# Math 308: Principal Component Analysis

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Task:

Perform a principal component analysis of this data using your preferred function. As part of this analysis, please be sure complete the following tasks: - Report the eigenvalues for all 11 principal components. - For the first two principal components, plot and interpret components in terms of the original variables. In particular, explain which variables are most highly correlated with each of these two components and how these components are different from each other. - Choose the smallest number of principal components that you believe can be used to summarize the information from the data and justify your choice.

Solution:

```
library(tinytex)
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v dplyr     1.1.2     v readr     2.1.4
## v forcats   1.0.0     v stringr   1.5.0
## v ggplot2   3.4.2     v tibble    3.2.1
## v lubridate 1.9.2     v tidyr     1.3.0
## v purrr     1.0.1
## -- Conflicts ---------------------------------------- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library(factoextra)
```

```
## Welcome! Want to learn more? See two factoextra-related books at https://goo.gl/ve3WBa
```

```
library(FactoMineR)
library(GGally)
```

```
## Registered S3 method overwritten by 'GGally':
##   method from
##   +.gg   ggplot2
```

```
## Rows: 1599 Columns: 12
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## dbl (12): fixed.acidity, volatile.acidity, citric.acid, residual.sugar, chlo...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
## # A tibble: 6 x 12
##   fixed.acidity volatile.acidity citric.acid residual.sugar chlorides
##           <dbl>            <dbl>       <dbl>          <dbl>     <dbl>
## 1           7.4             0.7         0               1.9     0.076
## 2           7.8             0.88        0               2.6     0.098
## 3           7.8             0.76        0.04            2.3     0.092
## 4          11.2             0.28        0.56            1.9     0.075
## 5           7.4             0.7         0               1.9     0.076
## 6           7.4             0.66        0               1.8     0.075
## # i 7 more variables: free.sulfur.dioxide <dbl>, total.sulfur.dioxide <dbl>,
## #   density <dbl>, pH <dbl>, sulphates <dbl>, alcohol <dbl>, quality <dbl>
```

```
## Rows: 1,599
## Columns: 12
## $ fixed.acidity        <dbl> 7.4, 7.8, 7.8, 11.2, 7.4, 7.4, 7.9, 7.3, 7.8, 7.5~
## $ volatile.acidity     <dbl> 0.700, 0.880, 0.760, 0.280, 0.700, 0.660, 0.600, ~
## $ citric.acid          <dbl> 0.00, 0.00, 0.04, 0.56, 0.00, 0.00, 0.06, 0.00, 0~
## $ residual.sugar       <dbl> 1.9, 2.6, 2.3, 1.9, 1.9, 1.8, 1.6, 1.2, 2.0, 6.1,~
## $ chlorides            <dbl> 0.076, 0.098, 0.092, 0.075, 0.076, 0.075, 0.069, ~
## $ free.sulfur.dioxide  <dbl> 11, 25, 15, 17, 11, 13, 15, 15, 9, 17, 15, 17, 16~
## $ total.sulfur.dioxide <dbl> 34, 67, 54, 60, 34, 40, 59, 21, 18, 102, 65, 102,~
## $ density              <dbl> 0.9978, 0.9968, 0.9970, 0.9980, 0.9978, 0.9978, 0~
## $ pH                   <dbl> 3.51, 3.20, 3.26, 3.16, 3.51, 3.51, 3.30, 3.39, 3~
## $ sulphates            <dbl> 0.56, 0.68, 0.65, 0.58, 0.56, 0.56, 0.46, 0.47, 0~
## $ alcohol              <dbl> 9.4, 9.8, 9.8, 9.8, 9.4, 9.4, 9.4, 10.0, 9.5, 10.~
## $ quality              <dbl> 5, 5, 5, 6, 5, 5, 5, 7, 7, 5, 5, 5, 5, 5, 5, 5, 7~
```

```
wine_chem<-wine%>%select(-quality)
head(wine_chem)
```

```
## # A tibble: 6 x 11
##   fixed.acidity volatile.acidity citric.acid residual.sugar chlorides
##           <dbl>            <dbl>       <dbl>          <dbl>     <dbl>
## 1           7.4             0.7         0               1.9     0.076
## 2           7.8             0.88        0               2.6     0.098
## 3           7.8             0.76        0.04            2.3     0.092
## 4          11.2             0.28        0.56            1.9     0.075
## 5           7.4             0.7         0               1.9     0.076
## 6           7.4             0.66        0               1.8     0.075
## # i 6 more variables: free.sulfur.dioxide <dbl>, total.sulfur.dioxide <dbl>,
## #   density <dbl>, pH <dbl>, sulphates <dbl>, alcohol <dbl>
```

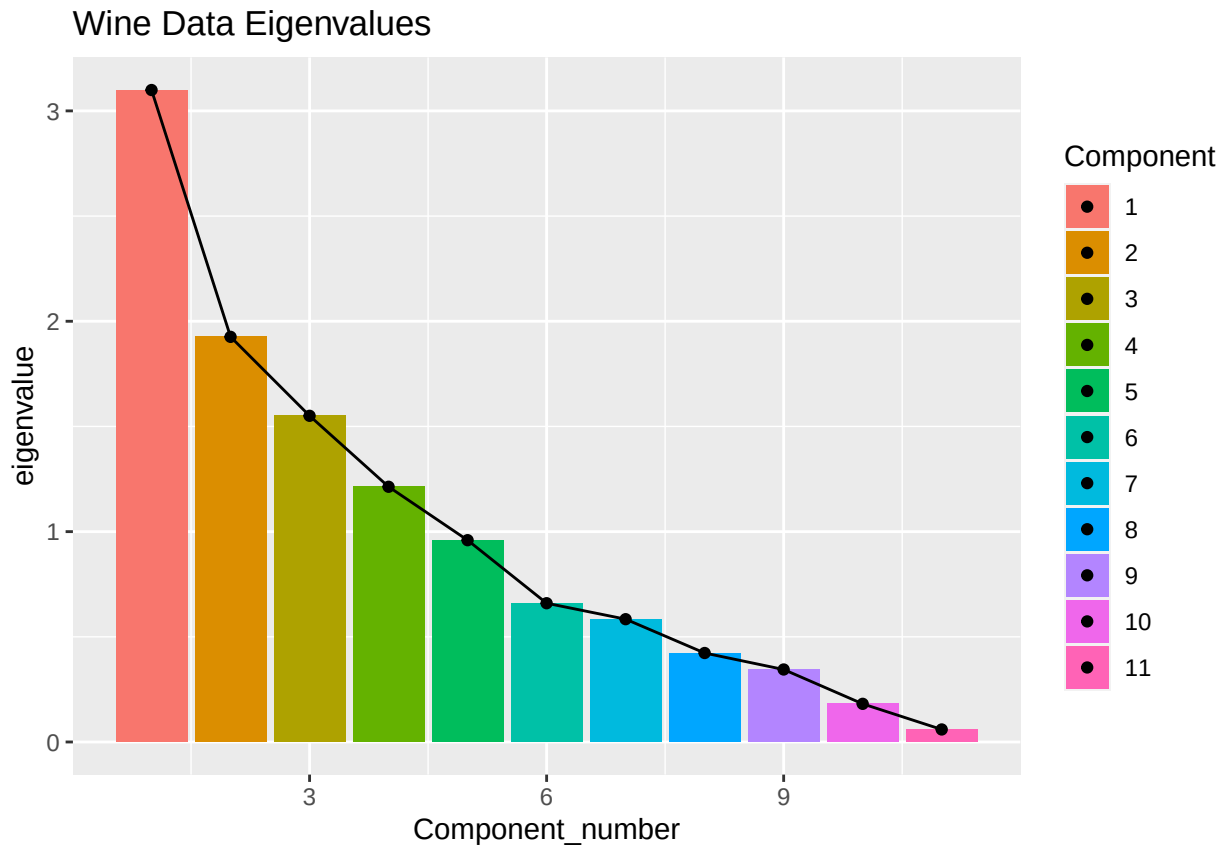
Eigenvalues:

```
wine_PCA<-PCA(wine_chem,graph=FALSE)
wine_PCA$eig[,"eigenvalue"]
```

```
##     comp 1     comp 2     comp 3     comp 4     comp 5     comp 6     comp 7
## 3.09913244 1.92590969 1.55054349 1.21323253 0.95929207 0.65960826 0.58379122
##     comp 8     comp 9    comp 10    comp 11
## 0.42295670 0.34464212 0.18133317 0.05955831
```

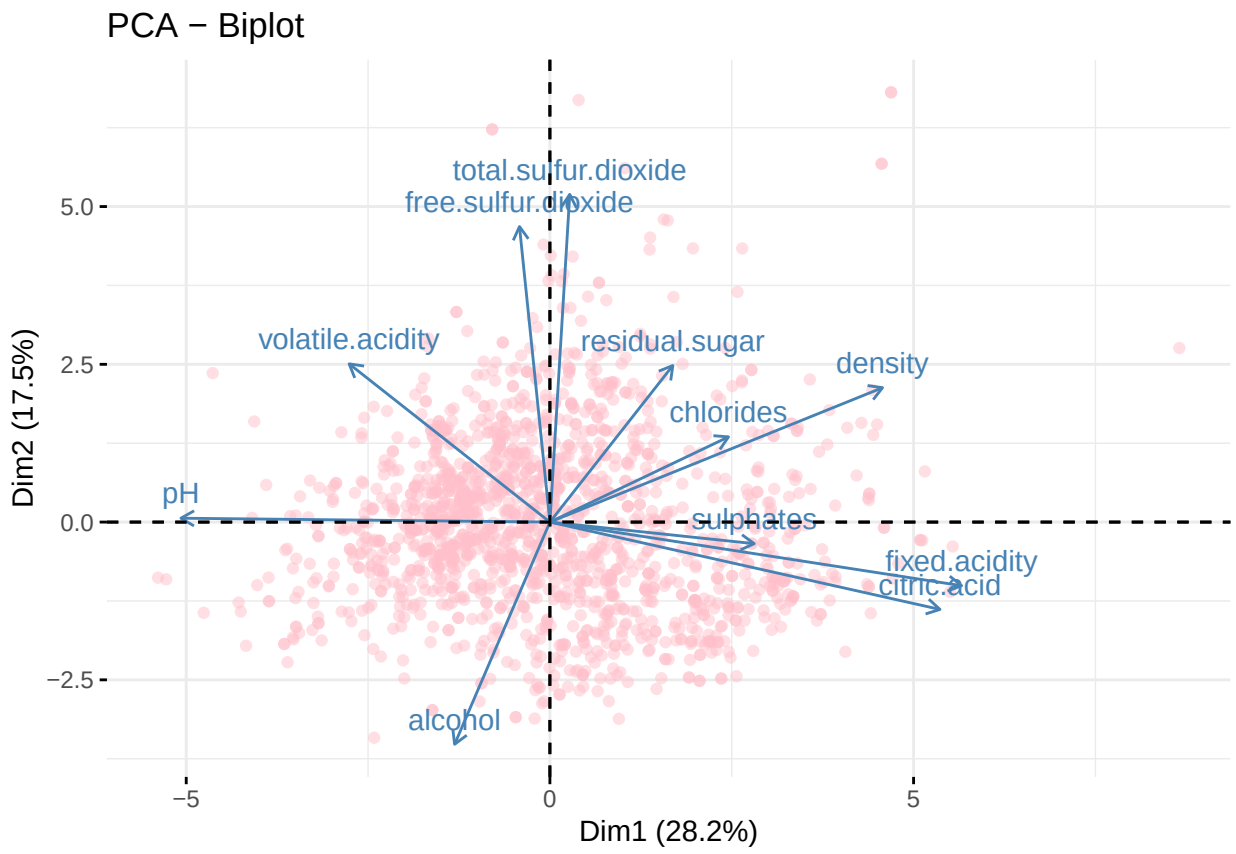
```
eigenvalues_wine<-as.data.frame(wine_PCA$eig) %>%
 rownames_to_column(var="Component")
eigenvalues_wine <- eigenvalues_wine %>%
 mutate(Component=map_chr(Component,~str_split(.x," ")[[1]][2]),
 Component_number=as.integer(Component),
 Component=factor(Component_number))


ggplot(eigenvalues_wine,aes(y=eigenvalue,x=Component_number,fill=Component)) +
 geom_bar(stat="identity") + geom_line(aes(fill=NULL)) + geom_point() +ggtitle("Wine Data Eigenvalues")
```




Component Analysis:

```
fviz_pca_biplot(wine_PCA,col.ind="pink",
 fill.ind="pink",label="var",
 alpha.ind=c(0.5))
```




```
round(wine_PCA$var$coord[,c(1:2)],2)
```

```
##                       Dim.1 Dim.2
## fixed.acidity          0.86 -0.15
## volatile.acidity      -0.42  0.38
## citric.acid            0.82 -0.21
## residual.sugar         0.26  0.38
## chlorides              0.37  0.21
## free.sulfur.dioxide   -0.06  0.71
## total.sulfur.dioxide   0.04  0.79
## density                0.70  0.32
## pH                    -0.77  0.01
## sulphates              0.43 -0.05
## alcohol               -0.20 -0.54
```

From the plot and the table above, we see that the first component mostly is a contrast between the pH level and the fixed acidity, citric acid and density values. These are the variables with the strongest correlations with the first component. The second component mostly depends on the free and total sulfur dioxide measures, which are contrasted mostly with the alcohol content, as these three variables are most strongly associated with the second component.

Number of Components:

```
wine_PCA$eig
```

```
##         eigenvalue percentage of variance cumulative percentage of variance
## comp 1  3.09913244             28.1739313                            28.17393
## comp 2  1.92590969             17.5082699                            45.68220
## comp 3  1.55054349             14.0958499                            59.77805
## comp 4  1.21323253             11.0293866                            70.80744
## comp 5  0.95929207              8.7208370                            79.52827
## comp 6  0.65960826              5.9964388                            85.52471
## comp 7  0.58379122              5.3071929                            90.83191
## comp 8  0.42295670              3.8450609                            94.67697
## comp 9  0.34464212              3.1331102                            97.81008
## comp 10 0.18133317              1.6484833                            99.45856
## comp 11 0.05955831              0.5414392                           100.00000
```

We see that using an 80% cumulative variance rule, we would decide to use either 5 or 6 components depending on how strictly we wanted to use the cutoff.