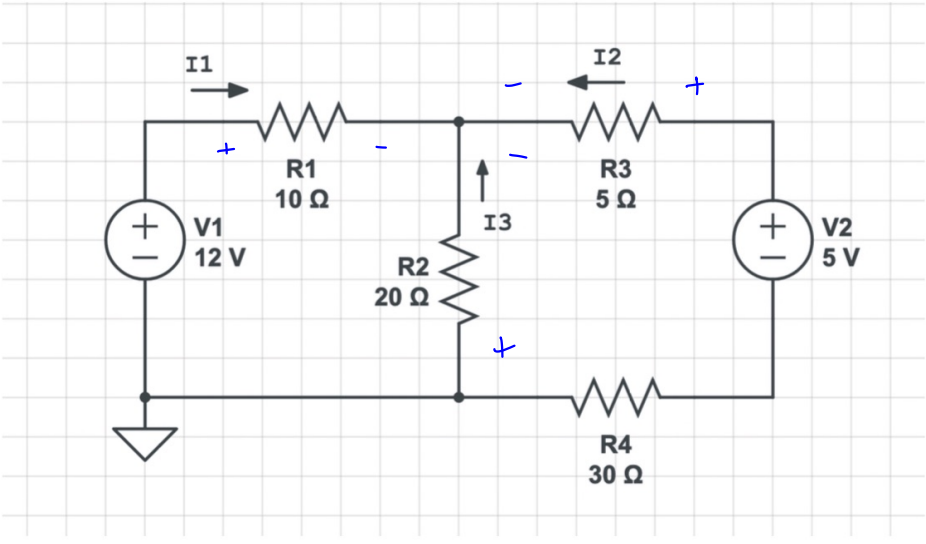
Lily Davoren HW 3 09/25

PROBLEM 1: OBTAINING LINEAR SYSTEMS FROM ENGINEERING PROBLEMS

a)



$I_1 + I_2 + I_3 = 0$

$12\text{ V} - 10I_1 + 20I_3 = 0$

$5\text{ V} - 5I_2 + 20I_3 - 30I_2 = 0$

b) $I_1 + I_2 + I_3 = 0$

$-10I_1 + 0I_2 + 20I_3 = -12$

$0I_1 - 35I_2 + 20I_3 = -5$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -10 & 0 & 20 & -12 \\ 0 & -35 & 20 & -5 \end{array}\right]$$

$$\begin{bmatrix} 1 & 1 & 1 \\ -10 & 0 & 20 \\ 0 & -35 & 20 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -12 \\ -5 \end{bmatrix}$$

c) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -10 & 0 & 20 & -12 \\ 0 & -35 & 20 & -5 \end{array}\right]$ $10R_1 + R_2 \rightarrow$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 10 & 30 & -12 \\ 0 & -35 & 20 & -5 \end{array}\right]$ $R_2/10 \rightarrow$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & -6/5 \\ 0 & -35 & 20 & -5 \end{array}\right]$ $35R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & -6/5 \\ 0 & 0 & 125 & -47 \end{array}\right]$ $-R_2 + R_1 \rightarrow$ $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 6/5 \\ 0 & 1 & 3 & -6/5 \\ 0 & 0 & 125 & -47 \end{array}\right]$ $R_3/125 \rightarrow$ $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 6/5 \\ 0 & 1 & 3 & -6/5 \\ 0 & 0 & 1 & -47/125 \end{array}\right]$ $-3R_3 + R_2$, $2R_3 + R_1$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 56/125 \\ 0 & 1 & 0 & -9/125 \\ 0 & 0 & 1 & -47/125 \end{array}\right]$

$I_1 = 0.448\text{ A}$

$I_2 = -0.072\text{ A}$

$I_3 = -0.376\text{ A}$

PROBLEM 2: 'GAUSSIAN ELIMINATION' BY HAND

$$A = \left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 1 \\ 1 & 1 & 2 & 0 & -1 \\ -1 & 2 & 1 & 1 & 0 \\ 6 & 1 & 1 & -2 & 2 \end{array}\right] \quad R_2 = R_2 - \tfrac{1}{2}R_1$$

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & -1 \\ 0 & 1/2 & 5/2 & 0 & -3/2 \\ -1 & 2 & 1 & 1 & 0 \\ 6 & 1 & 1 & -2 & 2 \end{array}\right] \quad R_3 = R_3 + \tfrac{1}{2}R_1$$

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & -1 \\ 0 & 1/2 & 5/2 & 0 & -3/2 \\ 0 & 5/2 & 1/2 & 1 & -1/2 \\ 6 & 1 & 1 & -2 & 2 \end{array}\right] \quad R_4 = R_4 - 3R_1$$

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & -1 \\ 0 & 1/2 & 5/2 & 0 & -3/2 \\ 0 & 5/2 & 1/2 & 1 & -1/2 \\ 0 & -2 & 4 & -2 & 7 \end{array}\right] \quad R_3 = R_3 - 5R_2$$

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & -1 \\ 0 & 1/2 & 5/2 & 0 & -3/2 \\ 0 & 0 & -12 & 1 & 7 \\ 0 & -2 & 4 & -2 & 7 \end{array}\right] \quad R_4 = R_4 + 2R_2$$

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & -1 \\ 0 & 1/2 & 5/2 & 0 & -3/2 \\ 0 & 0 & -12 & 1 & 7 \\ 0 & 0 & 9 & -2 & 4 \end{array}\right] \quad R_4 = R_4 + \tfrac{3}{4}R_3$$

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & -1 \\ 0 & 1/2 & 5/2 & 0 & -3/2 \\ 0 & 0 & -12 & 1 & 7 \\ 0 & 0 & 0 & -5/4 & 53/4 \end{array}\right]$$

PROBLEM 4: LU DECOMPOSITION

$A = LU \quad A \cdot x = c \quad [A][x] = [c]$

$[L][U][x] = [c]$

$$\begin{bmatrix} 2 & 1 & -1 & 0 \\ 1 & 1 & 2 & 0 \\ -1 & 2 & 1 & 1 \\ 0 & 1 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ -1/2 & 5 & 1 & 0 \\ 3 & -4 & -7/6 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & -1 & 0 \\ 0 & 1/2 & 5/2 & 0 \\ 0 & 0 & -12 & 1 \\ 0 & 0 & 0 & -5/6 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} y_1 & y_2 & y_3 & y_4 & \\ 1 & 0 & 0 & 0 & -1 \\ 1/2 & 1 & 0 & 0 & 1 \\ 1/2 & 5 & 1 & 0 & 2 \\ 3 & -4 & -7/6 & 1 & 0 \end{array}\right]$$

$y_1 = -1$

$1/2 y_1 + y_2 = 1$

$1/2 y_1 + 5 y_2 + y_3 = 2$

$3y_1 - 4y_2 - \frac{7}{6} y_3 + y_4 = 0$

$$y = \begin{bmatrix} -1 \\ 3/2 \\ -5 \\ 19/6 \end{bmatrix} \qquad x = \begin{bmatrix} -34/20 \\ 5/2 \\ 1/10 \\ 19/5 \end{bmatrix}$$

$-1/2 + y_2 = 1 \quad y_2 = 3/2$

$1/2(-1) + 5(3/2) + y_3 = 2$

$-1/2 + 15/2 + y_3 = 2$

$y_3 = -5$

$3(-1) - 4(3/2) - \frac{7}{6}(-5) + y_4 = 0$

$-3 - 6 + \frac{35}{6} + y_4 = 0$

$-9 + \frac{35}{6} + y_4 = 0$

$-\frac{54}{6} + \frac{35}{6} + y_4 = 0$

$-\frac{19}{6} + y_4 = 0 \quad y_4 = 19/6$

$$\left[\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ 2 & 1 & -1 & 0 & -1 \\ 0 & 1/2 & 5/2 & 0 & 3/2 \\ 0 & 0 & -12 & 1 & -5 \\ 0 & 0 & 0 & -5/6 & 19/6 \end{array}\right]$$

$2x_1 + \frac{25}{10} - \frac{1}{10} = -1$

$2x_1 + \frac{24}{10} = -1$

$\frac{2x_1}{2} = \frac{-10}{10} - \frac{24}{10} = \frac{-34}{10}$

$-\frac{5}{6} x_4 = 19/6 \qquad x_4 = 19/6 \cdot \frac{-6}{5} = -\frac{19}{5} \quad -\frac{34}{20}$

$-12x_3 - \frac{19}{5} = -5 \qquad -12x_3 + x_4 = -5$

$-12x_3 - \frac{19}{5} = -5$

$\frac{-12x_3}{12} = \frac{-25}{5} + \frac{19}{5} = \frac{-6}{5} \cdot \frac{1}{12_2} = \frac{-1}{10} = x_3$

$1/2 x_2 + 5/2 x_3 = 3/2$

$1/2 x_2 + 5/2 \left(\frac{1}{10}_2\right) = 3/2$

$1/2 x_2 + 1/4 = 3/2 \quad \left(\frac{1}{2} x_2 = \frac{6}{4} - \frac{1}{4} = \frac{5}{4} \cdot 2 = \frac{5}{2}\right.$