Boxplots of 100 Training Error Rates and Test Error Rates for 6 methods

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1 Introduction

In this project, I repeat the following 100 times:

Randomly split the data in half, that is train and test.

Fitted the models with 6 methods:

- 1. LDA
- 2. QDA
- 3. KNN and tune K using 10-fold CV
- 4. LASSO logistic and tune lambda using 10-fold CV
- 5. Ridge logistic and tune lambda using 10 fold CV
- 6. Random Forest with m = sqrt(p) and 300 bootstrapped trees

Later use the models to predict the responses for the TRAIN data and TEST data, and calculate the TRAINING error rate and TEST error rate.

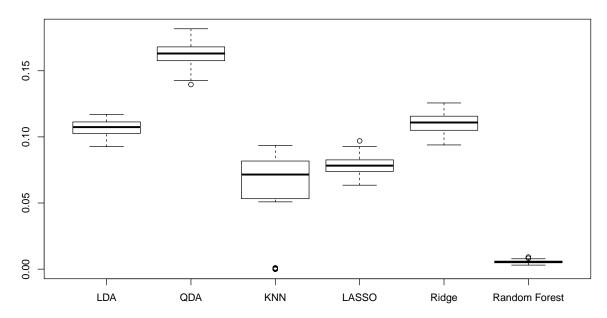
At last, I created side-by-side boxplots of the 100 Training Error Rates and Test Error Rates for the methods mentioned above.

Data Source: Spam Dataset https://web.stanford.edu/~hastie/ElemStatLearn/

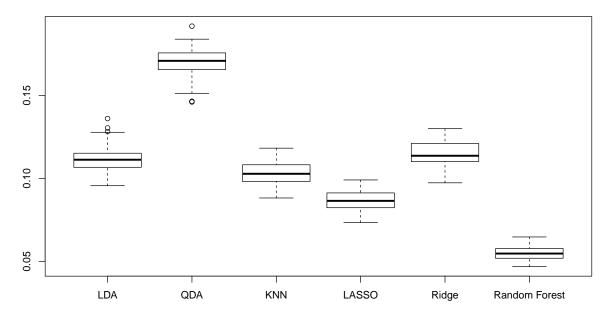
2 Boxplots

Side-by-side boxplots of the 100 Training Error Rates and Test Error Rates for LDA, QDA with noise, KNN, Lasso, Ridge, Random Forest methods shown here.

100 TRAINING ERROR RATES



100 TEST ERROR RATES



3 Conclusion

Within one method, we see that a model's training error rate is lower than test error rate, test error rate is true error rate, while the training error rate is often overly optimistic.

From the Boxplots, we see that **Random Forest seems works best** for this dataset, test error rate and training error rate are all very low. While **QDA perform worst**, actually this is already adding noise to QDA, the error rate is still highest. Other methods perform between QDA and Random Forest.

Lasso and ridge are similar methods, while Lasso perform better in this dataset. LDA and KNN perform ok. Also since I used Train data and tune.knn() function to choose optimal k, k value finally use are different (most k are from 1-10), so the variance of training error rate of KNN is larger than others.

4 Appendix

4.1 Matrix of Training Error Rates and Test Error Rates

Training Error Rate Matrix:

```
##
                 LDA
                           QDA
                                         KNN
                                                  LASSO
                                                             Ridge Random Forest
##
     [1,] 0.10652174 0.1621739 0.0682608696 0.07173913 0.11304348
                                                                      0.005217391
##
     [2,] 0.11304348 0.1652174 0.0717391304 0.08130435 0.11217391
                                                                      0.005217391
     [3,] 0.10565217 0.1569565 0.0000000000 0.07521739 0.10913043
##
                                                                      0.006086957
##
     [4,] 0.10086957 0.1608696 0.0691304348 0.07130435 0.10304348
                                                                      0.003913043
     [5.] 0.11173913 0.1713043 0.0913043478 0.08043478 0.11086957
##
                                                                      0.005652174
##
     [6,] 0.10391304 0.1643478 0.0839130435 0.08260870 0.10869565
                                                                      0.006521739
##
     [7,] 0.09913043 0.1626087 0.0730434783 0.07043478 0.09608696
                                                                      0.005217391
##
     [8,] 0.11521739 0.1630435 0.0826086957 0.07695652 0.11826087
                                                                      0.009130435
##
     [9,] 0.11130435 0.1608696 0.0534782609 0.07434783 0.11434783
                                                                      0.004347826
    [10,] 0.11608696 0.1665217 0.0856521739 0.08478261 0.11347826
##
                                                                      0.004782609
##
    [11,] 0.10173913 0.1617391 0.0765217391 0.08086957 0.10347826
                                                                      0.005217391
##
    [12,] 0.10739130 0.1617391 0.0865217391 0.08521739 0.12173913
                                                                      0.006086957
##
    [13,] 0.11565217 0.1647826 0.0008695652 0.07826087 0.11086957
                                                                      0.005217391
##
    [14,] 0.09695652 0.1495652 0.0556521739 0.07000000 0.10043478
                                                                      0.005217391
    [15,] 0.10956522 0.1617391 0.0700000000 0.08652174 0.11347826
##
                                                                      0.005217391
    [16,] 0.11043478 0.1691304 0.0008695652 0.08260870 0.11347826
##
                                                                      0.004782609
##
    [17,] 0.09391304 0.1604348 0.0752173913 0.06739130 0.09695652
                                                                      0.006086957
    [18,] 0.11608696 0.1760870 0.0821739130 0.08391304 0.12043478
##
                                                                      0.004782609
    [19,] 0.10217391 0.1591304 0.0556521739 0.08043478 0.10217391
##
                                                                      0.004347826
##
    [20,] 0.09826087 0.1600000 0.0004347826 0.07086957 0.10304348
                                                                      0.005652174
    [21,] 0.09956522 0.1660870 0.0800000000 0.07521739 0.10782609
##
                                                                      0.004782609
    [22,] 0.11000000 0.1560870 0.0856521739 0.07043478 0.10434783
##
                                                                      0.004347826
    [23,] 0.10304348 0.1495652 0.0852173913 0.07260870 0.10739130
##
                                                                      0.004782609
##
    [24,] 0.10478261 0.1426087 0.0004347826 0.07434783 0.11521739
                                                                      0.004782609
##
    [25,] 0.10739130 0.1552174 0.0665217391 0.07913043 0.11478261
                                                                      0.005217391
##
    [26,] 0.10913043 0.1756522 0.0513043478 0.08521739 0.11565217
                                                                      0.005652174
    [27,] 0.10869565 0.1730435 0.0000000000 0.08130435 0.11130435
##
                                                                      0.003913043
                                                                      0.005217391
##
    [28,] 0.09782609 0.1573913 0.0000000000 0.07043478 0.09652174
##
    [29,] 0.11434783 0.1769565 0.0826086957 0.08391304 0.12347826
                                                                      0.007826087
    [30,] 0.10956522 0.1452174 0.0860869565 0.06695652 0.11086957
##
                                                                      0.006086957
##
    [31,] 0.11043478 0.1717391 0.0913043478 0.08086957 0.12260870
                                                                      0.006521739
##
    [32,] 0.11173913 0.1682609 0.0556521739 0.07782609 0.11521739
                                                                      0.006086957
    [33,] 0.10260870 0.1630435 0.0769565217 0.08782609 0.10913043
                                                                      0.005217391
##
##
    [34,] 0.10478261 0.1634783 0.0826086957 0.08173913 0.10739130
                                                                      0.005652174
    [35,] 0.10043478 0.1652174 0.0804347826 0.07826087 0.11347826
##
                                                                      0.007391304
##
    [36,] 0.11347826 0.1652174 0.0004347826 0.07608696 0.11043478
                                                                      0.005652174
    [37,] 0.10956522 0.1652174 0.0543478261 0.08043478 0.11695652
                                                                      0.004347826
    [38,] 0.10478261 0.1395652 0.0900000000 0.07304348 0.10739130
##
                                                                      0.005652174
```

```
[39,] 0.10826087 0.1473913 0.0773913043 0.07565217 0.10521739
                                                                      0.005652174
##
    [40,] 0.10086957 0.1513043 0.0821739130 0.06956522 0.10652174
                                                                      0.006521739
##
    [41,] 0.10521739 0.1600000 0.0660869565 0.07739130 0.10217391
                                                                      0.007391304
##
    [42,] 0.10826087 0.1691304 0.0008695652 0.08173913 0.11434783
                                                                      0.006521739
##
    [43,] 0.10521739 0.1586957 0.0000000000 0.07826087 0.11913043
                                                                      0.003913043
    [44,] 0.10260870 0.1678261 0.0008695652 0.07391304 0.11086957
##
                                                                      0.006956522
    [45,] 0.11478261 0.1717391 0.0817391304 0.09260870 0.11739130
                                                                      0.005217391
##
    [46,] 0.10739130 0.1604348 0.0582608696 0.07695652 0.10391304
                                                                      0.005652174
##
    [47,] 0.10260870 0.1678261 0.0886956522 0.08173913 0.11347826
                                                                      0.003913043
##
    [48,] 0.11000000 0.1530435 0.0778260870 0.08304348 0.11565217
                                                                      0.005652174
    [49,] 0.10391304 0.1491304 0.0530434783 0.07695652 0.12130435
                                                                      0.006521739
    [50,] 0.09260870 0.1773913 0.0665217391 0.07434783 0.10173913
##
                                                                      0.004782609
##
    [51,] 0.10565217 0.1617391 0.0008695652 0.07217391 0.10347826
                                                                      0.005652174
    [52,] 0.11652174 0.1695652 0.0752173913 0.08478261 0.11608696
                                                                      0.007391304
##
##
    [53,] 0.10043478 0.1552174 0.0000000000 0.07565217 0.10391304
                                                                      0.003478261
##
    [54,] 0.10347826 0.1765217 0.0004347826 0.07608696 0.11086957
                                                                      0.006521739
    [55,] 0.11217391 0.1682609 0.0004347826 0.08826087 0.11913043
##
                                                                      0.006521739
##
    [56,] 0.10434783 0.1556522 0.0900000000 0.08391304 0.10956522
                                                                      0.004347826
    [57,] 0.11217391 0.1656522 0.0730434783 0.08739130 0.12304348
##
                                                                      0.003913043
##
    [58,] 0.10130435 0.1491304 0.0826086957 0.07608696 0.10565217
                                                                      0.005652174
##
    [59,] 0.10260870 0.1617391 0.0926086957 0.07652174 0.10347826
                                                                      0.006956522
    [60,] 0.10434783 0.1508696 0.0660869565 0.07086957 0.10608696
##
                                                                      0.004782609
##
    [61,] 0.09782609 0.1582609 0.0569565217 0.08043478 0.09434783
                                                                      0.004347826
    [62.] 0.09695652 0.1569565 0.0791304348 0.06782609 0.10173913
##
                                                                      0.006086957
    [63,] 0.10869565 0.1808696 0.0700000000 0.08217391 0.11217391
                                                                      0.006086957
##
    [64,] 0.11565217 0.1630435 0.0734782609 0.08391304 0.12217391
                                                                      0.003913043
##
    [65,] 0.09304348 0.1556522 0.0704347826 0.06826087 0.09565217
                                                                      0.004347826
    [66,] 0.09652174 0.1634783 0.0000000000 0.07391304 0.10652174
##
                                                                      0.004782609
##
    [67,] 0.10739130 0.1647826 0.0573913043 0.08434783 0.11260870
                                                                      0.006521739
##
    [68,] 0.10304348 0.1604348 0.0000000000 0.07173913 0.10434783
                                                                      0.003478261
##
    [69,] 0.10956522 0.1547826 0.0734782609 0.08130435 0.11652174
                                                                      0.004782609
##
    [70,] 0.11173913 0.1686957 0.0769565217 0.08521739 0.10826087
                                                                      0.003913043
##
    [71,] 0.11043478 0.1552174 0.0843478261 0.08000000 0.11391304
                                                                      0.007391304
    [72,] 0.11391304 0.1660870 0.0843478261 0.07913043 0.11652174
##
                                                                      0.003913043
##
    [73,] 0.10434783 0.1517391 0.0547826087 0.07173913 0.10391304
                                                                      0.005217391
##
    [74,] 0.10434783 0.1608696 0.0778260870 0.07782609 0.11739130
                                                                      0.006086957
##
    [75,] 0.11347826 0.1782609 0.0508695652 0.08826087 0.12000000
                                                                      0.005652174
##
    [76,] 0.11130435 0.1552174 0.0643478261 0.07869565 0.12565217
                                                                      0.005217391
    [77,] 0.09478261 0.1578261 0.0817391304 0.06347826 0.09608696
##
                                                                      0.004782609
    [78,] 0.10086957 0.1660870 0.0752173913 0.06869565 0.10608696
##
                                                                      0.006956522
    [79,] 0.10956522 0.1717391 0.0004347826 0.08000000 0.11173913
                                                                      0.006086957
##
    [80,] 0.10782609 0.1669565 0.0000000000 0.08608696 0.11391304
                                                                      0.006086957
    [81.] 0.11086957 0.1704348 0.0726086957 0.07565217 0.10608696
##
                                                                      0.007826087
##
    [82,] 0.10391304 0.1465217 0.0660869565 0.07173913 0.10521739
                                                                      0.004782609
    [83,] 0.11086957 0.1617391 0.0795652174 0.07739130 0.11391304
                                                                      0.005652174
    [84,] 0.10652174 0.1617391 0.0713043478 0.07869565 0.10304348
##
                                                                      0.004347826
##
    [85,] 0.11000000 0.1743478 0.0591304348 0.07260870 0.11086957
                                                                      0.006086957
    [86,] 0.11217391 0.1669565 0.0004347826 0.08565217 0.11260870
##
                                                                      0.005217391
                                                                      0.006956522
    [87,] 0.11000000 0.1600000 0.0521739130 0.08173913 0.11826087
##
    [88,] 0.10869565 0.1665217 0.0678260870 0.07826087 0.10782609
                                                                      0.003913043
##
    [89,] 0.11521739 0.1678261 0.0782608696 0.09695652 0.12173913
                                                                      0.005652174
##
    [90,] 0.10043478 0.1817391 0.0921739130 0.08043478 0.10391304
                                                                      0.006956522
    [91,] 0.11652174 0.1647826 0.0843478261 0.09086957 0.12130435
##
                                                                      0.005652174
    [92,] 0.11043478 0.1639130 0.0678260870 0.07521739 0.11043478
                                                                      0.004782609
```

```
[93,] 0.11391304 0.1608696 0.0730434783 0.08000000 0.10826087
                                                                     0.003043478
##
    [94,] 0.11000000 0.1704348 0.0782608696 0.07826087 0.11217391
                                                                     0.004347826
    [95,] 0.09782609 0.1660870 0.0773913043 0.08043478 0.10478261
##
                                                                     0.006086957
##
   [96,] 0.09956522 0.1460870 0.0682608696 0.07260870 0.09391304
                                                                     0.003478261
##
    [97,] 0.11434783 0.1673913 0.0934782609 0.08260870 0.11565217
                                                                     0.008260870
    [98,] 0.10695652 0.1686957 0.0830434783 0.07217391 0.10739130
##
                                                                     0.003913043
    [99,] 0.11434783 0.1700000 0.0808695652 0.08391304 0.11739130
                                                                     0.005652174
## [100,] 0.11695652 0.1782609 0.0004347826 0.08260870 0.12304348
                                                                     0.007826087
```

Test Error Rate Matrix:

```
##
                 LDA
                           QDA
                                       KNN
                                                LASSO
                                                           Ridge Random Forest
##
     [1,] 0.09778357 0.1616688 0.09821817 0.08344198 0.09908735
                                                                     0.05432421
##
     [2,] 0.10908301 0.1642764 0.09995654 0.08387658 0.11082138
                                                                     0.05649718
##
     [3,] 0.12168622 0.1755758 0.10908301 0.08952629 0.12690135
                                                                     0.05736636
##
     [4,] 0.11386354 0.1751412 0.10039113 0.09343764 0.11169057
                                                                     0.05432421
     [5,] 0.13037810 0.1547153 0.11386354 0.09387223 0.12733594
##
                                                                     0.05388961
##
     [6,] 0.11212516 0.1581921 0.10777923 0.09039548 0.11299435
                                                                     0.05953933
     [7,] 0.10647545 0.1720991 0.09734898 0.07779226 0.10995219
##
                                                                     0.05345502
##
     [8,] 0.10908301 0.1677532 0.10039113 0.09169926 0.11255976
                                                                     0.06040852
     [9,] 0.12646675 0.1694915 0.10560626 0.08996089 0.12429379
##
                                                                     0.05736636
##
    [10,] 0.10734463 0.1673186 0.10256410 0.08822251 0.10691004
                                                                     0.04997827
##
    [11,] 0.10212951 0.1655802 0.10343329 0.08474576 0.10473707
                                                                     0.06214689
    [12,] 0.10126032 0.1773142 0.09734898 0.07866145 0.09952195
##
                                                                     0.05258583
##
    [13,] 0.11169057 0.1464581 0.11212516 0.08604954 0.11473272
                                                                     0.05780096
    [14,] 0.12820513 0.1742721 0.10647545 0.09083007 0.12255541
##
                                                                     0.06388527
##
    [15,] 0.10430248 0.1738375 0.09213385 0.07953064 0.10691004
                                                                     0.05388961
##
    [16,] 0.10777923 0.1629726 0.09734898 0.08604954 0.10691004
                                                                     0.05215124
##
    [17,] 0.12125163 0.1825293 0.10386788 0.09691439 0.12168622
                                                                     0.05649718
##
    [18,] 0.11690569 0.1651456 0.10082573 0.09213385 0.12081704
                                                                     0.04737071
##
    [19,] 0.12125163 0.1673186 0.10299870 0.09865276 0.12559757
                                                                     0.05910474
    [20,] 0.10473707 0.1621034 0.11386354 0.08126901 0.11994785
                                                                     0.05736636
##
##
    [21,] 0.10604085 0.1755758 0.09865276 0.08170361 0.11038679
                                                                     0.06214689
    [22,] 0.10647545 0.1773142 0.10691004 0.08300739 0.11082138
                                                                     0.05953933
##
##
    [23,] 0.10951760 0.1586267 0.10821382 0.07648848 0.11212516
                                                                     0.05302043
##
    [24,] 0.11907866 0.1751412 0.11429813 0.09213385 0.12299000
                                                                     0.06475445
    [25,] 0.11342894 0.1586267 0.09517601 0.09474142 0.12516297
##
                                                                     0.05258583
    [26,] 0.12777053 0.1755758 0.11038679 0.09213385 0.12603216
                                                                     0.05432421
    [27,] 0.11864407 0.1707953 0.10821382 0.08474576 0.11169057
##
                                                                     0.05258583
##
    [28,] 0.10777923 0.1707953 0.11212516 0.08474576 0.10691004
                                                                     0.05823555
##
    [29,] 0.10821382 0.1690569 0.10299870 0.08126901 0.11994785
                                                                     0.05432421
    [30,] 0.12299000 0.1760104 0.11820947 0.09821817 0.12472838
                                                                     0.05649718
    [31,] 0.09647979 0.1616688 0.09517601 0.07344633 0.10430248
##
                                                                     0.04823990
##
    [32,] 0.11864407 0.1707953 0.10082573 0.09865276 0.12429379
                                                                     0.05432421
##
    [33,] 0.11342894 0.1712299 0.10647545 0.08996089 0.11386354
                                                                     0.05084746
    [34,] 0.10734463 0.1794872 0.10560626 0.09083007 0.11082138
##
                                                                     0.05171664
##
    [35,] 0.10691004 0.1742721 0.09995654 0.08648414 0.11082138
                                                                     0.05388961
##
    [36,] 0.10430248 0.1673186 0.10343329 0.07909605 0.11386354
                                                                     0.05475880
    [37,] 0.10691004 0.1664494 0.11212516 0.07909605 0.11169057
##
                                                                     0.04823990
    [38,] 0.11125598 0.1603651 0.11255976 0.08213820 0.11255976
##
                                                                     0.05084746
##
    [39,] 0.11125598 0.1738375 0.09908735 0.09734898 0.12125163
                                                                     0.05910474
##
   [40,] 0.10951760 0.1807910 0.11473272 0.07953064 0.11169057
                                                                     0.05606258
    [41,] 0.11516732 0.1773142 0.09517601 0.09083007 0.11516732
                                                                     0.05606258
    [42,] 0.12125163 0.1703607 0.09604520 0.08648414 0.12168622
                                                                     0.05606258
##
```

```
[43,] 0.10212951 0.1577575 0.09995654 0.07909605 0.11734029
                                                                     0.05693177
##
    [44,] 0.10821382 0.1616688 0.10560626 0.08604954 0.11386354
                                                                     0.05649718
    [45,] 0.09561060 0.1760104 0.08909170 0.07431551 0.09821817
                                                                     0.04737071
##
##
    [46,] 0.11994785 0.1677532 0.11038679 0.09387223 0.11994785
                                                                     0.06388527
##
    [47,] 0.11864407 0.1616688 0.11342894 0.09561060 0.12212082
                                                                     0.05693177
    [48,] 0.10604085 0.1716645 0.10777923 0.08257279 0.11038679
##
                                                                     0.05084746
    [49,] 0.10864841 0.1690569 0.09778357 0.08083442 0.12559757
                                                                     0.05171664
##
    [50,] 0.10169492 0.1651456 0.10169492 0.08387658 0.10517166
                                                                     0.05867014
##
    [51,] 0.10995219 0.1686223 0.11212516 0.07996523 0.10908301
                                                                     0.05128205
##
    [52,] 0.11299435 0.1707953 0.10343329 0.08604954 0.10777923
                                                                     0.05084746
    [53,] 0.09995654 0.1734029 0.11429813 0.08691873 0.11038679
                                                                     0.05475880
    [54,] 0.10734463 0.1768796 0.10647545 0.08735332 0.11386354
##
                                                                     0.05475880
##
    [55,] 0.11342894 0.1690569 0.10995219 0.09169926 0.11560191
                                                                     0.06214689
    [56,] 0.11777488 0.1677532 0.10777923 0.08518036 0.12125163
                                                                     0.05432421
##
##
    [57,] 0.11734029 0.1512386 0.10039113 0.09908735 0.12950891
                                                                     0.05823555
##
    [58,] 0.10951760 0.1690569 0.11299435 0.08952629 0.11864407
                                                                     0.06345067
##
    [59,] 0.10517166 0.1760104 0.09952195 0.07909605 0.10647545
                                                                     0.04693611
##
    [60,] 0.11516732 0.1812256 0.10212951 0.09083007 0.11907866
                                                                     0.05780096
    [61,] 0.11429813 0.1777488 0.10604085 0.09430682 0.11777488
##
                                                                     0.05171664
##
    [62,] 0.09821817 0.1712299 0.09691439 0.07779226 0.09734898
                                                                     0.05780096
##
    [63,] 0.11169057 0.1720991 0.09474142 0.09213385 0.12168622
                                                                     0.05258583
    [64,] 0.11125598 0.1664494 0.09604520 0.08822251 0.12081704
##
                                                                     0.05084746
##
    [65,] 0.11212516 0.1677532 0.10951760 0.08170361 0.11038679
                                                                     0.05693177
    [66.] 0.10691004 0.1707953 0.11777488 0.08300739 0.10951760
##
                                                                     0.05606258
    [67,] 0.11994785 0.1755758 0.10343329 0.09430682 0.12168622
##
                                                                     0.05780096
    [68,] 0.11342894 0.1655802 0.11429813 0.08996089 0.11125598
                                                                     0.06214689
##
    [69,] 0.09908735 0.1799218 0.08865711 0.08170361 0.10864841
                                                                     0.05388961
##
    [70,] 0.11647110 0.1807910 0.10169492 0.08952629 0.11212516
                                                                     0.04954368
##
    [71,] 0.10126032 0.1651456 0.10082573 0.08778792 0.11125598
                                                                     0.05606258
##
    [72,] 0.12429379 0.1664494 0.10430248 0.09039548 0.12516297
                                                                     0.05432421
##
    [73,] 0.11603651 0.1742721 0.11082138 0.09604520 0.11820947
                                                                     0.05345502
##
    [74,] 0.11038679 0.1777488 0.10212951 0.08257279 0.11212516
                                                                     0.06084311
##
    [75,] 0.11473272 0.1734029 0.10647545 0.09604520 0.12299000
                                                                     0.04693611
    [76,] 0.11429813 0.1460235 0.10039113 0.08648414 0.12994350
##
                                                                     0.05953933
##
    [77,] 0.10386788 0.1720991 0.11820947 0.08431117 0.10560626
                                                                     0.05867014
##
    [78,] 0.10908301 0.1655802 0.09604520 0.07779226 0.10777923
                                                                     0.05475880
##
    [79,] 0.10647545 0.1564537 0.09647979 0.08648414 0.11212516
                                                                     0.05171664
##
    [80,] 0.11473272 0.1734029 0.11516732 0.08387658 0.11647110
                                                                     0.05171664
##
    [81,] 0.11299435 0.1668840 0.08822251 0.09083007 0.11342894
                                                                     0.05823555
##
    [82,] 0.10734463 0.1594959 0.09821817 0.08518036 0.11473272
                                                                     0.05867014
    [83,] 0.10386788 0.1755758 0.09343764 0.07953064 0.10517166
                                                                     0.05128205
##
    [84,] 0.11125598 0.1816601 0.09995654 0.09083007 0.10777923
                                                                     0.04954368
##
    [85.] 0.11299435 0.1738375 0.10082573 0.08909170 0.11690569
                                                                     0.05997392
##
    [86,] 0.11082138 0.1720991 0.09474142 0.08344198 0.11820947
                                                                     0.04780530
    [87,] 0.11603651 0.1816601 0.10647545 0.09213385 0.12516297
                                                                     0.05258583
##
    [88,] 0.12125163 0.1738375 0.10169492 0.09734898 0.12255541
                                                                     0.05388961
##
    [89,] 0.11255976 0.1534116 0.09083007 0.09213385 0.11777488
                                                                     0.05258583
    [90,] 0.09821817 0.1664494 0.09995654 0.08778792 0.10691004
##
                                                                     0.05693177
    [91,] 0.11082138 0.1760104 0.08996089 0.07953064 0.10951760
                                                                     0.05302043
##
    [92,] 0.11255976 0.1703607 0.09647979 0.08778792 0.10691004
                                                                     0.05171664
##
    [93,] 0.11169057 0.1694915 0.10256410 0.08778792 0.10430248
                                                                     0.05519339
##
    [94,] 0.11125598 0.1712299 0.09734898 0.08518036 0.11473272
                                                                     0.05953933
##
    [95,] 0.10647545 0.1916558 0.10560626 0.09039548 0.11125598
                                                                     0.05475880
    [96,] 0.13602781 0.1734029 0.10386788 0.09691439 0.12299000
                                                                     0.06431986
```

```
## [97,] 0.11038679 0.1534116 0.10082573 0.08213820 0.11690569 0.04823990  
## [98,] 0.11342894 0.1803564 0.11255976 0.07909605 0.11082138 0.05215124  
## [99,] 0.11255976 0.1681877 0.10647545 0.08474576 0.11603651 0.04954368  
## [100,] 0.10647545 0.1838331 0.09300304 0.08648414 0.11255976 0.05519339
```

Matrix Summary:

Training Error Rate Matrix Summary:

```
LDA
                                                               LASSO
##
                           QDA
                                             KNN
##
    Min.
           :0.09261
                              :0.1396
                                               :0.00000
                                                                  :0.06348
                      Min.
                                        Min.
                                                          Min.
##
    1st Qu.:0.10261
                      1st Qu.:0.1577
                                        1st Qu.:0.05337
                                                           1st Qu.:0.07391
   Median :0.10739
                      Median :0.1630
                                        Median :0.07152
                                                          Median :0.07826
##
    Mean
           :0.10683
                      Mean
                            :0.1628
                                        Mean
                                               :0.05866
                                                           Mean
                                                                  :0.07836
    3rd Qu.:0.11130
                      3rd Qu.:0.1679
                                        3rd Qu.:0.08174
                                                           3rd Qu.:0.08261
##
    Max.
           :0.11696
##
                      Max.
                             :0.1817
                                        Max.
                                               :0.09348
                                                           Max.
                                                                  :0.09696
        Ridge
                      Random Forest
##
   Min.
           :0.09391
                      Min.
                              :0.003043
##
   1st Qu.:0.10511
                      1st Qu.:0.004783
                      Median : 0.005435
  Median :0.11087
## Mean
           :0.11046
                      Mean
                              :0.005478
    3rd Qu.:0.11565
                      3rd Qu.:0.006087
##
    Max.
           :0.12565
                      Max.
                              :0.009130
```

Test Error Rate Matrix Summary:

```
LASSO
##
                                             KNN
         LDA
                           QDA
##
    Min.
           :0.09561
                      Min.
                              :0.1460
                                        Min.
                                               :0.08822
                                                          Min.
                                                                  :0.07345
##
    1st Qu.:0.10680
                      1st Qu.:0.1656
                                        1st Qu.:0.09822
                                                           1st Qu.:0.08246
   Median :0.11126
                      Median :0.1708
                                        Median :0.10278
                                                          Median :0.08648
   Mean
           :0.11143
##
                      Mean
                            :0.1698
                                        Mean
                                               :0.10340
                                                           Mean
                                                                  :0.08705
    3rd Qu.:0.11517
                      3rd Qu.:0.1756
                                        3rd Qu.:0.10821
                                                           3rd Qu.:0.09105
##
##
    Max.
           :0.13603
                      Max.
                             :0.1917
                                        Max.
                                               :0.11821
                                                           Max.
                                                                  :0.09909
##
        Ridge
                      Random Forest
##
  Min.
           :0.09735
                      Min.
                              :0.04694
##
    1st Qu.:0.11028
                      1st Qu.:0.05204
  Median :0.11365
                      Median : 0.05476
## Mean
           :0.11461
                      Mean
                              :0.05508
##
    3rd Qu.:0.12093
                      3rd Qu.:0.05780
    Max.
           :0.12994
                              :0.06475
                      Max.
```

4.2 R Code

```
knitr::opts_chunk$set(echo = TRUE)
set.seed(1)
# load library
library(MASS)
library(class)
library(glmnet )
library(tree)
library(e1071)
```

```
library(randomForest)
# Exploratory Data Analysis
spam = read.table("D:/d/Courses/STA/STA 9891/HW/HW4/spam.data.txt")
# Matrix create
train_err.matrix = matrix(0, nrow = 100, ncol = 6)
colnames(train_err.matrix) = c("LDA","QDA","KNN","LASSO","Ridge","Random Forest")
test_err.matrix = matrix(0, nrow = 100, ncol = 6)
colnames(test_err.matrix) = c("LDA","QDA","KNN","LASSO","Ridge","Random Forest")
# Standardize the data
n = nrow(spam)
p = ncol(spam) - 1
spam[ ,1:p] = data.frame(scale(spam[ ,1:p]))
# add noise for QDA
noise = matrix(rnorm(n=131100, mean=0, sd=0.01), nrow=2300, ncol=57)
# for loop
for (i in 1:100) {
  # Split in half
 train = sample(n, n/2)
  spam.train = spam[train, ]
  spam.test = spam[-train, ]
  V58.train = spam[,58][train]
  V58.test = spam[,58][-train]
  x.train = spam.train[,-58]
  x.test = spam.test[,-58]
  # LDA
  fit.lda = lda(V58 ~ ., data = spam, subset = train)
  lda.pred.train = predict(fit.lda, spam.train)
  lda.class.train=lda.pred.train$class
  train_err.matrix[i,1] = mean(lda.class.train != V58.train)
  lda.pred.test = predict(fit.lda, spam.test)
  lda.class.test=lda.pred.test$class
  test_err.matrix[i,1] = mean(lda.class.test != V58.test)
  # QDA with noise
  spam.train.noise = data.frame(data.matrix(spam.train[,-58])+noise, 'V58'=spam.train[,58])
  fit.qda = qda(V58 ~ ., data = spam.train.noise)
  # train
  qda.pred.train = predict(fit.qda, spam.train)
  qda.class.train=qda.pred.train$class
  train_err.matrix[i,2] = mean(qda.class.train != V58.train)
  # test
  qda.pred.test = predict(fit.qda, spam.test)
  qda.class.test=qda.pred.test$class
  test_err.matrix[i,2] = mean(qda.class.test != V58.test)
```

```
# KNN
# tune K using 10-fold CV for KNN
optimal k = tune.knn(as.matrix(x.train), as.factor(V58.train), k=1:20,
                     tunecontrol=tune.control(sampling="cross"), cross=10)
best_k = optimal_k$best.parameters$k
knn.pred.train = knn(x.train, x.train, V58.train, k=best_k)
train err.matrix[i,3] = mean(knn.pred.train != V58.train)
knn.pred.test = knn(x.train, x.test, V58.train, k=best_k)
test_err.matrix[i,3] = mean(knn.pred.test != V58.test)
# LASSO and Ridge
x = model.matrix(V58 ~ ., spam)[,-1]
y = spam $V58
# Lasso
# Find the optimal lambda value via cross validation
cv.out.lasso = cv.glmnet(x[train,], y[train], alpha=1, family="binomial",
                         intercept=TRUE, standardize=FALSE, type.measure="class")
bestlam.lasso = cv.out.lasso$lambda.min
# Fit a lasso regression model
lasso.fit = glmnet(x[train,],y[train], alpha=1, lambda=bestlam.lasso, family="binomial",
                   intercept=TRUE, standardize=FALSE)
# Compute the train error
lasso.prob.train = predict(lasso.fit, s=bestlam.lasso, newx=x[train,])
lasso.pred.train = V58.train
lasso.pred.train[lasso.prob.train>0.5] = 1
lasso.pred.train[lasso.prob.train<0.5] = 0</pre>
train_err.matrix[i,4] = mean(y[train] != lasso.pred.train)
# Compute the test error
lasso.prob.test = predict(lasso.fit, s=bestlam.lasso, newx=x[-train,])
lasso.pred.test = V58.test
lasso.pred.test[lasso.prob.test>0.5] = 1
lasso.pred.test[lasso.prob.test<0.5] = 0</pre>
test_err.matrix[i,4] = mean(y[-train] != lasso.pred.test)
# Ridge
# Find the optimal lambda value via cross validation
cv.out.ridge = cv.glmnet(x[train,], y[train], alpha=0, family="binomial",
                         intercept=TRUE, standardize=FALSE, type.measure="class")
bestlam.ridge = cv.out.ridge$lambda.min
# Fit a ridge regression model
ridge.fit = glmnet(x[train,],y[train], alpha=0, lambda=bestlam.ridge, family="binomial",
                   intercept=TRUE, standardize=FALSE)
# Compute the train error
ridge.prob.train = predict(ridge.fit, s=bestlam.ridge, newx=x[train,])
ridge.pred.train = V58.train
ridge.pred.train[ridge.prob.train>0.5] = 1
ridge.pred.train[ridge.prob.train<0.5] = 0</pre>
train_err.matrix[i,5] = mean(y[train] != ridge.pred.train)
# Compute the test error
ridge.prob.test = predict(ridge.fit, s=bestlam.ridge, newx=x[-train,])
```

```
ridge.pred.test = V58.test
  ridge.pred.test[ridge.prob.test>0.5] = 1
  ridge.pred.test[ridge.prob.test<0.5] = 0
  test_err.matrix[i,5] = mean(y[-train] != ridge.pred.test)
  # Random Forest
  fit.rf = randomForest(V58 ~ ., data=spam, subset=train,
                        mtry=sqrt(p), ntree=300, importance=TRUE)
  # train
  rf.prod.train = predict(fit.rf, newdata=spam.train)
  rf.pred.train = V58.train
  rf.pred.train[rf.prod.train>0.5] = 1
  rf.pred.train[rf.prod.train<0.5] = 0</pre>
  train_err.matrix[i,6] = mean(V58.train != rf.pred.train)
  # test
 rf.prod.test = predict(fit.rf, newdata=spam.test)
  rf.pred.test = V58.test
 rf.pred.test[rf.prod.test>0.5] = 1
 rf.pred.test[rf.prod.test<0.5] = 0
 test_err.matrix[i,6] = mean(V58.test != rf.pred.test)
train_err.matrix
test_err.matrix
# boxplots
par(mfrow=c(2,1))
boxplot(train_err.matrix, main = "100 TRAINING ERROR RATES")
boxplot(test_err.matrix, main = "100 TEST ERROR RATES")
# Training Error Rate Matrix
train_err.matrix
# Test Error Rate Matrix
test_err.matrix
# Training Error Rate Matrix Summary
summary(train_err.matrix)
# Test Error Rate Matrix Summary
summary(test_err.matrix)
```