# III. Lab 3. Inductors and *LC* Filter Circuits

## *1.* Introduction

### a) Real Inductors

The impedance of an ideal inductor is $Z_L = j\omega L$. Thus, if we plot $|Z_L|$ as a function of frequency $\omega$ on a log-log scale, we expect a straight line with a slope of 1 as shown in the left graph in Figure III-1. However, a real inductor has an impedance that departs from this ideal, as shown in the right graph in Figure III-1.

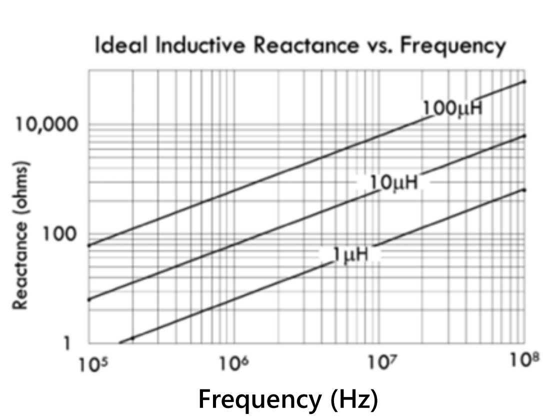


Figure III-1. Ideal and real inductor impedance $|Z_L|$ *vs.* frequency. Figure is from *Practical Electronics for Inventors,* by P. Scherz and S. Monk.

Real inductors are made of coils of wire, usually wound around a ferromagnetic core. For the inductors in this lab, the core is made of ferrite. An equivalent circuit for a real inductor is shown in Figure III-2(a). In addition to its inductance *L*, a real inductor has a series resistance $R_{DC}$ due to the resistance of the wires used to wind the coil. Furthermore, there is a small amount of capacitance between each winding and the next. This effect produces an overall parallel capacitance $C_P$. Finally, if the coil is wound on a ferromagnetic core, that core will exhibit energy loss due to eddy currents, hysteresis losses, or both. This is equivalent to a parallel resistance $R_P$.

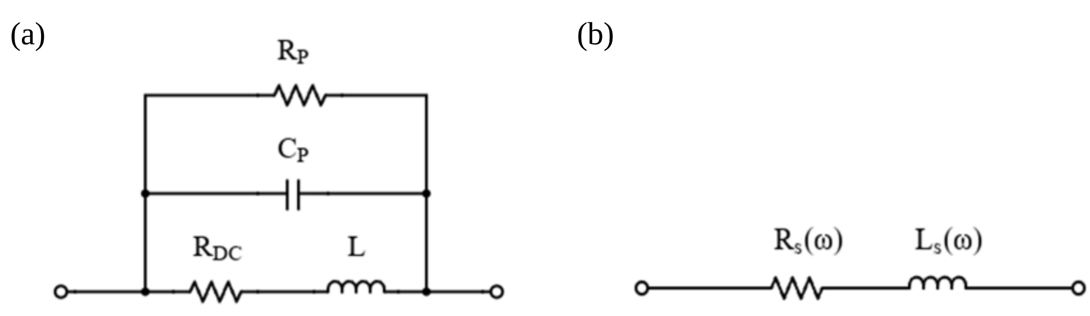


Figure III-2. a) Equivalent circuit for a real inductor. b) Alternative equivalent circuit.

As can be seen from the right graph in Figure III-1, a real inductor has a resonance in its impedance. This occurs at frequency $f = \dfrac{1}{2\pi\sqrt{LC_P}}$, which is referred to as the "self-resonant frequency (SRF)." For frequencies well above the SRF the component actually behaves more like a capacitor. For frequencies well below the SRF, the capacitance doesn't have a very significant effect.

At any given frequency $\omega$, it is possible to model the inductor with the alternative equivalent circuit in Figure III-2(b). In this case the inductance $L_S$ and resistance $R_S$ will depend on frequency $\omega$. The subscript *S* indicates that we are using an equivalent circuit with a series combination of *L* and *R*. (It is also possible to use an equivalent circuit with a parallel combination of *L* and *R*.)

There is a further nonideality associated with the magnetization curve of the inductor core material, which is a graph of the magnetic field *B* in the material as a function of the field *H* (Amps/meter of the windings) driving its magnetization. Example curves for N87 ferrite are shown in Figure III-3. The different color curves correspond to different amplitudes of the sinusoidal variation of *H*.

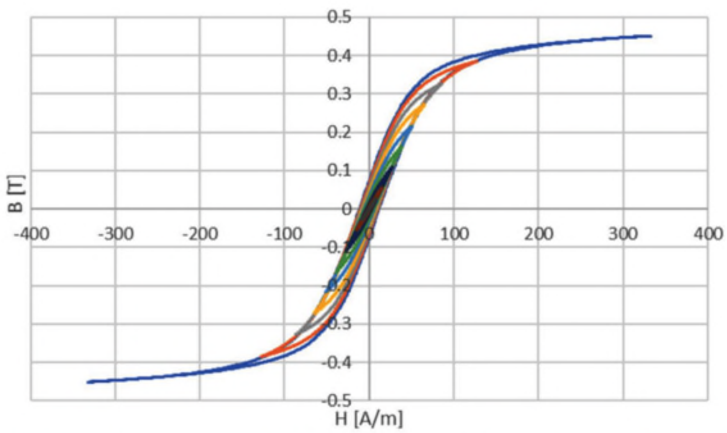


*Fig. 5: Low frequency BH loops (excitation at 850 Hz, $N_p$ = 26, $N_s$ = 26)*

Figure III-3. Magnetization curves for N87 ferrite. Figure is from
https://www.netl.doe.gov/sites/default/files/netl-file/Core-Loss-Datasheet---MnZn-Ferrite---N87%5B1%5D.pdf

What we want to produce a near-ideal inductor is a straight line. But that is not at all what we get. For amplitudes less than about 50 A/m, the curve is sort of straight but does show hysteresis. This causes the voltage *vs.* current to be somewhat nonlinear, and it causes distortion in what would otherwise be a sinusoidal response. Furthermore this results in energy dissipation in the core material, which is accounted for in the above model by the resistance $R_P$. If the driving field *H* is increased up to or past about 100 A/m, the core material *saturates.* For modest saturation the inductor behaves as through its inductance is reduced compared to its low saturation inductance. For high saturation the component will have a behavior that is very far from the ideal inductor. In practice, inductors have a maximum current specification. This is determined either by the current at which the core starts to saturate, or by the current at which the heating of the device becomes too large.

Another thing to notice about the real inductor is that its resistive impedance $R_{DC}$ is always present while its inductive impedance $j\omega L$ goes to zero as $\omega \to 0$. Thus there is always some frequency $\sim R_{DC} / L$ that is a dividing line for the real inductor behavior. For much lower frequencies the real inductor looks more like a resistor with impedance $R_{DC}$. For much higher frequencies it will look more like an ideal inductor, assuming that the frequency is still small compared to the SRF and that the losses from $R_P$ are sufficiently small.

b) <u>LCR meter</u>


Figure III-4. IET labs DE-5000 LCR meter with TL-21 test leads.

In this lab, we have a few of the LCR meters shown in Figure III-4. An LCR meter applies an AC voltage with complex amplitude (*i.e.* amplitude and phase) $\tilde{V}$ to the component under test and measures the complex current amplitude $\tilde{I}$ through the component. The complex impedance of the component is then determined from $Z = \dfrac{\tilde{V}}{\tilde{I}}$. The unit can display the results in terms of either a series or parallel equivalent circuit. Here, we'll focus on the results for the series equivalent circuit shown in Figure III-5(b). The component impedance is $Z = R_S + jX_S$. It can be represented in the complex plane as shown in Figure III-5(a). As a vector in this plane, we can specify the impedance by giving its real part $R_S$ and imaginary part $X_S$, or by giving the magnitude of the vector $|Z|$ and the angle $\theta$ that it makes with the real axis. If $\theta$ is positive, the imaginary part of the impedance is inductive, *i.e.* $X_S = \omega L_S$. If $\theta$ is negative, the imaginary part of the impedance is capacitive, *i.e.* $X_S = -\frac{1}{\omega C_S}$. Depending on the setting of the meter, it will read out $L_S$, $C_S$, $R_S =$ ESR = Equivalent Series Resistance, $\theta$, $Q = \left|\frac{X_S}{R_S}\right|$, and/or $D = \frac{1}{Q} = \left|\frac{R_S}{X_S}\right|$. When operating the meter the LCR Auto button toggles between different measurement modes (Auto, *L*, *C*, *R*, and ESR). You will mainly want to stay with Auto, *L*, or *C*. You toggle between parallel and series equivalent circuit models with the SER/PAL button. You should stick with series. If you are in series mode, the subscript S appears on the displayed quantities. Finally the D/Q/ESR/$\theta$ button toggles between the display of different quantities when in *L* or *C* mode.

(a) (b)

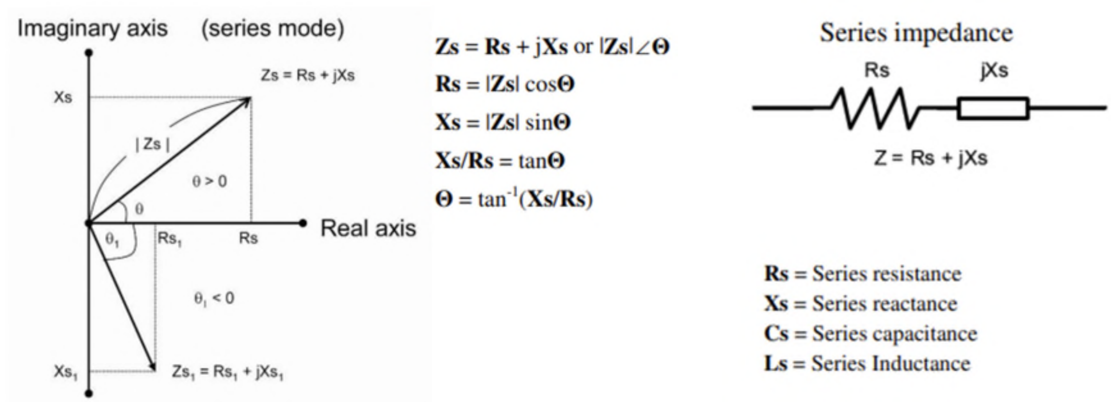


Figure III-5. (a) Representation of impedance in the complex plane for the case of a series equivalent circuit. (b) The series equivalent circuit. Figures are from the DE-5000 User Manual.

Since the impedance of a component is a function of frequency, the LCR meter allows you to select from a number of frequencies to use for the measurement. You do this with the FREQ button. In the case of the DE-5000, the available frequencies are 100 Hz, 120 Hz, 1 kHz, 10 kHz, and 100 kHz.

The TL-21 test lead fixture is designed to keep parasitic capacitances and inductances from affecting the measurements. To measure the impedance of a component, just clip onto it with the two clips on the TL-21. Don't use any other wires to connect. For the most accurate measurements it is best to zero the meter first. But you shouldn't need to do that for this lab because we'll zero the meters at the start of the lab. You can find further instructions in the DE-5000 manual, which is readily available online.

## 2. Procedure

### a) <u>Impedance measurement with the LCR meter</u>

Take one of the 1 mH inductors from the parts bin. Specification sheets for the inductors are attached to the end of this chapter. Take note of whether you have a Bourns model RL181S-102J-RC inductor or a Signal Transformer model DRC-0406-102J-UL inductor. Using the meter, measure the following properties of the inductor at each of the frequencies 100 Hz, 1 kHz, 10 kHz, and 100 kHz: $L_S$, $Q$, ESR, and $\theta$.

For your report: Give a table of your results. Look up the SRF of your inductor in its specification sheet. Using this value and your measurement results, over what range of frequencies does your component have properties that are reasonably close to an ideal inductor? Also, using the measured value of $L_S$ and $\theta$ at frequency 1 kHz, check that the measured 1 kHz values of ESR and $Q$ are correct.

Using the LCR meter, measure the values of $C_S$, $D$, $ESR$, and $\theta$ for a 100 nF ceramic capacitor at each of the frequencies 100 Hz, 1 kHz, 10 kHz, and 100 kHz. Which component behaves closer to the ideal component in the frequency range 10 Hz to 100 kHz, the capacitor or the inductor? Include these results in your report.

b) Impedance measurements with an oscilloscope

Next, set-up the circuit shown in Figure III-6, with $R = 100\ \Omega$ and $L = 1$ mH.

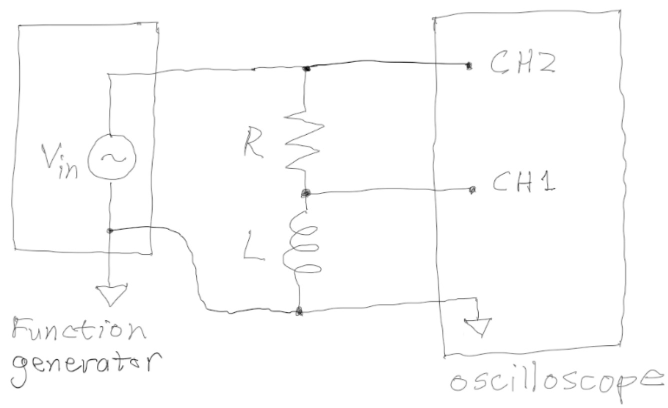


Figure III-6. Circuit for measurement of inductor properties.

Set the oscilloscope to display the voltage on channel 1, the voltage on channel 2, and the voltage difference $V_2 - V_1$. Don't forget to set Probe Setup to 1x for both channels. To display the voltage difference, press Math, set Source 1 to CH2, operator to -, and Source 2 to CH1 and then exit by pressing the Menu button.

CH1 measures the voltage across the inductor. Math measures the voltage across the resistor. Since the resistor obeys Ohm's law very accurately, the current through the resistor is just $I = V_{math} / R$. Since the resistor and the inductor are in series, this means that Math measures the current through the inductor. For quantitative measurements you just need to divide $V_{math}$ by $R$.

Set up the oscilloscope to measure the frequency and amplitude of the waveform on CH1, the amplitude of the waveform on CH2, the amplitude of the waveform on Math, and the phase difference between the waveform on CH1 and the waveform on Math.

Set the frequency to 1 kHz. (If the channel 1 signal is noisy, you can reduce the noise with signal averaging. To do that, press “Acquire”, then “Mode” and select “Average 16.”) Use the information displayed to deduce the values of $L_S$, $Q$, ESR, and $\theta$ for your inductor at frequency 1 kHz. Do your measurements agree with the LCR meter?

For your report: Be sure to include a copy of the oscilloscope display, and to reproduce your calculations of $L_S$, $Q$, ESR, and $\theta$, and comment on the agreement between the two measurement methods. Also, how close was your current in this measurement to the maximum allowed value for your inductor?

Next, scan the frequency from about 50 kHz up to about 1 MHz. Watch what happens to the voltage amplitude across the inductor and the current through the inductor as you do this. Do you see any evidence of a self-resonance in the impedance of the inductor?

For your report: If you see evidence of a self-resonance, describe what the evidence is and state what frequency the resonance occurs at.

c) <u>Notch filter</u>

Next, build the circuit shown in Figure III-7. Use $R = 1\ \mathrm{k\Omega}$ and $L = 1\ \mathrm{mH}$. Choose $C$ such that the resonance frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ is between 50 kHz and 150 kHz. This should be in the frequency range where your inductor properties are not too far from those of an ideal inductor. Use a function generator to produce a variable frequency sine wave for $V_{in}$, and measure $V_{out}$ with an oscilloscope.

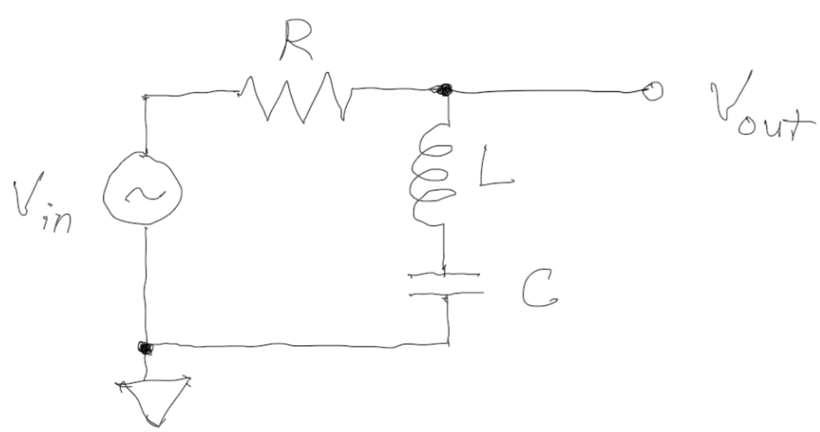


Figure III-7. Notch filter.

Measure the gain of this circuit $\left( \text{value of } \dfrac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|} \right)$ as a function of frequency. You should find that this circuit acts as a *notch filter*, *i.e.* it has a high transmission for all frequencies except those near the frequency $f_0$. Make sure to measure the ratio at 5 or 6 frequencies near $f_0$ so that you can clearly see the shape of the dip in the transmission function. Also measure the ratio at a few frequencies well below $f_0$ and a few frequencies well above $f_0$.

For your report: Include a representative scope trace and a graph of $\frac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|}$ as a function of frequency. What is the frequency at which $\frac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|}$ is minimum, and what is the value of $\frac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|}$ at that frequency? Compare your results to the values expected from the circuit theory. What is the quality factor *Q* of the resonance? What equivalent series resistance in your inductor would be needed to account for that value of *Q*?

d) <u>Second-order Butterworth filter</u>

A *low-pass Butterworth filter* is a filter that has gain $G(\omega)=\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}}$, where $\omega_0$ is the $-3\text{dB}$ frequency of the filter. A *second-order* Butterworth filter is a filter that has the above response with $n=2$.

It is possible to show that the circuit shown in Figure III-8 is a second-order Butterworth filter with $\omega_0=\sqrt{\frac{1}{LC}}$ provided that $R=\frac{1}{\sqrt{2}\omega_0 C}$.

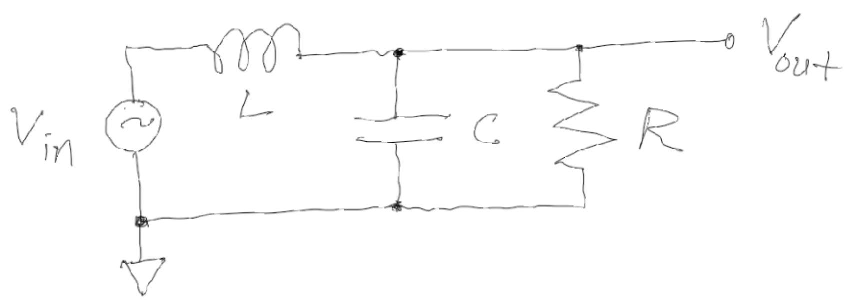


Figure III-8. A second-order Butterworth filter.

Build the circuit shown in Figure III-8. Choose $L=1\text{ mH}$ and a value of *C* that gives you a resonant frequency $f_0=\frac{\omega_0}{2\pi}$ somewhere in the range between 50 kHz and 150 kHz. You won't be able to use the exact value of *R* given by the formula above, but use a value of *R* that is reasonably close to that value. You may want to use two resistors in series to achieve that.

Measure the gain of this circuit at a series of different frequencies spanning the range from about 10 kHz to about 1 MHz. Use enough different frequencies that you can clearly see how the gain changes with frequency.

For your report: Make a plot of the gain *vs.* frequency on a log-log scale. Compare your data with the ideal Butterworth filter function. Does your measured gain conform closely to the Butterworth function? What should the slope of the function be at high frequencies on the log-log scale? How close does your data at high frequency come to having this slope?

e) <u>Specifications for 1 mH inductors in lab:</u>

First type:


BOURNS®

**Features**
- Formerly J.W. Miller® model
- Current rating up to 90 mA
- Inductance range: 1000 µH to 120,000 µH
- High Q
- Shielded construction
- RoHS compliant*

**Applications**
- DC/DC converters
- Power supplies
- Desktop notebooks
- Output chokes

**RL181S Series - Radial Lead RF Choke**

**Electrical Specifications (@ 25 °C)**

| Part Number | Inductance (µH) | Tol. | Q (Min.) | Test Frequency L | Test Frequency Q | SRF (MHz) Typ. | DCR (Ω) Max. | I dc (mA) |
|---|---|---|---|---|---|---|---|---|
| RL181S-102J-RC | 1000 | ±5 % | 70 | 1 KHz | 50 KHz | 0.77 | 3.4 | 90 |

**General Specifications**

Rated Current...... Inductance drop 10 %

Operating Temperature

................................-55 °C to +105 °C

Storage Temperature

Second type:

# DRC-0406
## Radial Coil Inductors

The DRC-0406 Series is the first of four new inductor series of Signal's line of unshielded radial drum chokes which are also known as fixed or bobbin core inductors. These through hole mount components are commonly used for general purpose filtering of high frequency signals and mitigating EMI/RFI. They frequently make their appearance in a wide variety of applications such as industrial, consumer, and medical products.

Utilizing the latest winding technology, copper windings and ferrite cores, Signal's DRC Series addresses the needs of modern filtering applications by providing the most effective electrical noise suppression via a combination of high impedance, large inductance, and elevated current capacity.

### General Features

- Wide inductance range
- Lowest DCR
- Tape packaging for auto-insertion
- Shrink tubing protected winding
- Fixed lead spacing
- High peak current support
- Large saturation current capacity
- Small footprint

### Specifications

- Inductance Range: 1.2 µH to 2200 µH
- Rated Current : It is either the inductance is 10% lower than its initial value in DC Saturation characteristics or temperature rise becomes ΔT = 20°C (Ta = 20°C), whichever is lower.
- Operating Temperature Range: -20°C to +85°C
- Storage Temperature Range (component): -40°C to +125°C

### Applications

- TV and Audio equipment
- Telecommunication devices
- Personal computers
- Switching power supplies
- Other noise filters

## ELECTRICAL SPECIFICATIONS

**Electrical specifications for all part numbers measured at 25°C unless stated otherwise.**

### DRC-0406 SERIES

| Part Number | L @ 1 kHz (µH) | Q Min. | Q Test Freq. (MHz) | SRF Min. (MHz) | DCR Max. (Ω) | Rated Current Max (mA) |
|---|---|---|---|---|---|---|
| DRC-0406-1R2J-UL | 1.2 | 100 | 7.96 | 120 | 0.058 | 1950 |
| DRC-0406-3R3M-UL | 3.3 | 100 | 7.96 | 75 | 0.130 | 1500 |
| DRC-0406-470K-UL | 47 | 70 | 2.52 | 7.8 | 0.76 | 550 |
| DRC-0406-101J-UL | 100 | 45 | 0.796 | 6.0 | 1.0 | 400 |
| DRC-0406-151J-UL | 150 | 65 | 0.796 | 4.2 | 1.3 | 350 |
| DRC-0406-331J-UL | 330 | 50 | 0.796 | 2.2 | 2.2 | 250 |
| DRC-0406-471J-UL | 470 | 50 | 0.796 | 1.7 | 3.6 | 200 |
| DRC-0406-681J-UL | 680 | 50 | 0.796 | 1.3 | 4.6 | 170 |
| DRC-0406-102J-UL | 1000 | 90 | 0. 252 | 1.0 | 6.7 | 150 |
| DRC-0406-152J-UL | 1500 | 80 | 0.252 | 0.8 | 13 | 120 |
| DRC-0406-222J-UL | 2200 | 80 | 0.252 | 0.8 | 17 | 100 |

Note: Rated Current is the $I_{sat}$ (Saturation Current) or the $I_{rms}$ (Temperature Rise Current). Whichever value is lower.