Numerical Methods, 2023 Spring

Assignment 4
Due on May 11

Note: you should explain how you obtain your solution in your submission. If you use MATLAB or any other software to compute your results, you should provide your code or describe your solving process. This is a good practice for you to explain things in a logical, organized, and concise way!

1. (10%) Make a divided difference table for $f(x) = 1 + log_{10} x$ with data points at $x_0 = 0.15, x_1 = 0.21, x_2 = 0.23, x_3 = 0.27, x_4 = 0.32, x_5 = 0.35$. Compute a value for $f'(0.268)$ from a quadratic interpolating polynomial that fits the table at the three points that should give the most accurate answer. Which points are these?

2. (20%) Make a function difference table for $f(x) = x + \sin(x)/3$ with data points at $x_0 = 0.3, x_1 = 0.5, \dots, x_6 = 1.5$. Use it to find
   (a)$f'(0.72)$ from a cubic polynomial.
   (b)$f'(1.33)$ from a quadratic polynomial.
   (c)$f'(0.50)$ from a 4th degree polynomial.
   In each part, choose the best starting data points.

3. (20%) Use the method of underdetermined coefficients to obtain the formula for $f''(x)$ and $f'''(x)$ at x_0 using five evenly spaced points from x_{-2} to x_2. Please also derive their error terms.

4. (15%) Simpson's 3/8 rule cannot be applied directly to the following table because the number of panels is not divisible by three. Still, you can use it in combination with the 1/3 rule over two panels. There are several choices of where to use the 1/3 rule. Which of these choices gives the most accurate answer?

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$f(x)$	1.543	1.669	1.811	1.971	2.151	2.352	2.577	2.828	3.107

5. (15%) Integrate $f(x) = 1/x^2$ over the interval [0.2, 1] using the trapezoidal rule with an adaptive integration scheme, i.e., halving the panel width *h* at each iteration until the integral value does not differ more than 0.02 between two iterations. At what value for *h* do the computation terminate?

6. (20%) Evaluate the following integral using Gaussian quadrature, three-term formulas and h=0.1 in both directions.

$$\int_{-0.2}^{1.4}\int_{0.4}^{2.6} e^x \sin(2y)\, dydx$$