

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign):

Key

Name (print): _____

Student Number: _____

Instructor's Name: _____

Class Time: _____

Problem Number	Points Possible	Points Made
1	18	
2	26	
3	16	
4	20	
5	22	
6	20	
7	15	
8	20	
9	15	
10	15	
11	15	
12	15	
13	30	
14	10	
15	15	
Total:	272	

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- You are only allowed to use a **TI-30XS Multiview** calculator. No other calculators are permitted, and sharing of calculators is not permitted.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.

1. Determine the following limits. If you answer with ∞ or $-\infty$, briefly explain your thinking.
Print your final answer in the box provided.

_____ (a) [5 pts] $\lim_{x \rightarrow 2} (3x^2 + 7x - 5)$

$$\begin{aligned} &= 3 \cdot 2^2 + 7 \cdot 2 - 5 \\ &= 12 + 14 - 5 \\ &= 26 - 5 \end{aligned}$$

answer:

21

_____ (b) [5 pts] $\lim_{x \rightarrow 1^-} \frac{2x}{x-1}$

$2x \rightarrow 2$ as $x \rightarrow 1^-$
 $x-1 \rightarrow 0$ and is negative as $x \rightarrow 1^-$

answer:

$-\infty$

_____ (c) [8 pts] $\lim_{x \rightarrow \infty} \frac{\ln(5x)}{x^3 + 1}$ IF $\frac{\infty}{\infty}$

$$\stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{5x} \cdot 5}{3x^2}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \frac{1}{3x^2}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{3x^3}$$

$$= 0$$

answer:

0

2. Determine the first derivative of each of the following functions. Print your answer in the box provided. *You do not have to simplify your answers or explain your steps.*

_____ (a) [4 pts] $f(x) = 8x^3 - 15x + 12$

$$f'(x) = 24x^2 - 15$$

_____ (b) [6 pts] $g(t) = \frac{\sin(t)}{t}$

$$g'(t) = \frac{t \cos(t) - \sin(t)}{t^2}$$

_____ (c) [6 pts] $f(x) = \frac{e^x}{2x+1}$

$$f'(x) = \frac{(2x+1)(e^x) - e^x \cdot 2}{(2x+1)^2} = \frac{e^x(2x-1)}{(2x+1)^2}$$

_____ (d) [10 pts] $h(x) = (4x-3)^2 \arctan(x)$

$$h'(x) = (4x-3)^2 \cdot \frac{1}{1+x^2} + \arctan(x) \cdot 2(4x-3) \cdot 4$$

$$h'(x) = \frac{(4x-3)^2}{1+x^2} + 8(4x-3)\arctan(x)$$

3. (a) [8 pts] Determine $\frac{dy}{dx}$ for the equation $y^3 - x^4y = 6$. Print your answer in the box provided. You do not have to simplify your answer.

$$3y^2 \frac{dy}{dx} - \left(x^4 \frac{dy}{dx} + y \cdot 4x^3 \right) = 0$$

$$3y^2 \frac{dy}{dx} - x^4 \frac{dy}{dx} - 4x^3y = 0$$

$$3y^2 \frac{dy}{dx} - x^4 \frac{dy}{dx} = 4x^3y$$

$$\frac{dy}{dx} (3y^2 - x^4) = 4x^3y$$

$$\frac{dy}{dx} = \frac{4x^3y}{3y^2 - x^4}$$

- (b) [8 pts] Determine an equation of the tangent line to the curve $y^3 - x^4y = 6$ at the point $(1, 2)$.

$$\frac{dy}{dx} \Big|_{(1,2)} = \frac{4 \cdot 1 \cdot 2}{3 \cdot 4 - 1} = \frac{8}{11}$$

Equation:

$$y - 2 = \frac{8}{11}(x - 1)$$

4. Determine the following indefinite integrals. Print your answer to each part in the box provided.

_____ (a) [4 pts] $\int (-4x^7 + 8x^5 + 12) dx$

$$= -\frac{4}{8}x^8 + \frac{8}{6}x^6 + 12x + C$$

Final answer:

$$-\frac{1}{2}x^8 + \frac{4}{3}x^6 + 12x + C$$

_____ (b) [6 pts] $\int \left(\sec^2(t) + \frac{1}{t} \right) dt$

Final answer:

$$\tan(t) + \ln|t| + C$$

_____ (c) [10 pts] $\int \frac{x^4}{\sqrt{x^5 + 3}} dx$

$$= \int (x^5 + 3)^{-1/2} \cdot x^4 dx$$

$$= \frac{1}{5} \int (x^5 + 3)^{-1/2} \cdot 5x^4 dx \quad u = x^5 + 3 \\ du = 5x^4 dx$$

$$= \frac{1}{5} \int u^{-1/2} du$$

$$= \frac{1}{5} \cdot 2u^{1/2} + C$$

Final answer:

$$\frac{2}{5} \sqrt{x^5 + 3} + C$$

5. Evaluate the following definite integrals. Print your answer in the box provided.

_____ (a) [6 pts] $\int_1^8 \left(x^{2/3} - \frac{1}{x^{4/3}} \right) dx$

$$\begin{aligned} &= \int_1^8 (x^{2/3} - x^{-4/3}) dx \\ &= \left[\frac{3}{5}x^{5/3} + 3x^{-1/3} \right]_1^8 \\ &= \frac{3}{5}(8)^{5/3} + 3 \cdot 8^{-1/3} - \left(\frac{3}{5} + 3 \right) \end{aligned}$$

$$\begin{aligned} &\rightarrow = \frac{3 \cdot 32}{5} + \frac{3}{2} - \frac{3}{5} - \frac{6}{2} \\ &= \frac{96}{5} - \frac{3}{5} - \frac{3}{2} \\ &= \frac{93}{5} - \frac{3}{2} \end{aligned}$$

Value: $= \frac{186}{10} - \frac{15}{10} = \frac{171}{10} = 17.1$

_____ (b) [6 pts] $\int_0^{1/2} \frac{-1}{\sqrt{1-x^2}} dx$

$$\begin{aligned} &= [\arccos(x)]_0^{1/2} \\ &= \arccos(1/2) - \arccos(0) \\ &= \frac{\pi}{3} - \frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} &= [-\arcsin(x)]_0^{1/2} \\ &= -\arcsin(1/2) + \arcsin(0) \\ &= -\frac{\pi}{6} + 0 \end{aligned}$$

Value: $-\frac{\pi}{6}$

_____ (c) [10 pts] $\int_0^{\pi/4} \sin(4x)e^{\cos(4x)} dx = \int_1^{-1} -\frac{1}{4}e^u du = \left[-\frac{1}{4}e^u \right]_1^{-1}$

$$\begin{aligned} u &= \cos(4x) \\ du &= -4\sin(4x)dx \\ u(0) &= 1 \\ u(\pi/4) &= -1 \end{aligned}$$

Value: $-\frac{1}{4}e^{-1} - \left(-\frac{1}{4}e^1 \right) = \frac{-1}{4e} + \frac{e}{4}$
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6. (a) [5 pts] State the limit definition of the derivative of $f(x)$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

- (b) [10 pts] Use the limit definition of the derivative to show that the derivative of $f(x) = 12x - 2x^2$ is $f'(x) = 12 - 4x$. (You will receive 0 points for using the power rule.)

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{12(x+h) - 2(x+h)^2 - 12x + 2x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{12x + 12h - 2(x^2 + 2xh + h^2) - 12x + 2x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{12x + 12h - 2x^2 - 4xh - 2h^2 - 12x + 2x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{12h - 4xh - 2h^2}{h} \\ &= \lim_{h \rightarrow 0} (12 - 4x - 2h) \\ &= 12 - 4x \end{aligned}$$

- (c) [5 pts] Determine all values of x for which the graph of $f(x) = 12x - 2x^2$ has a horizontal tangent line.

$$\begin{aligned} 12 - 4x &= 0 \\ 12 &= 4x \\ 3 &= x \end{aligned}$$

7. [15 pts] Determine the absolute maximum and absolute minimum values of $f(x) = 2x\sqrt{9-x}$ on the interval $[-1, 9]$.

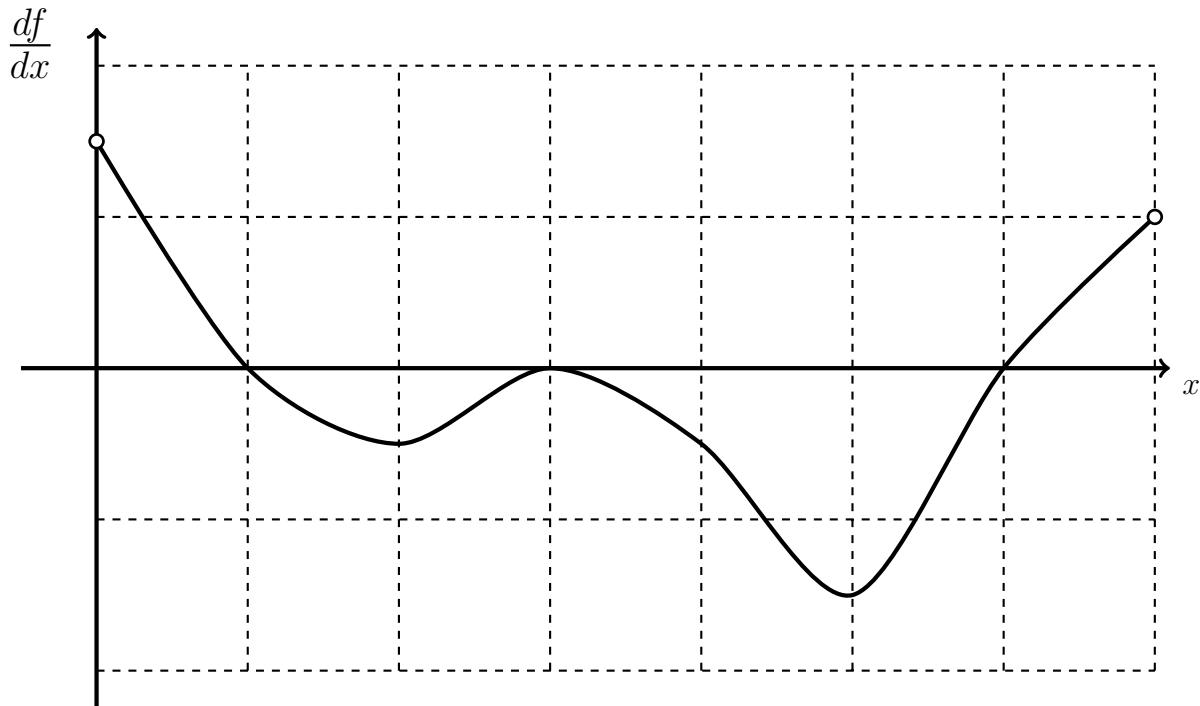
$$\begin{aligned}
 f'(x) &= 2x \cdot \frac{1}{2}(9-x)^{-\frac{1}{2}} \cdot -1 + \sqrt{9-x} \cdot 2 \\
 &= \frac{-x}{\sqrt{9-x}} + 2\sqrt{9-x} \cdot \frac{\sqrt{9-x}}{\sqrt{9-x}} \\
 &= \frac{-x + 2(9-x)}{\sqrt{9-x}} \\
 &= \frac{-x + 18 - 2x}{\sqrt{9-x}} \\
 &= \frac{18-3x}{\sqrt{9-x}}
 \end{aligned}$$

f' dne: $x=9$ not a critical number (domain)

$$f'(x)=0 : \quad x=6$$

x	f(x)
-1	$-2\sqrt{10}$ ← absolute min is $-2\sqrt{10}$
6	$12\sqrt{3}$ ← absolute max is $12\sqrt{3}$
9	0

8. The graph below is the graph of the derivative of $f(x)$. Use it to answer the questions that follow. The grid lines are one unit apart, and the domain of f is $(0, 7)$.



- _____ (a) [5 pts] Determine all critical numbers (critical points) of f .

$$x = 1, 3, 6$$

- _____ (b) [5 pts] Determine the intervals on which f is increasing.

$$(0, 1] \cup [6, 7)$$

also accepted: $(0, 1) \cup (6, 7)$

- _____ (c) [5 pts] Determine all values of x at which f has a local minimum.

f' changes sign from - to + at $x = 6$

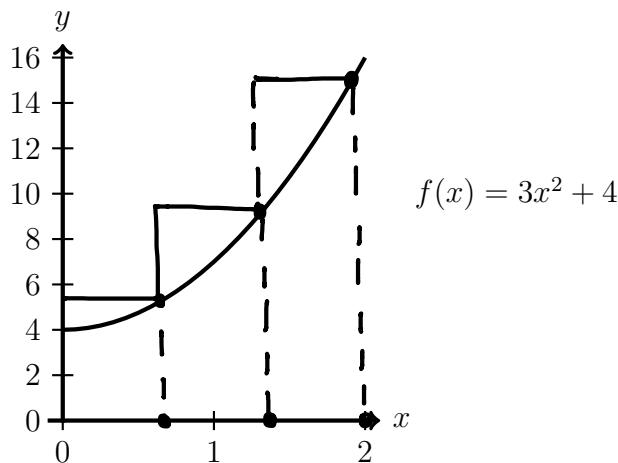
- _____ (d) [5 pts] Determine the intervals on which f is concave up.

f is concave up where f' is increasing :

$$[2, 3] \cup [5, 7]$$

also accepted: $(2, 3) \cup (5, 7)$

9. For this problem, use $f(x) = 3x^2 + 4$ on the interval $[0, 2]$. Its graph is provided to the right.



- (a) [5 pts] Determine a Riemann sum for f on the interval $[0, 2]$ using 3 subintervals of equal width and using right endpoints on each subinterval.

$$\Delta x = \frac{2-0}{3} = \frac{2}{3} \quad [0, \frac{2}{3}] \left[\frac{2}{3}, \frac{4}{3} \right] \left[\frac{4}{3}, 2 \right]$$

Rie. sum: $f\left(\frac{2}{3}\right) \cdot \frac{2}{3} + f\left(\frac{4}{3}\right) \cdot \frac{2}{3} + f(2) \cdot \frac{2}{3}$ 20.4

$$= \left(3\left(\frac{2}{3}\right)^2 + 4\right) \cdot \frac{2}{3} + \left(3\left(\frac{4}{3}\right)^2 + 4\right) \cdot \frac{2}{3} + \left(3(2)^2 + 4\right) \cdot \frac{2}{3}$$

- (b) [5 pts] Is your Riemann sum above an over- or under-estimate of the integral $\int_0^2 f(x) dx$?

Explain how you can tell, *without doing any calculations or working out the answers*, whether it's an over-estimate or an under-estimate. (You may want to illustrate the Riemann sum on the graph of f provided above.)

It's an over-estimate because f is increasing on $[0, 2]$

or: ... because the rectangles above contain more area than the region under the curve.

- (c) [5 pts] Use summation (sigma) notation to write an expression for a Riemann sum for f on the interval $[0, 2]$ using n subintervals of equal width and using right endpoints on each subinterval. You do not have to work out the value of the sum, but your sum should involve only $\sum_{k=1}^n$, the variables k and n , and numbers.

$$\Delta x = \frac{2}{n}$$

$$c_k = \frac{2k}{n}$$

$$f(c_k) = 3\left(\frac{2k}{n}\right)^2 + 4 = 3\left(\frac{4k^2}{n^2}\right) + 4$$

$$\text{Rie. sum} = \sum_{k=1}^n \left[3\left(\frac{2k}{n}\right)^2 + 4 \right] \cdot \frac{2}{n} = \sum_{k=1}^n \left(\frac{12k^2}{n^2} + 4 \right) \cdot \frac{2}{n}$$

$$= \boxed{\sum_{k=1}^n \left(\frac{24k^2}{n^3} + \frac{8}{n} \right)}$$

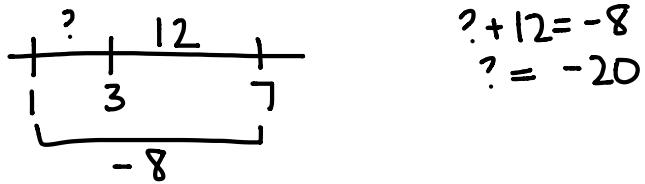
10. Use the values of the given definite integrals to determine the quantities below.

$$\int_1^7 f(x) \, dx = -8, \quad \int_3^7 f(x) \, dx = 12, \quad \int_1^7 g(x) \, dx = 9$$

_____ (a) [5 pts] $\int_1^7 (2f(x) - 5g(x)) \, dx$

$$\begin{aligned} &= 2(-8) - 5(9) \\ &= -16 - 45 \\ &= -61 \end{aligned}$$

_____ (b) [5 pts] $\int_1^3 f(x) \, dx = -20$



(c) [5 pts] $\int_1^7 (g(t) - t^2) \, dt$

$$\begin{aligned} &= \int_1^7 g(t) \, dt - \int_1^7 t^2 \, dt \\ &= 9 - \left[\frac{1}{3} t^3 \right]_1^7 \\ &= 9 - \left(\frac{1}{3} \cdot 7^3 - \frac{1}{3} \cdot 1^3 \right) \leftarrow \begin{matrix} \text{ok final} \\ \text{answer} \end{matrix} \\ &= \frac{27}{3} - \frac{343}{3} + \frac{1}{3} = -\frac{315}{3} = 105 \end{aligned}$$

11. The charts below contain information about a function f and its derivative. Assume that f is differentiable on $[-2, 1]$. Use the charts to answer the questions that follow.

x	-2	-1	0	1
$f(x)$	3	2	0	-1

x	-2	-1	0	1
$f'(x)$	$-\frac{1}{8}$	$-\frac{1}{3}$	-1	0

- _____ (a) [5 pts] Determine the linearization of f at $x = -1$.

$$\begin{aligned}
 L(x) &= f(-1) + f'(-1)(x+1) \\
 &= \boxed{2 + -\frac{1}{3}(x+1)} \leftarrow \text{ok final answer} \\
 &= 2 - \frac{1}{3}x - \frac{1}{3} \\
 &= \frac{5}{3} - \frac{1}{3}x
 \end{aligned}$$

- _____ (b) [5 pts] Use your linearization above to estimate the value of $f(-1.5)$.

$$\begin{aligned}
 f(-1.5) &\approx L(-1.5) = \boxed{2 - \frac{1}{3}(-1.5 + 1)} \leftarrow \text{ok final answer} \\
 &= 2 - \frac{1}{3}\left(-\frac{1}{2}\right) \\
 &= 2 + \frac{1}{6} \\
 &= \frac{13}{6} \\
 &= 2.\overline{16}
 \end{aligned}$$

- _____ (c) [5 pts] Suppose you also know that f' is continuous on $[-2, 1]$. Explain why the graph of f must have an inflection point somewhere in the interval $[-2, 1]$.

Based on the chart for f' , there must be an inflection point in $[-2, 1]$ since f' changes from decreasing to increasing (at least once).

12. [15 pts] A diesel truck develops an oil leak. The oil drips onto the dry ground in the shape of a circular puddle. Assuming that the leak begins at time $t = 0$ and that the radius of the oil slick increases at a constant rate of .05 meters per minute, determine the rate of change of the area of the puddle 4 minutes after the leak begins.

goal: $\frac{dA}{dt}$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dr}{dt} = .05 \text{ meters/minute}$$

$$r = \frac{.05 \text{ meters}}{1 \text{ min}} \times 4 \text{ minutes} = 0.2 \text{ m} \quad \text{since } \frac{dr}{dt} \text{ is constant}$$

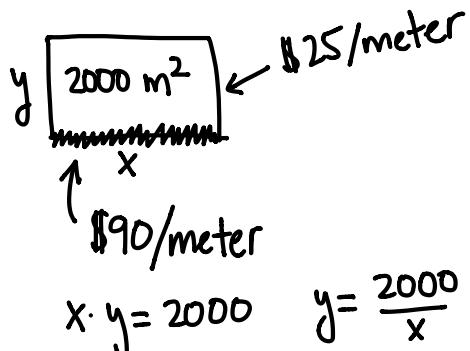
$$\frac{dA}{dt} = 2\pi(0.2)(.05) \text{ square meters/minute}$$

$$= .02\pi \text{ square meters/minute}$$

$$\approx .06283 \text{ square meters/minute}$$

13. A landscape designer plans to construct a rectangular garden whose area is 2000 square meters. One side will consist of a wrought iron fence which costs \$90 per meter. The remaining three sides will be constructed from chain link fence costing \$25 per meter.

- (a) [15 pts] Determine a function for the total cost $C(x)$ of the garden, where x is the length of wrought iron fence used (in meters).



$$C(x) = 90x + 25\left(\frac{2000}{x}\right) + 25x + 25\left(\frac{2000}{x}\right)$$

$$C(x) = 115x + \frac{100000}{x}$$

- (b) [15 pts] What dimensions of the garden will minimize the total cost? Use calculus techniques to show that the dimensions result in the minimum possible cost.

domain: $(0, \infty)$

$$C'(x) = 115 - \frac{100000}{x^2}$$

$$C' \text{ due: none } C' = 0: 115 = \frac{100000}{x^2}$$

$$x^2 = \frac{100000}{115}$$

$$x^2 = \frac{20000}{23}$$

$$x = \sqrt{\frac{20000}{23}} = 100 \cdot \sqrt{\frac{2}{23}}$$

Note: The first derivative test is also fine to use.
See the last page.

$$\text{Second derivative test: } C''(x) = \frac{200000}{x^3}$$

$$C''\left(\sqrt{\frac{20000}{23}}\right) = \frac{200000}{\sqrt{\frac{20000}{23}}} = 2000 \cdot \sqrt{\frac{23}{2}} > 0 \text{ so } C(x) \text{ has a local min at } x = 100 \cdot \sqrt{\frac{2}{23}}$$

Since there is only one critical number in $(0, \infty)$, the local min is an absolute min. The dimensions are $x = 100 \cdot \sqrt{\frac{2}{23}} \approx 29.4884$ meters

and $y = \frac{2000}{100 \sqrt{\frac{2}{23}}} \approx 67.8233$ meters.

14. [10 pts] Let $y = \ln(x)$. Show that $\frac{dy}{dx} = \frac{1}{x}$ by solving the equation $y = \ln(x)$ for x and then using implicit differentiation. Your final answer should be $\frac{dy}{dx}$, given as a function of x .

$$y = \ln(x)$$

$$e^y = x \quad (\text{solve for } x)$$

$$e^y \frac{dy}{dx} = 1 \quad (\text{differentiate both sides with respect to } x)$$

$$\frac{dy}{dx} = \frac{1}{e^y} \quad (\text{solve for } \frac{dy}{dx})$$

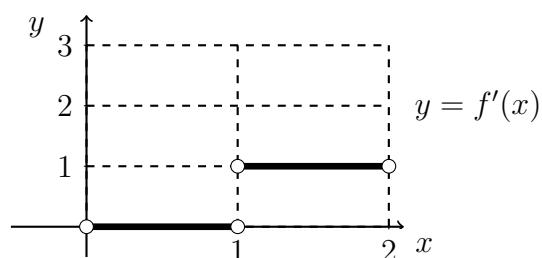
$$\frac{dy}{dx} = \frac{1}{x} \quad (\text{sub in } x \text{ for } e^y \text{ since } e^y = x)$$

15. Information about a function, f , and its derivative is given below. Use the information to answer the questions that follow.

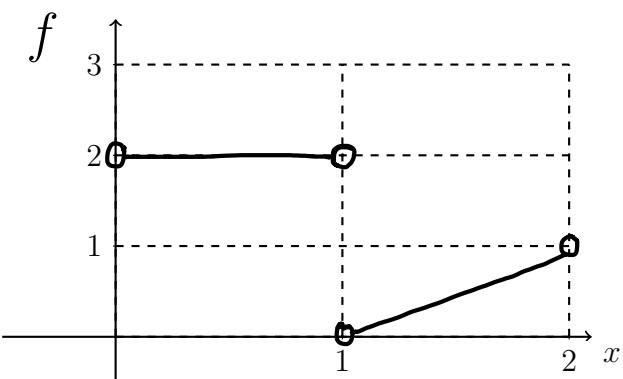
Information about f :

$$\begin{aligned}\lim_{x \rightarrow 0^+} f(x) &= 2, \\ \lim_{x \rightarrow 2^-} f(x) &= 1,\end{aligned}$$

This is the graph of the **derivative** of f :



- (a) [5 pts] Make a rough sketch of the graph of $y = f(x)$. (Hint: Think about slopes.)



function values at
 $x = 0, 1, 2$ are not
needed
* Does your graph pass the
vertical line test at $x = 1$?

- (b) [5 pts] Determine $\lim_{x \rightarrow 1^-} f(x)$.

2

- (c) [5 pts] Determine $\lim_{x \rightarrow 1^+} f(x)$.

0

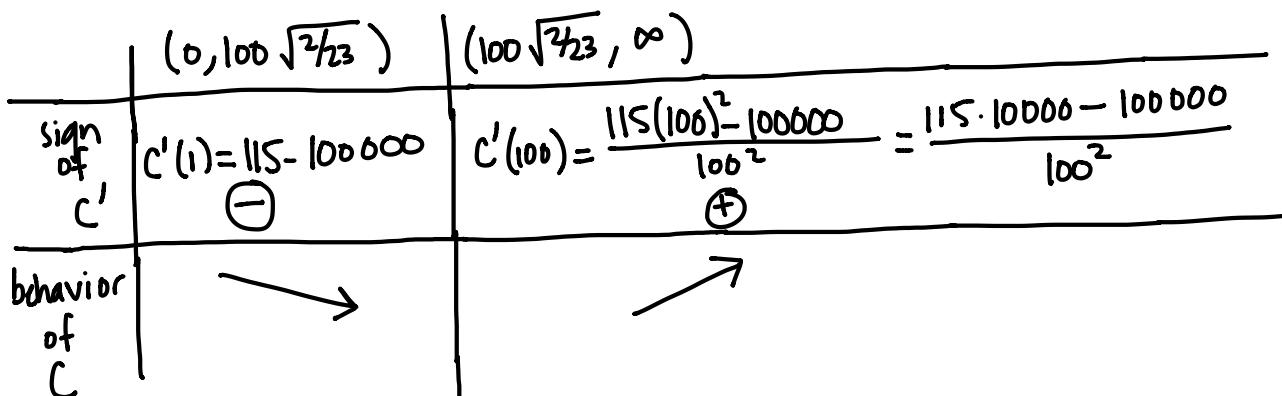
Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____

#13 - 1st derivative test

$$C'(x) = 115 - \frac{100,000}{x^2} = \frac{115x^2 - 100,000}{x^2}$$

Critical number: $x = 100\sqrt{\frac{2}{23}} \approx 29.49$



Therefore $C(x)$ has a local min at $x = 100\sqrt{\frac{2}{23}}$

Since there is only one critical number in $(0, \infty)$, the local min is an absolute min. The dimensions are $x = 100 \cdot \sqrt{\frac{2}{23}} \approx 29.4884$ meters

and $y = \frac{2000}{100\sqrt{\frac{2}{23}}} \approx 67.8233$ meters.