HW1

Q1. It's written in the given code.

Q2. Some contents are in the given code.

Newton: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$P(x) = (x-2)^3(x-4)^2$, $x_0 = 3$

$P'(x) = 3(x-2)^2(x-4)^2 + 2(x-2)^3(x-4)$

$$x_{n+1} = x_n - \frac{P(x_n)}{P'(x_n)} = x_n - \frac{(x_n-2)^3(x_n-4)^2}{3(x_n-2)^2(x_n-4)^2+2(x_n-2)^3(x_n-4)}$$

$$= x_n - \frac{(x_n-2)(x_n-4)}{3(x_n-4)+2(x_n-2)}$$

$x_1 = 3 - \frac{1\cdot(-1)}{3(-1)+2} = 3-1 = 2$

$x_2 = 2 - \frac{0\cdot(-2)}{3(-2)+0} = 2 \Rightarrow$ converged to the root 2

The error rate went from 1 to 0, so we can't exactly tell if the convergence is quadratic. However, if we alter the starting value to $x_0 = 5$ and run the program, we can see that the results are:

$x_1 = 4.6667$	$x_2 = 4.4242$	$x_3 = 4.2562$	$x_4 = 4.1468$
$x_5 = 4.0802$	$x_6 = 4.0423$	$x_7 = 4.0218$	$x_8 = 4.0111$
$x_9 = 4.0056$	$x_{10} = 4.0028$	$x_{11} = 4.0014$	$x_{12} = 4.0007$

As we can see from the results above, the error seems to converge at a linear rate of $\frac{1}{2}$ instead of quadratically. Newton's method is only quadratically convergent if it converges to a simple root. Neither 2 nor 4 is a simple root, thus they aren't quadratically convergent.

Q3. Some contents are in the given code.

$f(x) = x^3 - 4$

First, we should find a x_0.

$f(0) = -4 \qquad f(1) = -3 \qquad f(2) = 4$

Choose $x_0 = \frac{1+2}{2} = \frac{3}{2}$

(a). $x = \frac{(4+2x^3)}{x^2} - 2x$

$3x = \frac{(4+2x^3)}{x^2}$

$3x^3 = 4 + 2x^3$

$x^3 = 4$

$\Rightarrow f(x) = x^3 - 4 = 0$

$g(x) = \frac{(4+2x^3)}{x^2} - 2x$

$= \frac{4}{x^2} + 2x - 2x = \frac{4}{x^2}$

$g'(x) = \frac{-8}{x^3}$

$|g'(\frac{3}{2})| = |-8 \cdot (\frac{2}{3})^3| = \frac{64}{27} > 1$

$\Rightarrow$ (a) will diverge no matter the starting value.

(b) $x = \sqrt{\frac{4}{x}}$

$x^2 = \frac{4}{x}$

$x^3 = 4$

$\Rightarrow f(x) = x^3 - 4$

$g(x) = \sqrt{\frac{4}{x}} = \frac{2}{\sqrt{x}}$

$g'(x) = -\frac{1}{\sqrt[3]{x}}$

$|g'(\frac{3}{2})| = |-\sqrt[3]{\frac{2}{3}}| = \sqrt[3]{\frac{2}{3}} < 1$

$\Rightarrow$ (b) will converge.

$x_{n+1} = g(x_n)$: $\quad x_1 = 2^{1-\frac{1}{2}} = 2^{\frac{1}{2}} \qquad x_2 = 2^{1-\frac{1}{4}} = 2^{\frac{3}{4}} \qquad x_3 = 2^{1-\frac{3}{8}} = 2^{\frac{5}{8}}$

try $x_0 = 2 \qquad x_4 = 2^{1-\frac{5}{16}} = 2^{\frac{11}{16}} \qquad x_5 = 2^{1-\frac{11}{32}} = 2^{\frac{21}{32}} = 1.5759$

$\approx 2^{\frac{2}{3}} = 1.5874$

(c) $x = \frac{16+x^3}{5x^2}$

$5x^3 = 16 + x^3$

$4x^3 = 16$

$x^3 = 4$

$\Rightarrow f(x) = x^3 - 4$

$g(x) = \frac{16+x^3}{5x^2} = \frac{16}{5x^2} + \frac{x}{5}$

$g'(x) = \frac{-32}{5x^3} + \frac{1}{5}$

$|g'(\frac{3}{2})| = |-\frac{32}{5} \cdot (\frac{2}{3})^3 + \frac{1}{5}| = \frac{229}{135} > 1$

$\Rightarrow$ (c) will diverge no matter the starting value.

Q4. Execution is provided in the given code.

$$f(x) = \begin{bmatrix} f_1(x,y,z) \\ f_2(x,y,z) \\ f_3(x,y,z) \end{bmatrix} = \begin{bmatrix} x-3y-z^2+3 \\ 2x^3+y-5z^2+2 \\ 4x^2+y+z-7 \end{bmatrix} = 0$$

$$J(x) = \begin{bmatrix} 1 & -3 & -2z \\ 6x^2 & 1 & -10z \\ 8x & 1 & 1 \end{bmatrix} \qquad J(x_0) = \begin{bmatrix} 1 & -3 & -2 \\ 6 & 1 & -10 \\ 8 & 1 & 1 \end{bmatrix}$$

$x_0 = [1,1,1]$ $\quad x_n = [1.1114, 0.9882, 1.0709]$

$x_0 = [1.3, 0.9, -1.2]$ $\quad x_n = [1.3537, 0.9254, -1.2560]$

$x_0 = [30, 0, 0]$ $\quad x_n = [32.8846, -4434.0866, 115.4908]$

$x_0 = [50, 0, 0]$ $\quad x_n = [31.1514, -3768.1571, -106.4829]$

$x_0 = [-1+i, 0, 0]$ $\quad x_n = [-1.2506+0.4899i, 0.6651-0.0134i, 0.0886+0.5036i]$

since imaginary roots always comes in pairs, there's also another root

$x_n = [-1.2506-0.4899i, 0.6651+0.0134i, 0.0886-0.5036i]$