P1

100 Points Possible

9/26/2025

Attempt 1

In Progress
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Unlimited Attempts Allowed
8/25/2025 to 9/28/2025

Details

P1: Logistic Map

In your first project, you will compute and investigate the bifurcation diagram for a logistic map. Here is the theory of logistic maps.

A logistic map is a one-dimensional map whose general form is given by

$$x_{n+1} = f(x_n)$$

A solution of a map proceeds by iteration. Starting with an initial value of x_0, one generates the sequence x_1, x_2, x_3, and so on.

x_* is a fixed map point if $x_* = f(x_*)$. The stability of a fixed point can be determined by perturbation. We write $x_n = x_* + e_n$, and $x_{n+1} = x_* + e_{n+1}$. The quantities are e_n and e_{n+1} are small error terms.

Using a first-order Taylor series expansion, we obtain

$$x_* + e_{n+1} = f(x_* + e_n) = f(x_*) + e_n f'(x_*) = x_* + e_n f'(x_*)$$

which, for small perturbations, yields

$$|f'(x_*)| = |\frac{e_{n+1}}{e_n}|$$

The fixed point x_* is called stable if $|e_{n+1}| < |e_n|$ so the perturbation decays. Therefore,

- x_* is a stable fixed point if $|f'(x_*)| < 1$.
- x_* is an unstable fixed point if $|f'(x_*)| > 1$.

The logistic map that you will study is given by:

$$x_{n+1} = \mu * x_n(1 - x_n)$$

where you will assume that $0 < \mu < 4$ and $0 < x_0 < 1$.

A fixed point x_* of the logistic map satisfies the quadratic equation $x = \mu * x(1-x)$, which has two solutions given by $x_* = 0$ and $x_* = 1 - \frac{1}{\mu}$.

The stability of the fixed points is determined from the derivative $f'(x) = \mu(1-2x)$ evaluated at the fixed points. We find that $x_* = 0$ is stable for $0 < \mu < 1$ and $x_* = 1 - \frac{1}{\mu}$ is stable for $1 < \mu < 3$. For $\mu > 3$, there are no stable fixed points, and you will reveal the map's behavior numerically.

Consider the logistic map.

$$x_{n+1} = \mu * x_n(1 - x_n)$$

You aim to draw a diagram illustrating the logistic map's behavior as a function of the parameter μ. It would be best to discard the earliest iterations corresponding to transient behavior. The parameter μ should vary over the range

Your main computation will contain one outer and two inner loops. Here is an outline:

```
Loop 1: Start at μ = 2.4 and finish at μ = 4.
  Set x = x0.
  Loop 2: Iterate the logistic map a fixed number of times (transient).
    Compute x.
  Loop 2 (end)
  Loop 3: Iterate logistic map a fixed number of times (data).
    Compute x and save x.
  Loop 3 (end)
Loop 1 (end)
```

You must set some parameters, which can usually be adjusted after viewing a preliminary plot. Parameters include:

1. the resolution in μ.
2. the starting value of x_0.
3. the number of transient iterations.
4. the number of data points for each μ.

You will have to graph the bifurcation diagram using Matplotlib. The most straightforward approach is to plot the iterates of the logistic map as points on a graph. This can illustrate the bifurcation diagram but cannot ultimately result in a high-resolution image. Plotting more and more data, rather than resulting in a better picture, results in a blackened figure with all the fine details obscured.

You will research how best to draw the bifurcation diagram and implement that code.

Your research paper is a crucial part of this project. It will be a culmination of your efforts and a testament to your understanding of the logistic map. The tone of your paper is critical to us.

Browse papers in the Astrophysical Journal or Physical Review to get a feel of the writing style that we are expecting. You will be submitting a single paper for your group. We have enclosed the LaTeX template that you have to use. Here is the outline of your paper:

- Title
- Authors' Names
- Affiliation Name (The University of Texas at Austin)
- Abstract
- Keywords
- In your introduction, you will delve into the historical background of logistic maps. This is an exciting opportunity to uncover the origins and evolution of this fascinating concept. Your summary of the importance of this computation will add a contemporary perspective to this historical journey. We want references to papers in published journals, not blogs on the Internet.
- One of the critical elements of your paper will be a formal statement of the problem you will solve. This formality is crucial to academic writing and ensures clarity and precision. Your ability to articulate the problem will testify to your understanding of the logistic map. Use the logistic map that we discussed in class.
- Discuss the numerical method that you chose to solve the problem.
- Summarize how chaos starts.
- Include the bifurcation diagram of the solution.
- Explain and interpret your results. Discuss any sources of errors that you may have in your computation.
- Conclusion - Reflect on how well you showed the onset of chaos using a simple logistic map. Discuss any future work you could have done to extend your work and contribute to chaos theory.
- Acknowledgment - acknowledge any help that you got from your classmates or the TAs.
- References

Use the templates *APJ_Template.pdf* and *apj_template.tex* in Files-> Projects.

View Rubric

P1 Rubric

Criteria	Ratings		Points
Title view longer description	**Full Marks** **4 pts**	**No Marks** **0 pts**	/4 pts
Author view longer description	**Full Marks** **2 pts**	**No Marks** **0 pts**	/2 pts

Criteria	Ratings		Points
Affiliation view longer description	**Full Marks** **2 pts**	**No Marks** **0 pts**	/2 pts
Abstract	**Full Marks** **10 pts**	**No Marks** **0 pts**	/10 pts
Key	**Full Marks** **2 pts**	**No Marks** **0 pts**	/2 pts
Introduction view longer description	**Full Marks** **15 pts**	**No Marks** **0 pts**	/15 pts
Formal Statement view longer description	**Full Marks** **12.5 pts**	**No Marks** **0 pts**	/12.5 pts
Numerical Method view longer description	**Full Marks** **12.5 pts**	**No Marks** **0 pts**	/12.5 pts
Data view longer description	**Full Marks** **17.5 pts**	**No Marks** **0 pts**	/17.5 pts
Results view longer description	**Full Marks** **10 pts**	**No Marks** **0 pts**	/10 pts
Conclusion view longer description	**Full Marks** **7.5 pts**	**No Marks** **0 pts**	/7.5 pts
Acknowledgements/References	**Full Marks** **5 pts**	**No Marks** **0 pts**	/5 pts

Keep in mind, this submission will count for everyone in your P1 group.

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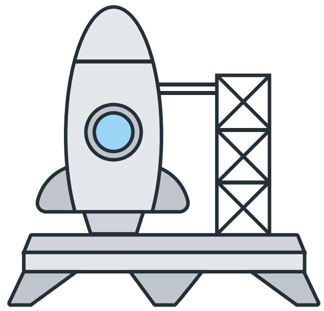

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