A Tantalizing Twist with Cubes

Nicholas Brandt – Thoa Ta – Yi Yi (Lily) Zhang

Faculty Mentor: **Satyanand Singh, Ph.D.**

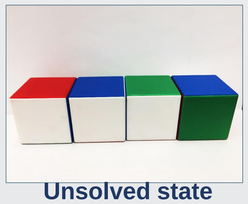
Division of Computer Science – College of Professional Studies

ST. JOHN'S UNIVERSITY

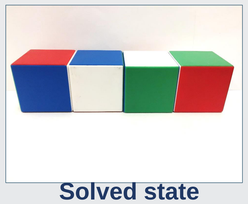
COLLEGE OF PROFESSIONAL STUDIES

SUMMARY OF THE PUZZLE

We start with 4 cubes, and each face on a given cube is shaded one of the 4 colors (R, W, B, G). The goal is to arrange the cubes into a rectangular log so that each long side of the log has one and only one occurrence of each color.



Unsolved state

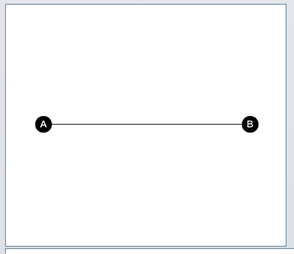


Solved state

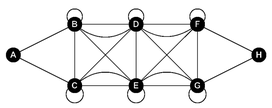
41,472 possible arrangements if done by hand! Is there a more efficient way?

OVERVIEW - GRAPH THEORY

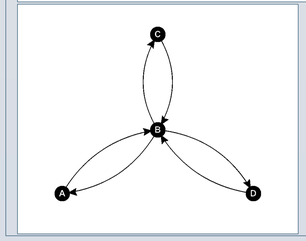
Graphs help **represent** a set of **data**, and any **relationships** inherent within that set, in a **more intuitive** way.



- **Vertices**: A, B
- **Edge**: the line connecting A and B
- **Degree of a vertex:** number of edges that touch that vertex
- **Simple graph**: unweighted, undirected, no loops or multiple edges



- **Loop:** an "edge" which joins a vertex to itself
- **Complex graph**: with loops and multiple edges



- **Directed graph:** edges have directions

← This is an example of an Euler cycle (visits every edge exactly once)

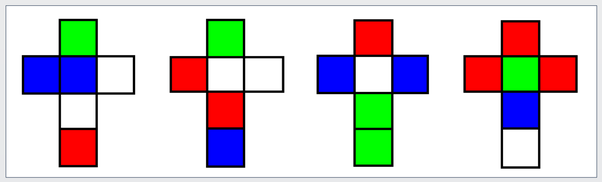
METHOD

- A graph with 4 vertices labelled R, B, W, G.
- To describe the relationship between two opposite faces, an edge is drawn between the vertex(es) corresponding to their color(s).

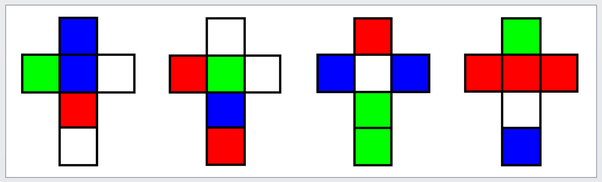
PROCEDURE

The procedure here features the set of cubes in the commercially sold "Instant Insanity."

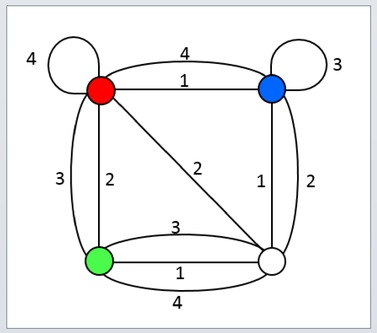
UNSOLVED



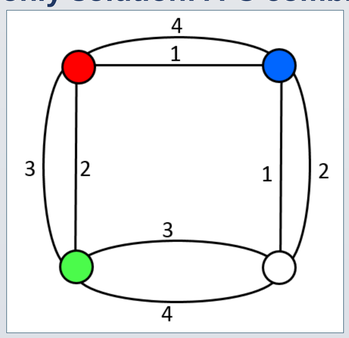
SOLVED



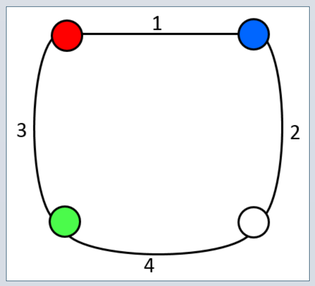
Master Graph



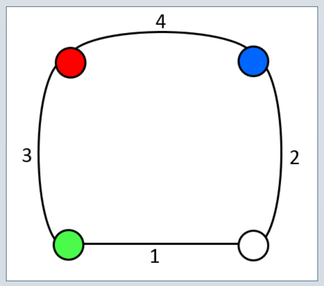
The only solution: A-C combined



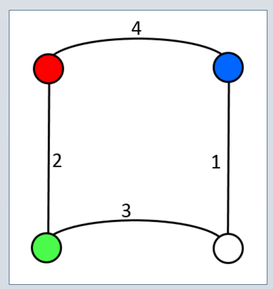
Acceptable subgraphs: subgraphs with degree-2



A



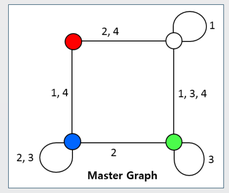
B

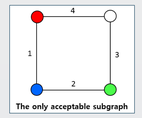


C

MORE EXAMPLES

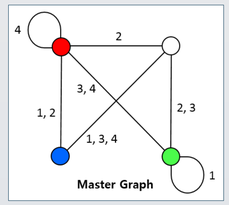
An example that does not work:

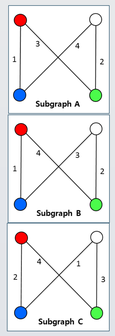




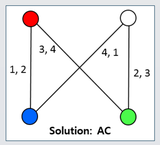
NO SOLUTION

An example that works (Arlinghaus, 1991):

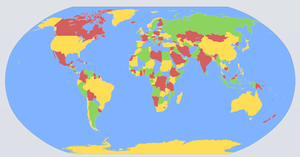


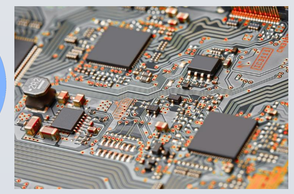


ONE SOLUTION

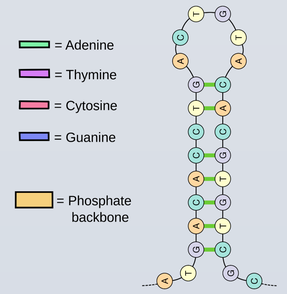


APPLICATIONS - GRAPH THEORY





DNA



REFERENCES

Arlinghaus, W. C. 1991. "The Tantalizing Four Cubes." In John G. Michaels and Kenneth H. Rosen., editors, *Applications of Discrete Mathematics*, chapter 16. McGraw-Hill Higher Education.