PHY338K Electronic Techniques

Fall 2025

Homework 2 - Revised

Due: Sept. 12, 2025

Revision is a correction of a typo in problem 2, as described in the footnote below.

For this homework please carry out all calculations analytically. You may refer to a circuit simulator to check your work, but please be sure to include analytical solutions to all of the problems.

1. (35 points) Resistive and capacitive loading of a signal by an oscilloscope

In the following I will adopt the notation that $V(t) = V_0 \cos(\omega t + \phi) = \mathrm{Re}\left(\tilde{V} e^{j\omega t}\right)$, where $j = \sqrt{-1}$ and $\tilde{V} = V_0 e^{j\phi}$ is the complex voltage amplitude, and similarly for the current. (Your textbook uses $V(t)$ for the voltage and $V(\omega)$ for the complex amplitude, which can be confusing at times.)

Suppose you measure the signal from a transducer that outputs a sinusoidal signal $V_{trans}(t) = V_{0,trans} \cos(2\pi ft)$ with complex amplitude $\tilde{V}_{trans} = V_{0,trans} = 100$ mV and variable frequency f. The transducer has a large output impedance $R_{trans} = 500$ kΩ. You connect the transducer to your Tektronix TBS1052C oscilloscope input, which has an input resistance $R_{scope} = 1$ MΩ in parallel with capacitance $C_{scope} = 14$ pF. You make the connection with a 3 ft length of RG-58A/U coaxial cable and a short pair of clip leads. The RG58A/U cable has a capacitance of 26.5 pF/ft, so the capacitance of the cable is $C_{cable} = 80.5$ pF. The overall circuit for your measurement is therefore as shown in Figure 1.

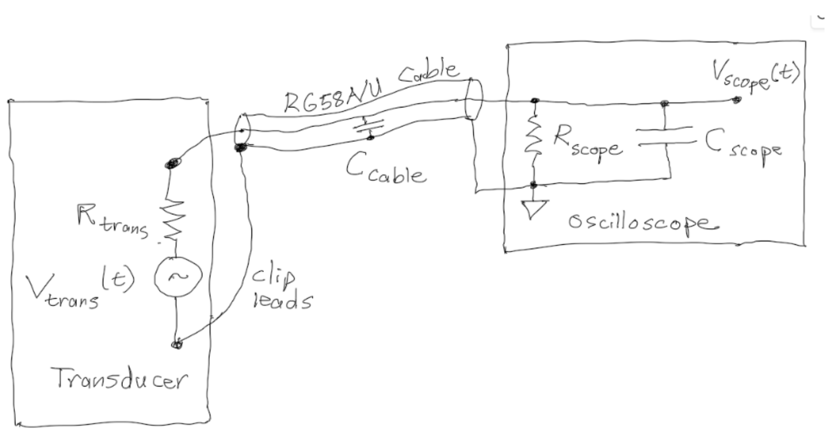

Figure 1. Circuit for measurement of a transducer output on an oscilloscope.

The above circuit is equivalent to the one shown in Figure 2. That is, the combination of the oscilloscope input resistive and capacitive impedances and the capacitive impedance of the cable acts as a load on the transducer with impedance Z_L.

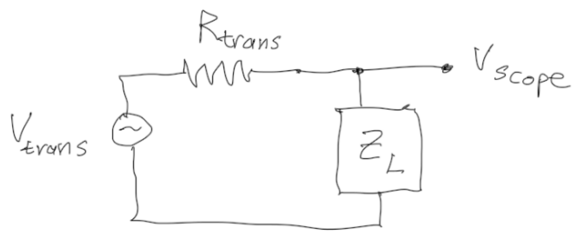

Figure 2. Equivalent circuit to the one shown in Figure 1.

a) Find the value of the complex impedance Z_L.

b) Give numerical values of $|Z_L|$ for frequencies (i) $f = 0.50$ kHz, (ii) $f = 3.0$ kHz, and (iii) $f = 20$ kHz. Explain why $|Z_L|$ changes as you increase the frequency.

c) Since the circuit is linear and driven by a sinusoidal input, the voltage displayed by the oscilloscope must also be sinusoidal with the same frequency, *i.e.* $V_{scope}(t) = \mathrm{Re}\left(\tilde{V}_{scope} e^{j\omega t}\right)$, where $\omega = 2\pi f$. Find the magnitude $|\tilde{V}_{scope}|$ of the voltage oscillation displayed on the oscilloscope and the phase shift between the displayed voltage $V_{scope}(t)$ and the signal $V_{trans}(t)$ for the cases that the frequency is (i) $f = 0.50$ kHz, (ii) $f = 3.0$ kHz, and (iii) $f = 20$ kHz. What effect does the loading of your signal by the impedance Z_L have on the signal displayed on the oscilloscope? Is this display a true and accurate measurement of $V_{trans}(t)$? If not, is the departure of the display from $V_{trans}(t)$ worse at some frequencies than at others? If so, why?

d) You should have found in your lab 2 that impedance loading effects had only a small effect on your measurements. Why were impedance loading effects in lab 2 so much smaller than the effects you just calculated in parts (a) through (c)?

2. (30 points) <u>Oscilloscope probe</u>

Many electronic engineers and technicians routinely use an *oscilloscope probe.* These come in both active and passive types. Passive probes include only passive components (*e.g.* resistors and capacitors). The main purpose of an oscilloscope probe is to reduce the loading effects discussed in problem 1.

Our oscilloscopes each came with the Tektronix TPP0100 10x passive oscilloscope probe shown in Figure 3. At one end it has a BNC connector to attach to the oscilloscope input. At the other end the tip of the probe has a small hook that you can connect to a wire in your circuit. The probe also has a small alligator clip that you connect to the ground of your circuit. When you do this, you have the circuit shown in Figure 4.

Figure 3. Tektronix TPP0100 passive oscilloscope probe.

10x means that the probe attenuates the signal by a factor of 10. For instance, if a 10 Volt sine wave is applied to the tip of the probe, the voltage at the oscilloscope input will be a 1 Volt sine wave, assuming loading effects are negligible. That is the purpose of the 10x setting on “Probe setup” on your oscilloscope, which is the default setting. In this setting, the oscilloscope multiplies the voltage at the oscilloscope input by a factor of 10 to make up for the factor of 10 attenuation of the probe.

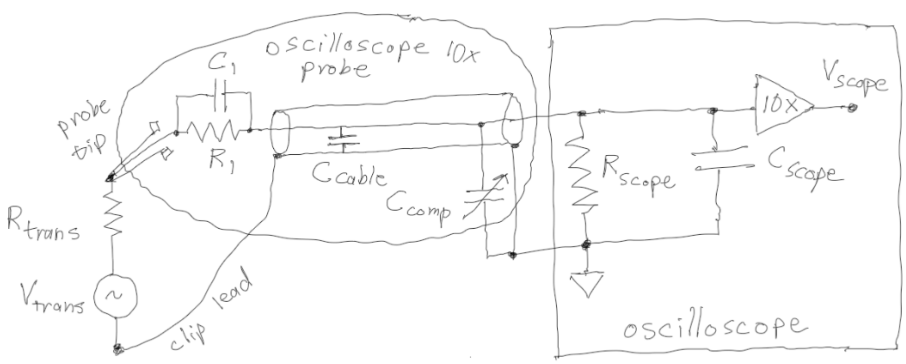

Figure 4. Circuit when a 10x scope probe is used to connect an oscilloscope to a transducer signal source.

The 10x probe has three additional components: a resistor $R_1 = 9\ \mathrm{M}\Omega,$ a fixed capacitor $C_1,$ and a variable capacitor C_{comp}. If capacitances can be neglected, R_{scope} and R_1 form a simple 10x voltage divider. So for this situation the voltage at the scope input is 1/10th of the voltage at the probe input, assuming loading effects can be neglected. The input impedance at the tip of the scope probe is now 10 MΩ, and is 10x higher than the scope input without the probe.

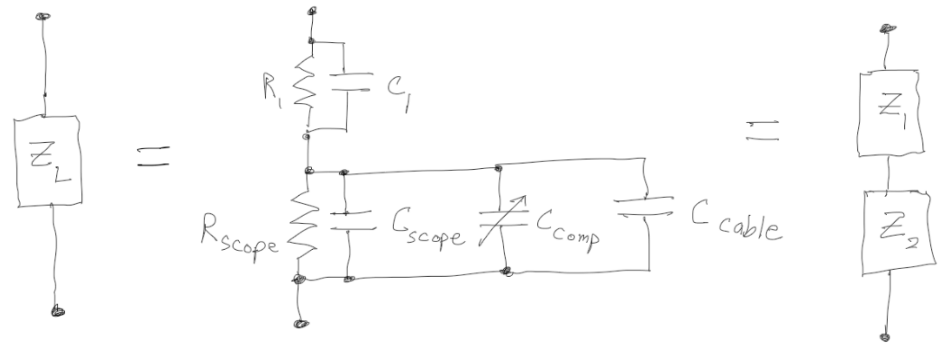

Figure 5. Load impedance for the circuit illustrated in Figure 4.

a) Taking into account the capacitances, the load at the probe tip now looks as shown in Figure 5. Let Z_1 be the impedance of the parallel combination of R_1 and C_1, and let Z_2 be the impedance of the parallel combination of R_{scope}, C_{scope}, C_{comp}, and C_{cable}. Suppose that we again have $R_1 = 9\ \text{M}\Omega$, $R_{scope} = 1\ \text{M}\Omega$, $C_{scope} = 14\ \text{pF}$, $C_{cable} = 80.5\ \text{pF}$, and that $C_1 = 12\ \text{pF}$. Your goal is to make the complex impedance Z_2 to be exactly 1/9 of the complex impedance Z_1.[1] To what value should you adjust the compensation capacitor C_{comp} to achieve this?

b) Show that for the parameters of part (a), the impedance Z_L is equivalent to the parallel combination of a single resistor R_L in parallel with a single capacitor C_L as shown in Figure 6.

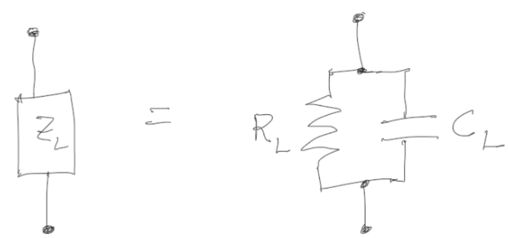

Figure 6. Equivalent circuit for the impedance shown in Figure 5.

What are the values of R_L and C_L? What is the value of Z_L?

c) Assuming again that $\tilde{V}_{trans} = 100\ \text{mV}$, $R_{trans} = 500\ \text{k}\Omega$, and the oscilloscope and probe parameters of part (b), and taking into account the 10x internal multiplication in the oscilloscope, find the magnitude $|\tilde{V}_{scope}|$ of the voltage oscillation displayed on the oscilloscope and the phase shift between the displayed voltage $V_{scope}(t)$ and the signal $V_{trans}(t)$ for the cases that the frequency is (i) $f = 0.50\ \text{kHz}$, (ii) $f = 3.0\ \text{kHz}$, and (iii) $f = 20\ \text{kHz}$. Has the effect of oscilloscope loading on your displayed signal changed from what you found in problem 1? If so, explain why that happens.

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[1] In the original version of this Homework, I incorrectly stated that the complex impedance Z_2 should be exactly 1/10 of the complex impedance Z_1.

3. (35 points) RLC bandpass filter

In Figure 7 we show a parallel *LRC* bandpass filter. This filter passes frequencies within a narrow range centered on the resonance frequency $\omega_0 = \sqrt{\dfrac{1}{LC}}$ and blocks frequencies outside of that range. We assume that the input voltage is $V_{in}(t) = \mathrm{Re}\left(\tilde{V}_{in} e^{j\omega t}\right)$. Since the filter contains only linear components, the output voltage must be $V_{out}(t) = \mathrm{Re}\left(\tilde{V}_{out} e^{j\omega t}\right)$.

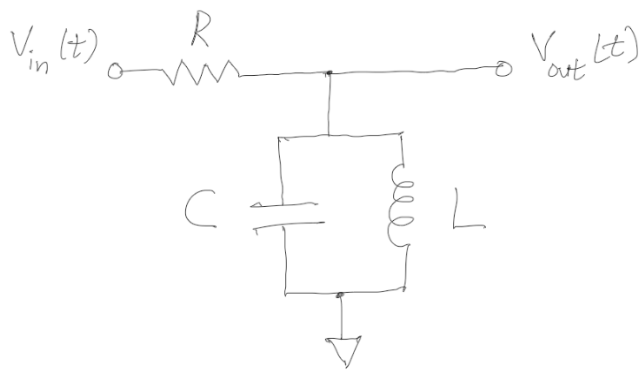

Figure 7. A parallel LRC bandpass filter.

For this problem, assume that $L = 2.5$ mH, $C = 10.0$ nF, and $R = 25$ kΩ.

a) Show that the amplitude ratio $\dfrac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|}$ of this filter is $\dfrac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|} = \dfrac{1}{\sqrt{1 + \left(\dfrac{R}{\omega L}\right)^2 \left(1 - \dfrac{\omega^2}{\omega_0^{\,2}}\right)^2}}$

b) Find the amplitude ratio (numerical value) if the frequency is tuned to exact resonance, *i.e.* $\omega = \omega_0$.

c) Find the two frequencies at which the amplitude ratio is $\dfrac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|} = \dfrac{1}{\sqrt{2}} = 0.707.$ Call the larger of the two frequencies ω_+ and the smaller of the two frequencies ω_-.

d) The *bandwidth* of the filter is defined as $\Delta\omega_{FWHM} = \omega_+ - \omega_-$. It is also called the Full-Width-at Half-Maximum (FWHM). This refers to the fact that the *power* transmitted by the filter as a function of frequency has a resonance curve for which the power transmitted at the frequencies ω_+ and ω_- is ½ of the maximum power transmitted on exact resonance. The FWHM is the width of the peak measured between these two frequencies.

e) The *quality factor* Q is defined as $Q = \dfrac{\omega_0}{\Delta\omega_{FWHM}}$. What is the Q of your filter?

f) Draw a qualitatively correct plot of the amplitude ratio $\dfrac{\left|\tilde{V}_{out}\right|}{\left|\tilde{V}_{in}\right|}$ of your filter as a function of frequency, with the frequency given in Hz.

Aside: The parallel combination of L and C is called a *tank circuit.* Tank circuits have a high impedance on resonance because for a given oscillating voltage amplitude, the currents through the L and C are equal and opposite. In other words the LC combination draws almost zero total current. Since impedance is the ratio of voltage to current, the tank circuit impedance is very high. In this filter application, the high impedance of the tank circuit on resonance effectively removes it from the circuit, so that the full input voltage appears at the output. But for other frequencies the impedance of the tank circuit becomes low, which pulls the oscillating voltage to a low amplitude at the output.