Lily Davoren HW 2 09/18

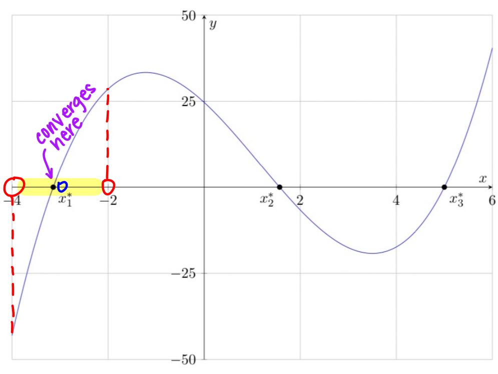
PROBLEM 1: BISECTION METHOD

a) To fix this bug in the code, change the first equals sign in line 26 to an exclamation point to indicate that if the sign of $f(x_3)$ and the sign of $f(x_2)$ are NOT the same, then bisection should be performed again on the new range $[x_3, x_2]$ by setting x_1 equal to x_3. Bisection cannot be performed if the signs of the y-values of the x-values in the brackets are the same.

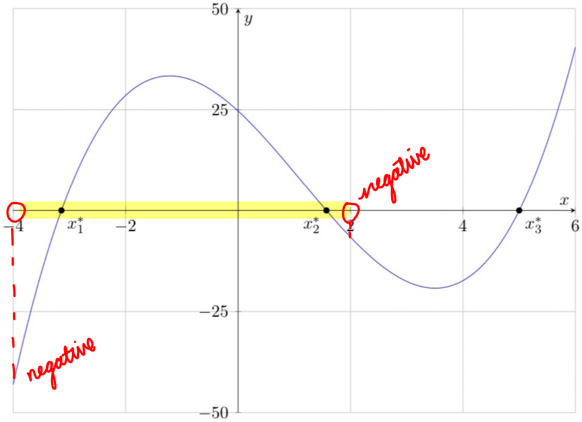
b) The next six brackets predicted by the algorithm are $[-3.5, -3]$, $[-3.25, -3]$, $[-3.25, -3.125]$, $[-3.188, -3.125]$, $[-3.156, -3.125]$, $[-3.156, -3.141]$.

It takes 20 steps to bracket the root within an interval of size 1×10^{-6}. This can be evaluated from the equation $n \approx \frac{\log\left(\frac{\Delta x}{\varepsilon}\right)}{\log(2)}$. For this problem, this equation will look like $n \approx \frac{\log\left(\frac{1}{1\times10^{-6}}\right)}{\log(2)}$ which rounds to approximately 20 iterations.

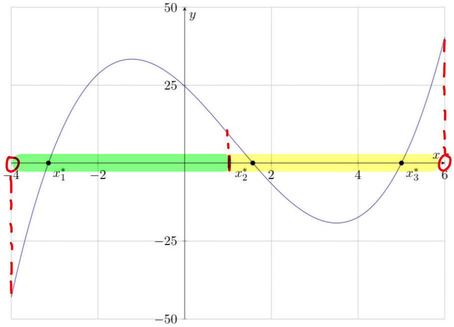
c) i) When starting with bracket $[-4, -2]$, the bisection algorithm will converge on the root $x_1 = -\pi$. The bisection algorithm will continue to bisect the range $[-4, -2]$ until the root on the x-axis is found ($f(x_3) = 0.0$).



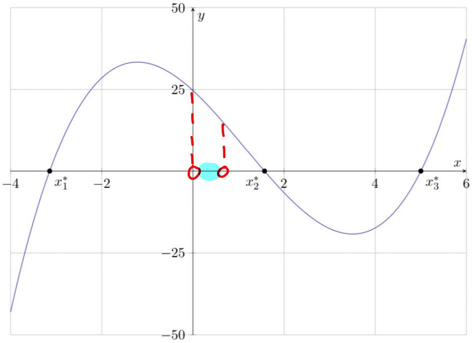
c) ii) When starting with the bracket [-4, +2], the bisection algorithm does not converge on a root because the y-values of -4 and +2 are both negative. According to the algorithm, if the signs of the y-values are the same, then the bisection algorithm fails.



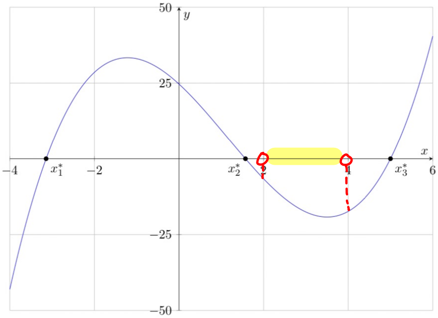
c) iii) When starting with bracket [-4, +6], the bisection algorithm converges at x_1 because, although there are 3 roots in between -4 and 6, the midpoint is at +1. The y-value at 1 is positive, so the range is reduced to [-4, 1] because the bracketed end that is negative that would allow for continuation of the bisection algorithm is at -4 which has a negative y-value. The only root between [-4, 1] is $x_1 = -\pi$, so the bisection algorithm will converge to $x_1 = -\pi$.



c)iv) When starting with bracket [0, +0.49], the bisection algorithm does not converge at a root because there is no root between [0, +0.49], and because f(x) signs are the same.



c)v) When starting with the bracket [+2, +4], the bisection algorithm does not converge on a root because the y-values of 2 and 4 are both negative.



PROBLEM 2 : SECANT METHOD

a)

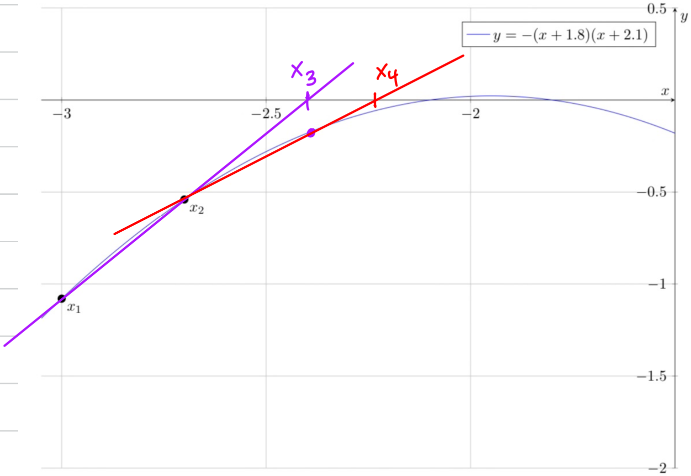


Figure 2:

c) i)

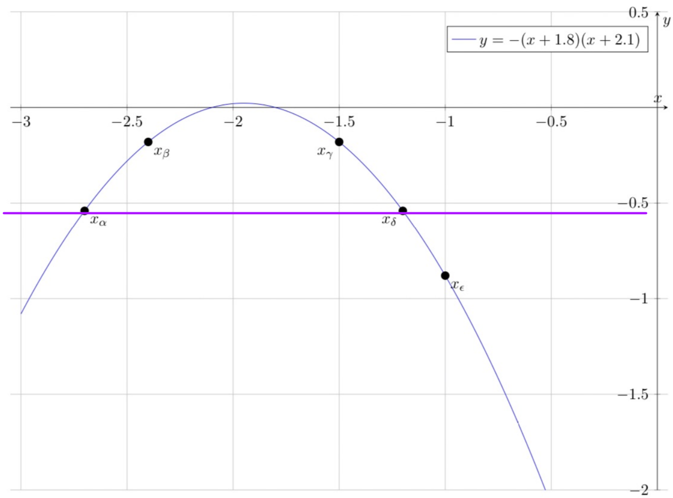


Figure 3:

i) Does not converge on a root

c)ii)

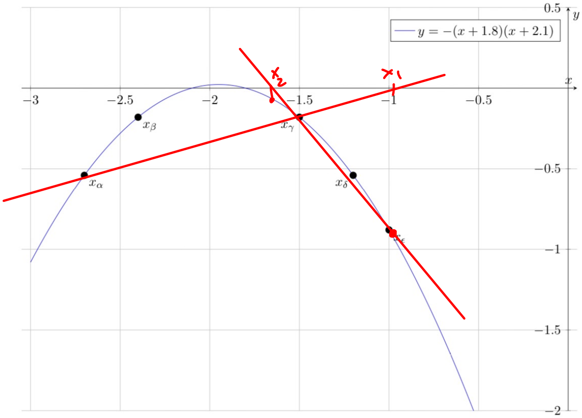


Figure 3:

Converges on -1.8

c)iii)

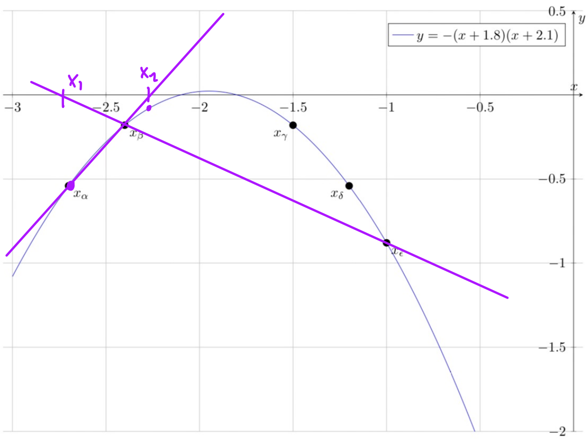


Figure 3:

converges on -2.1

PROBLEM 3: NEWTON'S METHOD

n	TRUE RELATIVE ERROR	APPROX RELATIVE ERROR
1	0.0453886273625 9986	0.3910209507695690
2	$7.7062590216 83052 \times 10^{-5}$	0.0454691939266758
3	$3.7643631629943756 \times 10^{-13}$	0.0000770625905932
4	0.0000000000000000	0.0000000000003764
5	0.0000000000000000	0.0000000000000000
6	0.0000000000000000	0.0000000000000000