HW 2

Q1. (a) $\left[\begin{array}{ccc|c} 2.51 & 1.48 & 4.53 & 0.05 \\ 1.48 & 0.93 & -1.30 & 1.03 \\ 2.68 & 3.04 & -1.48 & -0.53 \end{array}\right] = \left[\begin{array}{ccc|c} 2.51 & 1.48 & 4.53 & 0.0500 \\ 0 & 0.0593 & -3.97 & 1.00 \\ 0 & 1.46 & -6.32 & -0.583 \end{array}\right]$

$= \left[\begin{array}{ccc|c} 2.51 & 1.48 & 4.53 & 0.0500 \\ 0 & 0.0593 & -3.97 & 1.00 \\ 0 & 0 & 94.8 & -26.1 \end{array}\right]$ $\quad x = 1.46 \quad y = -1.60 \quad z = -0.275$

$2.51(1.46) + 1.48(-1.60) + 4.53(-0.275) = 0.05085$

$1.48(1.46) + 0.93(-1.60) - 1.30(-0.275) = 1.03030$

$2.68(1.46) + 3.04(-1.60) - 1.48(-0.275) = -0.54420$

They actually match quite well! The total error is

$|0.05085 - 0.05| + |1.03030 - 1.03| + |-0.54420 - (-0.53)|$

$= 0.00085 + 0.0003 + 0.0142 = 0.01535$

(b) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2.51 & 1.48 & 4.53 & 0.05 \\ 1.48 & 0.93 & -1.30 & 1.03 \\ 2.68 & 3.04 & -1.48 & -0.53 \end{bmatrix} = \left[\begin{array}{ccc|c} 2.68 & 3.04 & -1.48 & -0.53 \\ 2.51 & 1.48 & 4.53 & 0.05 \\ 1.48 & 0.93 & -1.30 & 1.03 \end{array}\right]$

$= \left[\begin{array}{ccc|c} 2.68 & 3.04 & -1.48 & -0.530 \\ 0 & -1.37 & 5.92 & 0.546 \\ 0 & -0.749 & -0.483 & 1.32 \end{array}\right] = \left[\begin{array}{ccc|c} 2.68 & 3.04 & -1.48 & -0.530 \\ 0 & -1.37 & 5.92 & 0.546 \\ 0 & 0 & -3.72 & 1.02 \end{array}\right]$

$x = 1.44 \quad y = -1.58 \quad z = -0.274$

$2.51(1.44) + 1.48(-1.58) + 4.53(-0.274) = 0.03478$

$1.48(1.44) + 0.93(-1.58) - 1.30(-0.274) = 1.018$

$2.68(1.44) + 3.04(-1.58) - 1.48(-0.274) = -0.53848$

Surprisingly, they did worse than (a), it may just be a coincidence though. The total error is

$|0.03478 - 0.05| + |1.018 - 1.03| + |-0.53848 - (-0.53)|$

$= 0.01522 + 0.012 + 0.00848 = 0.0357$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \left[\begin{array}{ccc|c} 2.51 & 1.48 & 4.53 & 0.05 \\ 1.48 & 0.93 & -1.30 & 1.03 \\ 2.68 & 3.04 & -1.48 & -0.53 \end{array}\right] = \left[\begin{array}{ccc|c} 2.68 & 3.04 & -1.48 & -0.53 \\ 2.51 & 1.48 & 4.53 & 0.05 \\ 1.48 & 0.93 & -1.30 & 1.03 \end{array}\right]$

$= \left[\begin{array}{ccc|c} 2.68 & 3.04 & -1.48 & -0.53 \\ 0 & -1.36 & 5.91 & 0.546 \\ 0 & -0.748 & -0.482 & 1.32 \end{array}\right] = \left[\begin{array}{ccc|c} 2.68 & 3.04 & -1.48 & -0.53 \\ 0 & -1.36 & 5.91 & 0.546 \\ 0 & 0 & -3.73 & 1.01 \end{array}\right]$

$x = 1.43 \quad y = -1.57 \quad z = -0.270$

$2.51(1.43) + 1.48(-1.57) + 4.53(-0.270) = 0.0426$
$1.48(1.43) + 0.93(-1.57) - 1.30(-0.270) = 1.0073$
$2.68(1.43) + 3.04(-1.57) - 1.48(-0.270) = -0.5408$

This result did even worse than (a) & (b), but this is not as surprising since it chops number off instead of rounding it up in some cases. The total error is
$|0.0426 - 0.05| + |1.0073 - 1.03| + |-0.5408 - (-0.53)|$
$= 0.0074 + 0.0227 + 0.0108 = 0.0409$

Q2.

(a) (b) are provided in the code.

(c). Modify matrix:
From $i = 2 \sim n$ with 2 operations yields $(n-1) \times 2 = 2(n-1)$.

Backward substitution:
From $i = n \sim 1$ with 1 operation yields $n \times 1 = n$

$2(n-1) + n = 3n-1$ operations in total.

Q3. is in the given code.

Q4.

Using the given code, here is the table:

w	iterations
1.0	112
1.1	90
1.2	70
1.3	53
1.4	35
1.5	29 ☆
1.6	44
1.7	95
1.8	x
1.9	x
2.0	x

w	iterations
1.43	27
1.44	24
1.45	25
1.46	23 ☆
1.47	26
1.48	26
1.49	29
1.51	29
1.52	32
1.53	32
1.54	32
1.55	31

Q5. The result is calculated in the given code.

(a). condition $= 10^{20} \rightarrow$ ill-conditioned

(b). condition $= 1 \rightarrow$ well-conditioned

(c). condition $= 1 \rightarrow$ well-conditioned

(d). condition $= \infty \rightarrow$ ill-conditioned