Assignment 1
Due on Mar 23

Note: you should explain how you obtain your solution in your submission. If you use MATLAB or any other software to compute your results, you should provide your code and describe your solving process. This is good practice for you to explain things in a logical, organized, and concise way! **Please hand in your assignment in the class**.

1. (30%) The function $f(x) = x^2 + \sin(x) - \frac{e^x}{4} - 1$ has zeros for two values near $x = 0$. Please compute both roots, starting with $[-2, 0]$ and $[0, 2]$, to attain an accuracy of 10^{-5} using the following methods: (a) the bisection method (b) the secant method, and (c) Newton's method.

2. (25%) Use Newton's method on the polynomial $P(x) = (x-2)^3(x-4)^2$ with $x_0 = 3$. Does it converge? To which root? Is convergence quadratic?

3. (30%) Below are three different g(x) functions. All are rearrangements of the same f(x). What is f(x)?
   (a) $(4 + 2x^3)/x^2 - 2x$
   (b) $\sqrt{4/x}$
   (c) $(16 + x^3)/(5x^2)$
   Which of them converge? What x-value is obtained? Are there starting values for which one or more diverge? Which diverge?

4. (30%) Solve the following system of nonlinear equations using Newton's method or fixed-point method.
$$x - 3y - z^2 = -3$$
$$2x^3 + y - 5z^2 = -2$$
$$4x^2 + y + z = 7$$

   Hints: There are six solutions to this system. Two of the real solutions are near (1, 1, 1) and (1.3, 0.9, -1.2). Your score of this question will depend on how many solutions you find.