## Week1 monday

Alphabet e.g. $\Sigma$ , $\Gamma$	non-empty finite set
Symbol over $\Sigma$	element of alphabet $\Sigma$
String over $\Sigma$	finite list of symbols from $\Sigma$
Language over $\Sigma$	set of strings over $\Sigma$
Empty set $\emptyset$	the empty language
Pages 3, 4, 13, 14	

With  $\Sigma_1 = \{0,1\}$  and  $\Sigma_2 = \{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z\}$  and  $\Gamma = \{0,1,x,y,z\}$ 

An example of a string of length 3 over  $\Sigma_1$  is

An example of a string of length 1 over  $\Sigma_2$  is

The number of distinct strings of length 2 over  $\Gamma$  is

An example of a language over  $\Sigma_1$  of size 1 is

An example of an infinite language over  $\Sigma_1$  is

An example of a finite language over  $\Gamma$  is

Empty string $\varepsilon$	the string of length 0
Reverse of a string $w, w^{\mathcal{R}}$	write $w$ in the opposite order, if $w = w_1 \cdots w_n$ then $w^{\mathcal{R}} = w_n \cdots w_1$
Concatenating strings $x$ and $y$	take $x = x_1 \cdots x_m$ , $y = y_1 \cdots y_n$ and form $xy = x_1 \cdots x_m y_1 \cdots y_n$
String $z$ is a substring of string $w$	there are strings $u, v$ such that $w = uzv$
String $x$ is a prefix of string $y$	there is a string z such that $y = xz$
String $x$ is a proper prefix of string $y$	x is a prefix of y and $x \neq y$
Pages 13, 14	

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\varepsilon \in \Sigma_1 True False \varepsilon is a string over \Sigma_1 True False \varepsilon is a language over \Sigma_1 True False \varepsilon is a prefix of some string over \Sigma_1 True False There is a string over \Sigma_1 that is a proper prefix of \varepsilon True False
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**String order** over alphabet  $\Sigma$ : Order strings over  $\Sigma$  first by length and then according to the dictionary order, assuming symbols in  $\Sigma$  have an ordering.

The first five strings over  $\Sigma_1$  in string order, using the ordering 0 < 1:

The first five strings over  $\Sigma_2$  in string order, using the usual alphabetical ordering for single letters:

Assuming A and B are languages over alphabet $\Sigma$		
The union $A$ and $B$	$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$	
The concatenation of $A$ and $B$	$A \circ B = \{ xy \mid x \in A \text{ and } y \in B \}$	
The star of $A$	$A^* = \{x_1 x_2 \cdots x_k \mid k \ge 0 \text{ and each } x_i \in A\}$	
Definition 1.23 page 44		

Assuming $\Sigma$ is the alphabet, recursive definition of regular expressions is		
a is a regular expression	for $a \in \Sigma$	$L(a) = \{a\}$
$\varepsilon$ is a regular expression		$L(\varepsilon) = \{\varepsilon\}$
$\emptyset$ is a regular expression		$L(\emptyset) = \{\} = \emptyset$
$(R_1 \cup R_2)$ is a regular expression	for $R_1$ , $R_2$ regular expressions	$L((R_1 \cup R_2)) = L(R_1) \cup L(R_2)$
$(R_1 \circ R_2)$ is a regular expression	for $R_1$ , $R_2$ regular expressions	$L((R_1 \circ R_2)) = L(R_1) \circ L(R_2)$
$(R_1^*)$ is a regular expression	for $R_1$ a regular expression	$L((R_1^*)) = (L(R_1))^*$
Definition 1.52 page 64		

Assuming $\Sigma$ is the alphabet, we use the following conventions		
$\sum$	regular expression describing language consisting of all strings of length 1 over $\Sigma$	
$*$ then $\circ$ then $\cup$	precedence order, unless parentheses are used to change it	
$R_1R_2$	shorthand for $R_1 \circ R_2$ (concatenation symbol is implicit)	
$R^+$	shorthand for $R^* \circ R$	
$R^k$	shorthand for $R$ concatenated with itself $k$ times	
Pages 63 - 65		

For the following examples assume the alphabet is  $\Sigma_1 = \{0, 1\}$ :

Regular expression, $R$	Language described by the regular expression, $L(R)$
0	{0}
1	{1}
arepsilon	$\{arepsilon\}$
Ø	Ø
$((0 \cup 1) \cup 1)$	
1+	
$\Sigma_1^*1$	
$(\Sigma_1\Sigma_1\Sigma_1\Sigma_1\Sigma_1)^*$	
$1^*\emptyset 0$	
	$\{00, 01, 10, 11\}$
	$\{0^n1 \mid n \text{ is even}\}$