,  $\alpha = 0.05$ .

#### Solution:

In 4-digit numbers, there are only five possibilities- four different, 3 of a kind, 4 of a kind, 1 pair and 2 pairs. Let's calculate the probabilities for each of these cases:

Case-I: P(4 different digits)

$$= P(2^{\text{nd}} \text{ diff. from } 1^{\text{st}}) * P(3^{\text{rd}} \text{ diff. from } 1^{\text{st}} \& 2^{\text{nd}}) * P(4^{\text{th}} \text{ diff. from } 1^{\text{st}}, 2^{\text{nd}} \& 3^{\text{rd}}) \\ = 0.9 * 0.8 * 0.7 = 0.504$$

Case-II: P(3 like digits)

$$= P(2^{\text{nd}} \text{ digit same as } 1^{\text{st}}) * P(3^{\text{rd}} \text{ digit same as } 1^{\text{st}}) * P(4^{\text{th}} \text{ digit diff. from } 1^{\text{st}}) \\ * {}^4C_3 \\ = 0.1 * 0.1 * 0.9 * 4 = 0.036$$

Case-III: P(4 like digits)

$$= P(2^{\text{nd}} \text{ digit same as } 1^{\text{st}}) * P(3^{\text{rd}} \text{ digit same as } 1^{\text{st}}) * P(4^{\text{th}} \text{ digit same as } 1^{\text{st}}) \\ = 0.1 * 0.1 * 0.1 = 0.001$$

Case-IV: P(1 pair)

$$= \text{No. of possible combinations for a pair from 4 digits} * P(2^{\text{nd}} \text{ digit same as } 1^{\text{st}} \text{ in the pair}) \\ * P(3^{\text{rd}} \text{ digit diff. from } 1^{\text{st}}) * P(4^{\text{th}} \text{ digit diff. from } 1^{\text{st}} \& 3^{\text{rd}}) \\ = {}^4C_2 * 0.1 * 0.9 * 0.8 = 0.432$$

Case-V: P(2 pairs)

$$= 1 - P(4 \text{ different digits}) - P(3 \text{ like digits}) - P(4 \text{ like digits}) - P(1 \text{ pair}) \\ = 1 - 0.504 - 0.036 - 0.001 - 0.432 = 0.027$$

With N = 1000, let's summarize the results for Poker's test in the following table:

<i>Combination Distribution (i)</i>	<i>Observed Frequency O<sub>i</sub></i>	<i>Expected Frequency E<sub>i</sub> = Prob.*N</i>	<i>(O<sub>i</sub> - E<sub>i</sub>)<sup>2</sup>/E<sub>i</sub></i>
4 different digits	540	504	2.571
3 like digits	50	36	5.44
4 like digits	20	1	361
1 pair	320	432	29.037
2 pairs	70	27	68.48

**1000**

**1000**

**466.528**

Calculated  $\chi^2$  value = 466.528

Degree of freedom =  $n - 1 = 5 - 1 = 4$

At  $\alpha = 0.05$ , acceptable value of  $\chi^2$  from table = 9.49

Since, the calculated value of  $\chi^2$  is greater than the acceptable value at four degree of freedom, the independence of the random numbers is rejected on the basis of this test.