Week1 monday

Alphabet e.g. Σ, Γ	non-empty finite set
Symbol over Σ	element of alphabet Σ
String over Σ	finite list of symbols from Σ
Language over Σ	set of strings over Σ
Empty set $\emptyset$	the empty language
Pages 3, 4, 13, 14	

With $\Sigma_1 = \{0, 1\}$ and $\Sigma_2 = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$ and $\Gamma = \{0, 1, x, y, z\}$

An example of a string of length 3 over Σ_1 is

An example of a string of length 1 over Σ_2 is

The number of distinct strings of length 2 over Γ is

An example of a language over Σ_1 of size 1 is

An example of an infinite language over Σ_1 is

An example of a finite language over Γ is

Empty string ε	the string of length 0
Reverse of a string w, $w^{\mathcal{R}}$	write w in the opposite order, if $w = w_1 \cdots w_n$ then $w^{\mathcal{R}} = w_n \cdots w_1$
Concatenating strings x and y	take $x = x_1 \cdots x_m$, $y = y_1 \cdots y_n$ and form $xy = x_1 \cdots x_m y_1 \cdots y_n$
String z is a substring of string w	there are strings u, v such that $w = uzv$
String x is a prefix of string y	there is a string z such that $y = xz$
String x is a proper prefix of string y	x is a prefix of y and $x \neq y$
Pages 13, 14	

$\varepsilon \in \Sigma_1$	True	False
ε is a string over Σ_1	True	False
ε is a language over Σ_1	True	False
ε is a prefix of some string over Σ_1	True	False
There is a string over Σ_1 that is a proper prefix of ε	True	False

String order over alphabet Σ: Order strings over Σ first by length and then according to the dictionary order, assuming symbols in Σ have an ordering.

The first five strings over Σ_1 in string order, using the ordering $0 < 1$:

The first five strings over Σ_2 in string order, using the usual alphabetical ordering for single letters:

Assuming A and B are languages over alphabet Σ	
The union A and B	$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
The concatenation of A and B	$A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$
The star of A	$A^* = \{x_1 x_2 \cdots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$
Definition 1.23 page 44	

Assuming Σ is the alphabet, recursive definition of regular expressions is		
a is a regular expression	for $a \in \Sigma$	$L(a) = \{a\}$
ε is a regular expression		$L(\varepsilon) = \{\varepsilon\}$
$\emptyset$ is a regular expression		$L(\emptyset) = \{\} = \emptyset$
$(R_1 \cup R_2)$ is a regular expression	for R_1, R_2 regular expressions	$L(\ (R_1 \cup R_2)\ ) = L(R_1) \cup L(R_2)$
$(R_1 \circ R_2)$ is a regular expression	for R_1, R_2 regular expressions	$L(\ (R_1 \circ R_2)\ ) = L(R_1) \circ L(R_2)$
(R_1^*) is a regular expression	for R_1 a regular expression	$L(\ (R_1^*)\ ) = (\ L(R_1)\ )^*$
Definition 1.52 page 64		

Assuming Σ is the alphabet, we use the following conventions	
Σ	regular expression describing language consisting of all strings of length 1 over Σ
$*$ then $\circ$ then $\cup$	precedence order, unless parentheses are used to change it
$R_1 R_2$	shorthand for $R_1 \circ R_2$ (concatenation symbol is implicit)
R^+	shorthand for $R^* \circ R$
R^k	shorthand for R concatenated with itself k times
Pages 63 - 65	

For the following examples assume the alphabet is $\Sigma_1 = \{0, 1\}$:

Regular expression, R	Language described by the regular expression, $L(R)$
0	$\{0\}$
1	$\{1\}$
ε	$\{\varepsilon\}$
$\emptyset$	$\emptyset$
$((0 \cup 1) \cup 1)$	
1^+	
$\Sigma_1^* 1$	
$(\Sigma_1 \Sigma_1 \Sigma_1 \Sigma_1 \Sigma_1)^*$	
$1^* \emptyset 0$	
	$\{00, 01, 10, 11\}$
	$\{0^n 1 \mid n \text{ is even}\}$