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STAT 512: Applied Linear Regression Analysis

Section 03: Team 06 Project Final Report

Introduction

Basketball is one of the prominent sports in the world today and some of the world's greatest athletes are basketball players (Akers, Wolff & Buttross, 1992). Since its invention in 1891 by Jim Naismith, basketball has progressed from being an indoor fun game, an official collegiate sport, a national professional sport, and finally an Olympic sport in 1936 (Oliver, 2005). The National Basketball Association (NBA) as the main men's professional basketball league organization in North America ensures that the sport of basketball is competitive and upheld to the highest performance standard while rightfully observing all protocols, ethics and fair dealings (Hofler & Payne, 1997). Consequently, the NBA as a corporation has fostered several systematic tactical and strategic measures to prevent money-losing/devaluing of franchises, low game patronage, television rating decline, and overall encouraged favorable national appeal towards basketball (Csataljay, O'Donoghue, Hughes & Dancs, 2009). These tactical strategies have included quantitative and scientific measures for interpretation of key game outputs such as scorings and free throws earned. These factors are required to compute sound logical and analytical metrices like the player efficiency ratings (PER) to explore gaming opportunities and monitor performance across games and individual players (Sampaio, & Janeira, 2003).

Consequently, exploring efficiency to gaming opportunities and player performance implies that the more teams win, the higher the game event patronage (more fans of teams pay to attend games, buy jerseys, and other paraphernalia) and associated increase in allied financial remunerations. Therefore, it is important to players, coaches, and owners to develop winning strategies.

Teams win by scorning more points than the opposing team. The point system works as follows: three points are awarded for shots made from behind the three-point line, two points are awarded for shots made inside the three-point line, and one point is awarded for free throws. However, basketball is more than just point scoring; coaches need to know what teamwork strategies lead to more points. Fortunately, various metrics regarding team performance are recorded for every NBA game (Mikołajec, Maszczyk & Zając, 2013).

We propose a statistical analysis on nba.games.stats.csv which can be found from https://www.kaggle.com/ionaskel/nba-games-stats-from-2014-to-2018 (Kelepouris, 2018). We would like to find out which team performance metrics are the most important considerations for coaches trying to develop a winning strategy. Due to the substantial amount of entries in the data set, we choose to analyze the data presented for the Indiana Pacers. Though analysis we present a linear model that represents the best combination of metrics to predict points scored.

Methods

Data Description

The aim of this project is to investigate the factors that are significantly vital in impacting

the amount of points obtained by NBA teams. Specifically, we are interested in how the amount

of points scored by an NBA team is influenced by the following seven variables categorized below.

By researching and analyzing these factors, we hope to understand which traits NBA scouts should

be looking for to increase their points scored for the next game season.

The following table provides the description, range and type of variables we used:

Variable	Description	Range	Variable Type
X3PointShots	Percentage of 3-point shots made	(0, 0.75)	Continuous
FieldGoals	Percentage of field goals made	(19, 58)	Continuous
Assists	Number of assists	(6, 47)	Discrete
Rebounds	Number of rebounds	(19, 58)	Discrete
Turnovers	Number of turnovers	(2, 29)	Discrete
TotalFouls	Number of fouls	(7, 42)	Discrete
Steals	Number of steals	(0, 21)	Discrete
TeamPoints / Point differential	Total number of points scored	(64,	Discrete
		149)	

Yellow: Predictor Variable

Green: Response Variable

4

Preliminary Analyses

The first step we took in analyzing our data was to better understand which predictor variables had significant linear impact on our response variable. This could be achieved through data visualization: generating boxplots, scatterplots and histograms for each of our predictors (Appendix A). Our scatter plots indicate a linear relationship between Team Points and variables Field Goals, Assists, and 3-Point Shots. It does not show any signs of multicollinearity as there are no patterns to the intersecting variables. However, it must be addressed that our data has a large sample size (n=328), which could potentially hinder the observation of a linear pattern in our scatterplots.

As for our boxplots, we can see that the distributions of our variables are not heavily skewed to either side, which indicates normality. The mean of the variables is also relatively close to the median, with the presence of only a few outliers. The only exception to this would be the variable FieldGoals as it has a large range, creating several outliers.

Finally, our histograms help visualize the distribution of each variable and we can notice the general pattern of unimodality with a normal-shaped distribution. The one variable that does not quite follow this trend is Fouls, which has a relatively right skewed distribution with an outlier on the far end.

Model Selection

Full model TeamPoints ~ X3PointShots. + FieldGoals. + Assists + TotalRebounds +
Turnovers + TotalFouls + Steals

After conducting a Type I ANOVA and Type II ANOVA test on our data set, we can see from the summary output (Appendix A) that the p-values for all of the predictor variables are significantly lower than 0.05. Thus, we can conclude that all the predictors have a significant linear impact on the response variable (Team Points). Therefore, we do not need to drop any insignificant variable and can keep our full model as it is with all the predictors (X3PointShots., FieldGoals., Assists, TotalRebounds, Turnovers, TotalFouls, Steals). See below for summary of the full model along with an ANOVA table.

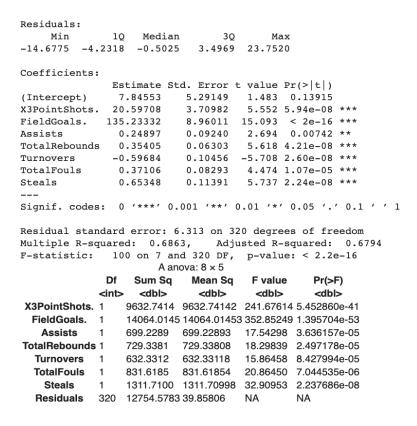


Figure 1 Model Summary and Type I ANOVA Results

```
Anova Table (Type II tests)
Response: TeamPoints
              Sum Sq Df F value
                                     Pr(>F)
X3PointShots.
              1228.6
                       1 30.8251 5.937e-08 ***
FieldGoals.
              9079.4
                       1 227.7931 < 2.2e-16 ***
Assists
               289.4
                           7.2596 0.007424 **
TotalRebounds 1257.8
                          31.5566 4.212e-08 ***
                       1
Turnovers
              1298.8
                       1
                          32.5854 2.603e-08 ***
TotalFouls
              798.0
                       1
                          20.0199 1.067e-05 ***
Steals
              1311.7
                          32.9095 2.238e-08 ***
                       1
Residuals
             12754.6 320
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
```

Figure 2 Full Model Type II ANOVA Results

Continuing with our model selection, we ran a BestSub analysis to identify predictors that could be dropped, while simultaneously supporting our ANOVA results. The criteria for our model selection are listed in the BestSub table output (Appendix A). From the output, we can see that Model 7 (with all parameters) has the best values for every criterion. Therefore, we do not drop any predictors and keep our full model as it is, as supported by the ANOVA output.

Diagnostics

As for the MLM diagnostics of our data, we ran several procedures to see if our model satisfies the assumption of normal error distribution, linearity and constant error variance (Appendix A).

Starting with the linearity assumption, we formulated a Pearson Residual plot, which indicated that our data had a linear pattern for each predictor apart from the variables FieldGoals and Fitted Values which have a slightly upward curving best fit line. Therefore, the Pearson Residual plot confirms that the linearity assumption was met.

To test the normality, we ran a Shapiro-Wilk normality test on our full model and concluded that our data is not normal. Therefore, we did a boxcox transformation and got a new transformed response variable (TeamPointsnew). The next two figures show the results of Type I and Type II ANOVA for the new model.

```
Analysis of Variance Table
Response: TeamPointsnew
                  Sum Sq Mean Sq F value
X3PointShots.
                1 0.92888 0.92888 241.121 < 2.2e-16 ***
                1 1.35050 1.35050 350.567 < 2.2e-16 ***
FieldGoals.
                1 0.06073 0.06073 15.765 8.859e-05 ***
Assists
TotalRebounds
                1 0.06314 0.06314 16.390 6.471e-05 ***
Turnovers
                1 0.06953 0.06953
                                  18.049 2.826e-05 ***
TotalFouls
                                   19.955 1.101e-05 ***
                1 0.07687 0.07687
Steals
                1 0.12954 0.12954
                                   33.627 1.602e-08 ***
Residuals
              320 1.23274 0.00385
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Figure 3 ANOVA Results for Model with Transformed Response Variable

```
Anova Table (Type II tests)
Response: TeamPointsnew
                           F value
               Sum Sq
                       Df
                                       Pr(>F)
X3PointShots. 0.12160
                           31.5650 4.196e-08
                        1
FieldGoals.
              0.87442
                        1 226.9857 < 2.2e-16
Assists
                            6.3191
              0.02434
                        1
                                      0.01244 *
TotalRebounds 0.11301
                           29.3351 1.198e-07 ***
                        1
Turnovers
              0.13720
                           35.6143 6.377e-09
                        1
TotalFouls
              0.07366
                        1
                           19.1207 1.662e-05
Steals
              0.12954
                           33.6267 1.602e-08 ***
                        1
Residuals
              1.23274 320
                0 '***, 0.001 '**, 0.01 '*, 0.05 '., 0.1 ', 1
Signif. codes:
```

Figure 4 Type II ANOVA Results for Model with Transformed Response Variable

After conducting a Type I ANOVA and Type II ANOVA test on our new model, we can see from the summary output (Appendix A) that the p-values for all of the predictor variables are significantly lower than 0.05. Thus, we can conclude that all the predictors have a significant linear impact on the response variable (ln(Team Points)). Therefore, we do not need to drop any insignificant variable and can keep our full model as it is with all the predictors (X3PointShots., FieldGoals., Assists, TotalRebounds, Turnovers, TotalFouls, Steals).

Lastly, we conducted several Brown-Forsythe Tests on each of our variables against our new response variable. From the output (Appendix A), we can see that the p-values for all the outputs are significantly higher than the alpha, indicating the constant variance assumption was met. This can also be supported by our observation of the normality QQ-plot where the fitted values do not increase in variance from left to right.

Outliers/Cook's Distance

To test for any extreme outliers that could be detrimental to our regression model, we ran an Influence plot highlighting studentized residuals vs. Hat-values. From our graph, we can see that there were 2 data values that were X outliers (Cases 8323 & 3365). There were no Y outliers present. Although these 2 cases were outliers, our Cook's Distance (Appendix A) values showed that they were not high enough to warrant them being dropped from our data set. Therefore, there were no cases dropped from our data set due to outliers.

Variance Inflation Factor (VIF) on Final Model

Once our final model met the necessary underlying assumptions, we ran VIFs to evaluate whether our predictors had any multicollinearity issues. Looking at our output (Appendix A), the maximum VIFs did not exceed 3, except for just one. Our results showed that there was not much of a collinearity issue between our predictors.

Ridge Regression

Although our VIF values were relatively small, we ran Ridge Regression tests to further reduce the multicollinearity of our model. From our plots and output, the best value of K to use is 0.04. At this value, we can see that the VIF values for every predictor is approximately close to 1.

Cross Validation

Since we do not need to drop any predictors from our model, we ran a Cross-Validation test on the final model. We obtain a rather small standard deviation of Rsquared and RMSE. We can also see from the output that our Rsquared adjusted is around 0.68, the same as our summary results, which further prove that we do not need to drop any predictors.

Results

The final model is shown below:

TeamPointsnew

= 3.707 + 0.204X3PointShots + 1.237FieldGoals + 0.002Assists

+ 0. 003TotalRebounds - 0. 006Turnovers + 0. 003TotalFouls + 0. 006Steals

Where TeamPointsnew = ln(Teampoints)

To make a prediction for team points based on this model, we find the confidence interval for the prediction of the TeamPointsnew variable and complete a back-transformation shown below.

CI for
$$Y' = (\widehat{Y'}_{lower}, \widehat{Y'}_{upper})$$

CI for
$$Y = (e^{\widehat{Y}_{lower}}, e^{\widehat{Y}_{upper}})$$

The final model shows that FieldGoals has the most influence on the number of points earned by a team. The second most influential variable is X3PointShots. Thus, teams with a high number of Field goals and three-point shots will score the most points. In addition, teams should avoid turnovers whenever possible, as the model shows that turnovers lead to a slight decrease in the number of points scored by a team. In general, basketball teams should employ strategies that maximize the amount of shots taken and minimize their rate of turnovers to the opposing team. The model makes intuitive sense; more shots leads to higher points while surrendering control of the ball to the opposing team leads to lower points. In addition, analysis of the final model shows promising results. Table 1 states that the final model has $R_{\rm adj}^2 = 0.678$ and the highest $p_{\rm value} =$

0.012 (for the Assists parameter). For a comprehensive view of the parameter estimates and confidence intervals see Table 2.

Table 1 ANOVA for Final Model

```
Call:
lm(formula = TeamPointsnew ~ X3PointShots. + FieldGoals. + Assists +
   TotalRebounds + Turnovers + TotalFouls + Steals, data = bball)
Coefficients:
  (Intercept) X3PointShots.
                               FieldGoals.
                                                  Assists TotalRebounds
                               1.327138
    3.707243
                 0.204908
                                                  0.002284
                                                                 0.003356
                 TotalFouls
                                    Steals
    Turnovers
    -0.006134
                   0.003565
                                  0.006494
                 A anova: 8 \times 4
             Sum Sq Df F value
                                       Pr(>F)
              <dbl> <dbl> <dbl>
                                       <dbl>
X3PointShots. 0.12159855 1 31.564981 4.195510e-08
 FieldGoals. 0.87442250 1 226.985670 3.854588e-39
   Assists
            0.02434320 1 6.319094 1.243506e-02
TotalRebounds 0.11300833 1 29.335100 1.198261e-07
  Turnovers 0.13719797 1 35.614333 6.377391e-09
  TotalFouls 0.07365933 1 19.120749 1.661645e-05
          0.12954106 1 33.626725 1.602261e-08
   Steals
  Residuals 1.23274390 320 NA
                                    NA
```

Table 2 Summary of Final Model

Analysis of Variance Table

```
Response: TeamPointsnew
              Df Sum Sq Mean Sq F value
                                           Pr(>F)
X3PointShots.
               1 0.92888 0.92888 241.121 < 2.2e-16 ***
FieldGoals.
              1 1.35050 1.35050 350.567 < 2.2e-16 ***
Assists
              1 0.06073 0.06073 15.765 8.859e-05 ***
TotalRebounds 1 0.06314 0.06314 16.390 6.471e-05 ***
Turnovers
               1 0.06953 0.06953 18.049 2.826e-05 ***
             1 0.07687 0.07687 19.955 1.101e-05 ***
TotalFouls
               1 0.12954 0.12954 33.627 1.602e-08 ***
Steals
Residuals 320 1.23274 0.00385
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
```

Discussion

The goal of this research was to investigate which factors significantly contribute to the amount of points scored by NBA teams. Specifically, the research concentrated on finding the factors that contributed to the amount of points scored by the Indiana Pacers over many seasons. The number of factors considered was 7; Percentage of 3-point shots, percentage of field goals made, number of assists, number of rebounds, number of turnovers, number of fouls and number of steals. Diagnostics showed that all the variables were linearly related to the response variable (team points), and upon analyzing their individual contribution, it was revealed that all the factors have a significant linear impact on the response. The response was transformed to a logarithmic function using the boxcox transformation method as the untransformed Y did not pass a normality test. Several other diagnostics were also run, and analysis was performed on the model.

The model shows great results. Analysis showed that percentage of field goals has the most influence on the number of points, followed by percentage of 3-point shots. The model therefore gives evidence for the importance of field goals and three-point shots to win games. Furthermore, number of turnovers is the only inversely related variable, which is expected as teams should avoid turnovers to retain control over the ball in a game. This research should be helpful to the NBA scouts looking to scout prospective players for their respective teams – they can look for offense players who score a lot of points and defense players who increase turnovers for the enemy team by stealing the ball and maintaining ball control. However, some limitations of the model exist – the effectiveness of the model is subject to research on a single NBA team, and so more research should be done on all NBA teams to come up with a global statistic.

The conclusions presented by the research can be supported by the prioritization of players like the ones suggested above, who can maintain a defensive zone through a point guard, while simultaneously controlling the flow of the game by giving more opportunities to the top scorers.

References

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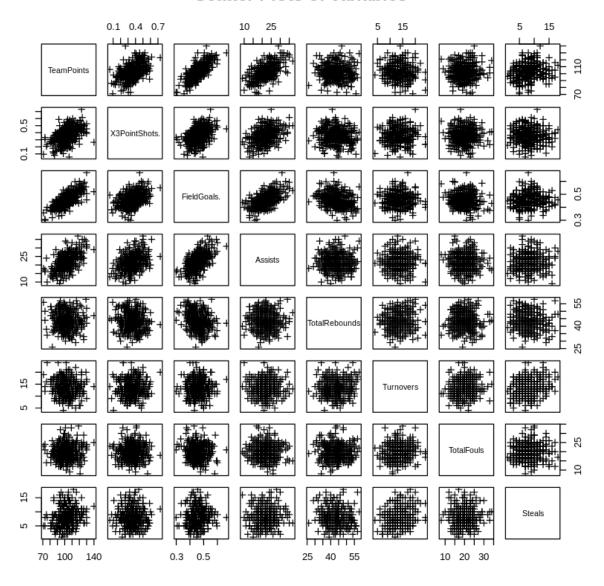
Hofler, R. A., & Payne, J. E. (1997). Measuring efficiency in the national basketball association. *Economics letters*, 55(2), 293-299.

Kelepouris, I. (2018). NBA Team Game Stats from 2014 to 2018. Retrieved from https://www.kaggle.com/ionaskel/nba-games-stats-from-2014-to-2018

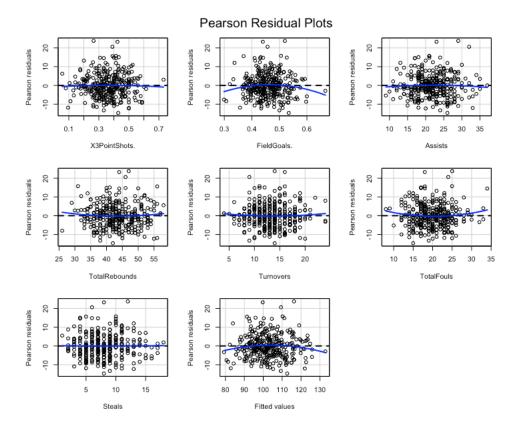
Mikołajec, K., Maszczyk, A., & Zając, T. (2013). Game Indicators Determining Sports Performance in the NBA. Journal of human kinetics, 37, 145–151

Appendix A

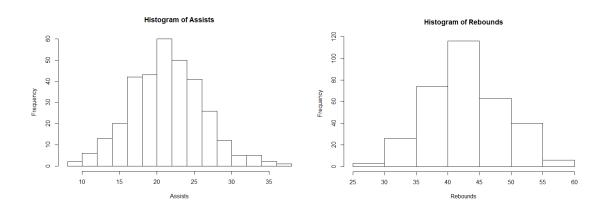
Scatter Plots of variables

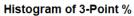


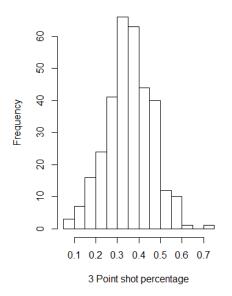
Residual Plots

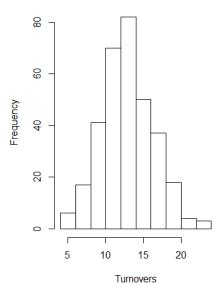


Histograms









Histogram of Turnovers

Histogram of Field Goals

120

9

8

90

40

20

0

0.3

0.4

0.5

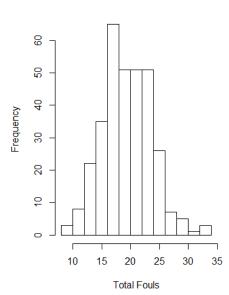
Field Goals

0.6

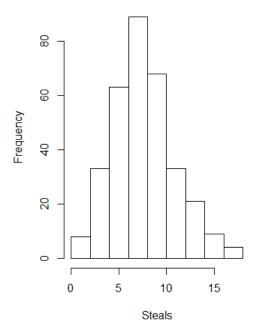
0.7

Frequency

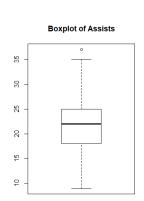
Histogram of Fouls

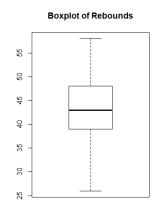


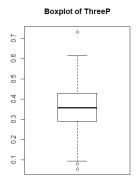
Histogram of Steals

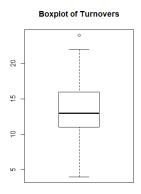


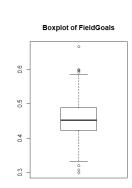
BoxPlots (Pre-Transform)

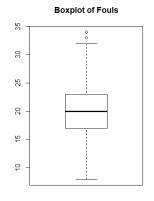




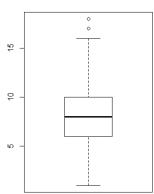




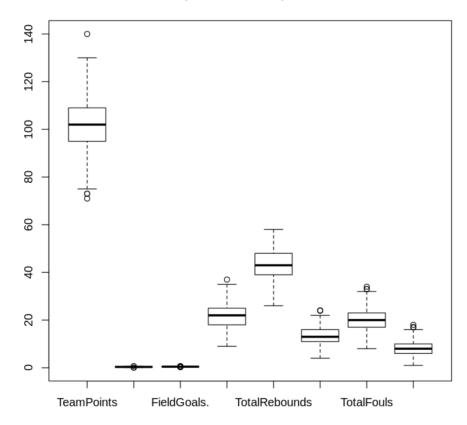




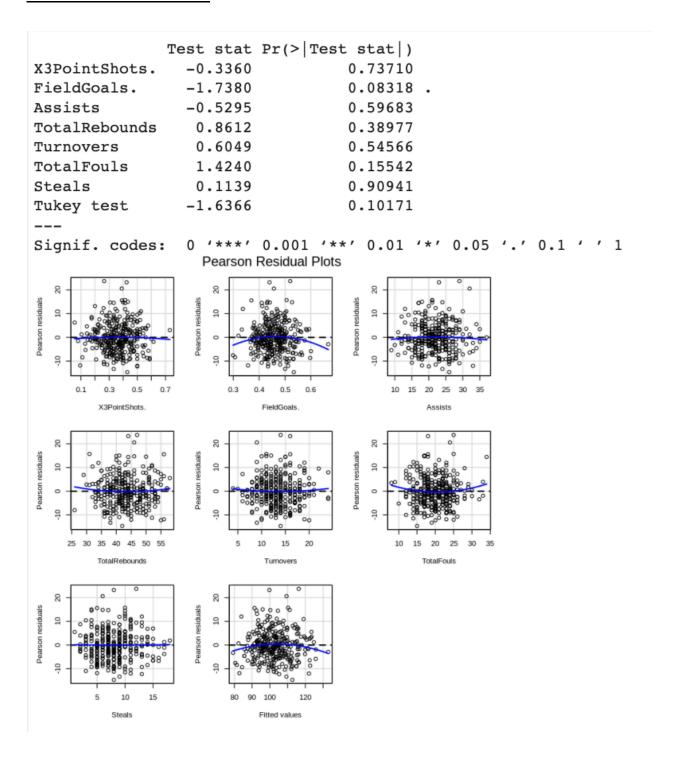
Boxplot of Steals



Box plots for each parameter



Residual Plots Pre-transform



Brown Forsythe Test Results (Pre-Transform)

```
Brown-Forsythe Test (alpha = 0.05)
data : resid and AssistsGroup
statistic : 0.2259788
num df : 4
denom df : 61.05583
p.value : 0.9228547
Result : Difference is not statistically significant.
Brown-Forsythe Test (alpha = 0.05)
data : resid and FieldGoals.Group
statistic : 0.8707002
num df : 4
denom df : 61.81626
p.value : 0.4866983
Result : Difference is not statistically significant.
Brown-Forsythe Test (alpha = 0.05)
data : resid and TotalFoulsGroup
statistic : 1.404006
num df : 4
denom df : 114.3023
p.value : 0.2371068
Result
          : Difference is not statistically significant.
Brown-Forsythe Test (alpha = 0.05)
data : resid and TotalReboundsGroup
statistic : 1.221902
num df : 4
denom df : 62.26163
p.value : 0.3106521
Result : Difference is not statistically significant.
Brown-Forsythe Test (alpha = 0.05)
data : resid and TurnoversGroup
 statistic : 0.9993378
num df : 4
denom df : 98.28106
p.value : 0.4117563
 Result : Difference is not statistically significant.
```

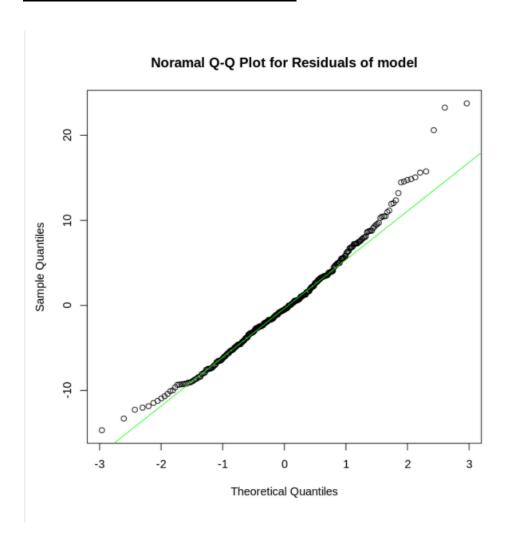
Brown-Forsythe Test (alpha = 0.05)

data : resid and StealsGroup

statistic : 0.9472937 num df : 4 denom df : 157.2211 p.value : 0.4383236

Result : Difference is not statistically significant.

QQNorm Plots for residual (Pre transform)



Shapiro Test Results (Pre-transform)

Shapiro-Wilk normality test

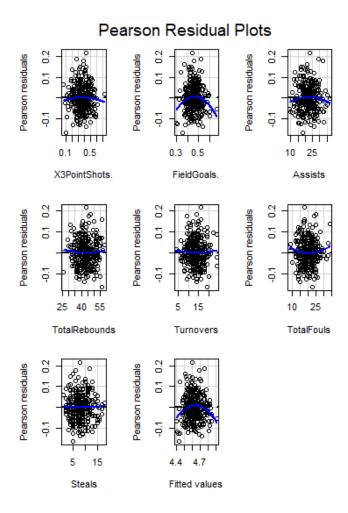
data: bball.fullmod\$residual
W = 0.97969, p-value = 0.0001361

Shapiro Test (Post-transform)

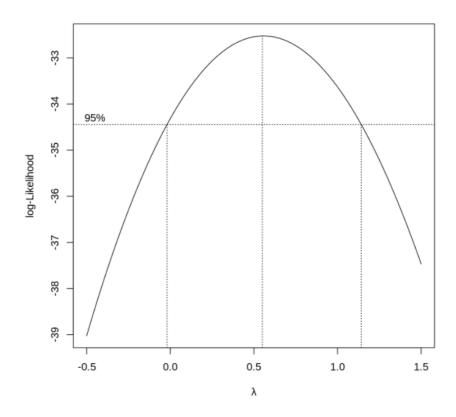
Shapiro-Wilk normality test

data: residuals(bball.newmod)
W = 0.99178, p-value = 0.06593

Person Residua Plots (Post Transform)



Boxcox Transformation



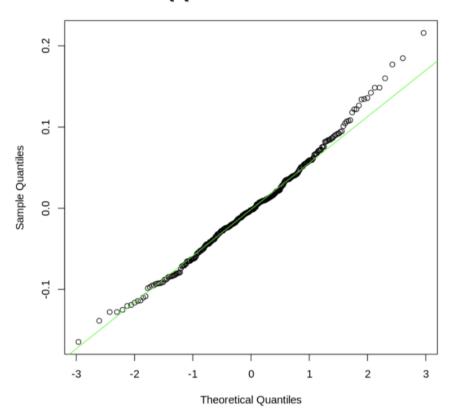
Brown Forsythe Test Results (Post-Transform)

```
Brown-Forsythe Test (alpha = 0.05)
data : residual and AssistsGroup
statistic : 0.4727327
          : 4
num df
denom df : 75.41799
p.value : 0.7555842
Result : Difference is not statistically significant.
Brown-Forsythe Test (alpha = 0.05)
data : residual and X3PointShots.Group
statistic : 0.1685243
num df
          : 4
denom df : 119.2134
p.value : 0.9540368
Result : Difference is not statistically significant.
Brown-Forsythe Test (alpha = 0.05)
data : residual and FieldGoals.Group
statistic : 1.772825
num df : 4
denom df : 41.04364
p.value : 0.1528177
Result : Difference is not statistically significant.
```

```
Brown-Forsythe Test (alpha = 0.05)
______
 data : residual and TotalFoulsGroup
 statistic : 1.544203
 num df : 4
denom df : 126.5893
         : 0.1933904
 p.value
 Result : Difference is not statistically significant.
 Brown-Forsythe Test (alpha = 0.05)
_____
 data : residual and TotalReboundsGroup
 statistic : 1.162524
 num df : 4
denom df : 65.59652
 p.value
        : 0.3354623
 Result : Difference is not statistically significant.
 Brown-Forsythe Test (alpha = 0.05)
_____
 data: residual and TurnoversGroup
 statistic : 0.8779067
 num df : 4
denom df : 72.88746
 p.value : 0.4815052
        : Difference is not statistically significant.
______
 Brown-Forsythe Test (alpha = 0.05)
 data : residual and StealsGroup
 statistic : 1.179605
 num df : 4
denom df : 170.2343
 p.value
         : 0.3215753
        : Difference is not statistically significant.
```

QQNorm Plots for residual (Post transform)

Noramal Q-Q Plot for Residuals of new model

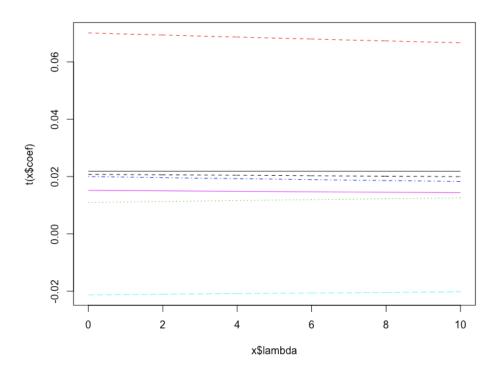


VIF Analysis

```
> VIF(lm(TeamPointsnew~X3PointShots.+FieldGoals.+Assists+TotalRebounds+Turnovers+TotalFouls+Steals, data = bball))
[1] 3.173359
> VIF(lm(TeamPointsnew~X3PointShots., data = bball))
[1] 1.311383
 VIF(lm(TeamPointsnew~FieldGoals., data = bball))
[1] 2.215877
> VIF(lm(TeamPointsnew~Assists, data = bball))
[1] 1.444195
> VIF(lm(TeamPointsnew~TotalRebounds, data = bball))
[1] 1.003771
> VIF(lm(TeamPointsnew~Turnovers, data = bball))
[1] 1.012345
 · VIF(lm(TeamPointsnew~TotalFouls, data = bball))
[1] 1.004073
 VIF(lm(TeamPointsnew~Steals, data = bball))
[1] 1.040086
  .
VIF(lm(TeamPointsnew~FieldGoals.+X3PointShots.+Assists+Turnovers+TotalFouls+TotalRebounds+Steals, data = bball))
[1] 3.173359
> VIF(lm(TeamPointsnew~Assists+FieldGoals.+X3PointShots., data=bball))
[1] 2.488778
> VIF(lm(TeamPointsnew~FieldGoals.+X3PointShots.+Assists, data=bball))
[1] 2.488778
> VIF(lm(TeamPointsnew~X3PointShots.+Assists+FieldGoals., data=bball))
[1] 2.488778
 • VIF(lm(TeamPointsnew~X3PointShots.+FieldGoals.+Assists, data=bball))
[1] 2.488778
```

Ridge Regression (MASS Library)

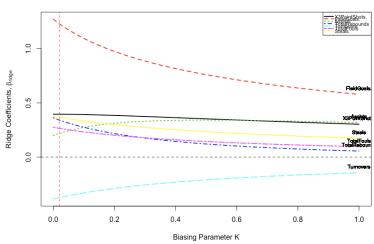
```
> library(MASS)
> bball.ridgelm = lm.ridge(TeamPointsnew~X3PointShots.+FieldGoals.+Assists+To
talRebounds+Turnovers+TotalFouls+Steals, data = bball, lambda=seq(0,10,0.02))
> plot(bball.ridgelm, main="Ridge Trace Plot")
> select(bball.ridgelm)
modified HKB estimator is 2.742669
modified L-W estimator is 2.358101
smallest value of GCV at 3.34
```



Ridge Regression (Imridge Library)

```
> bball.ridaemod2 = lmridae(TeamPointsnew~X3PointShots.+FieldGoals.+Assists+TotalRebounds+Turnovers+TotalFouls+Steal
s, data = bball, K=seq(0,1,0.02))
> plot(bball.ridgemod2)
> vif(bball.ridgemod2)
        k=0.02
k=0.04
                1.14615
                                  .52720 1.36609
                                                                                       0.94384
k=0.06
                1.08536
                                1.40123 1.26647
                                                           0.98633
                                                                        0.94501
                                                                                      0.90671 0.95227
                1.02960
0.97831
                                                                        0.90543
0.86846
k=0.08
                                1.29153 1.17874
                                                            0.93925
                                                                                      0.87184 0.91192
k=0.1
                                1.19537 1.10097
                                                            0.89610
                                                                                       0.83903 0.87424
k=0.12
                0.93100
                                1.11054 1.03163
                                                           0.85640
                                                                        0.83385
                                                                                      0.80810 0.83901
k=0.14
k=0.16
                0.88727
0.84674
                               1.03528 0.96948
0.96817 0.91350
                                                                        0.80140
0.77092
                                                                                      0.77892 0.80599
0.75134 0.77500
                                                            0.81972
                                                           0.78572
k=0.18
                0.80910
                               0.90804 0.86285
                                                           0.75410
                                                                        0.74225
                                                                                       0.72524 0.74587
                0.77407
0.74142
                                                                                      0.70051 0.71844
0.67706 0.69258
                                0.85392 0.81684
                                                            0.72461
k=0.22
                               0.80502 0.77488
                                                           0.69704
                                                                        0.68975
k=0.24
k=0.26
                0.71091
                               0.76065 0.73648
                                                           0.67121
0.64695
                                                                        0.66567
                                                                                      0.65479 0.66817
0.63363 0.64508
                0.68237
                               0.72026 0.70123
                                                                        0.64289
k=0.28
                0.65562
                               0.68337 0.66877
                                                           0.62413
                                                                        0.62132
                                                                                      0.61350 0.62324
k=0.3
k=0.32
                0.63051
0.60690
                               0.64956 0.63879
0.61850 0.61104
                                                           0.60261
0.58229
                                                                        0.60086
0.58145
                                                                                      0.59433 0.60253
0.57606 0.58289
k=0.34
                0.58468
                               0.58988 0.58528
                                                           0.56308
                                                                        0.56299
                                                                                       0.55863 0.56424
                0.56373
0.54395
                               0.56343 0.56131
0.53894 0.53896
                                                           0.54488
0.52763
                                                                        0.54545
0.52874
                                                                                      0.54200 0.54650
0.52611 0.52963
k=0.38
k=0.4
k=0.42
                               0.51620 0.51809
0.49506 0.49854
                                                                        0.51281
0.49763
                0 52526
                                                            0.51124
                                                                                       0.51091 0.51356
                0.50757
                                                            0.49566
                                                                                       0.49638 0.49823
                                                                        0.48313
                                                                                       0.48246 0.48361
k=0.44
                0.49082
                               0.47534 0.48022
                                                           0.48083
k=0.46
k=0.48
                0.47493
0.45985
                               0.45693 0.46300
0.43970 0.44680
                                                           0.46671
0.45323
                                                                        0.46928
0.45603
                                                                                      0.46913 0.46965
0.45635 0.45630
k=0.5
                0.44551
                               0.42354 0.43154
                                                           0.44037
                                                                        0.44336
                                                                                      0.44409 0.44354
                0.43187
0.41889
                               0.40838 0.41713
0.39411 0.40351
                                                                                      0.43232 0.43132
0.42102 0.41962
                                                            0.42807
                                                                        0.43123
                                                                        0.41960
k=0.54
                                                           0.41632
                0.40652
0.39472
                                                                        0.40845
0.39776
                                                                                      0.41017 0.40840
0.39973 0.39765
k=0.56
                               0.38068 0.39063
                                                            0.40506
k=0.58
                               0.36800 0.37842
                                                            0.39428
k=0.6
                0.38345
                               0.35603 0.36683
                                                           0.38395
                                                                        0.38749
                                                                                      0.38968 0.38732
k=0.62
k=0.64
                0.37269
0.36240
                               0.34471 0.35583
0.33399 0.34537
                                                           0.37404
0.36452
                                                                        0.37762
0.36814
                                                                                      0.38002 0.37740
0.37071 0.36788
k=0.66
                0.35255
                               0.32382 0.33541
                                                           0.35538
                                                                        0.35902
                                                                                      0.36174 0.35871
k=0.68
                0.34312
                               0.31416 0.32592
                                                            0.34659
                                                                        0.35024
                                                                                       0.35310 0.34990
                0.33409
                                                           0.33814
                                                                        0.34179
                                                                                      0.34477 0.34141
k=0.7
                               0.30499 0.31687
                                                                                      0.33673 0.33324
0.32897 0.32537
0.32147 0.31778
k=0.72
                0 32542
                               0.29626 0.30823
0.28795 0.29997
                                                            0 33000
                                                                        0.33365
k=0.74
                0.31711
                                                            0.32217
                                                                        0.32580
                               0.28002 0.29207
k=0.76
                0.30913
                                                           0.31462
                                                                        0.31823
k=0.78
k=0.8
                0.30146
0.29409
                               0.27246 0.28451
                                                           0.30734
                                                                        0.31093
0.30389
                                                                                      0.31423 0.31046
0.30724 0.30339
                               0.26524 0.27727
k=0.82
                0.28700
                               0.25833 0.27033
                                                           0.29355
                                                                        0.29708
                                                                                       0.30048 0.29657
                               0.25173 0.26368
0.24540 0.25728
                                                                        0.29051
0.28416
                0.28018
                                                            0.28701
                                                                                       0.29394 0.28998
k=0.86
                0.27361
                                                           0.28070
                                                                                       0.28761 0.28362
k=0.88
                0.26728
                               0.23934 0.25114
                                                           0.27459
                                                                        0.27801
                                                                                       0.28149 0.27746
                0.26118
                                                            0.26869
k=0.92
                0.25529
                               0.22794 0.23956
                                                           0.26298
                                                                        0.26632
                                                                                      0.26981 0.26576
k=0.94
k=0.96
                               0.22258 0.23409
0.21743 0.22883
                                                           0.25746
0.25212
                                                                        0.26076
0.25537
                0.24961
                                                                                      0.26425 0.26018
                0.24413
                                                                                       0.25885 0.25479
k=0.98
                0.23884
                               0.21248 0.22375
                                                           0.24694
                                                                        0.25015
                                                                                      0.25363 0.24956
                0.23372
                               0.20771 0.21886
                                                           0.24193
                                                                        0.24509
                                                                                      0.24855 0.24450
```

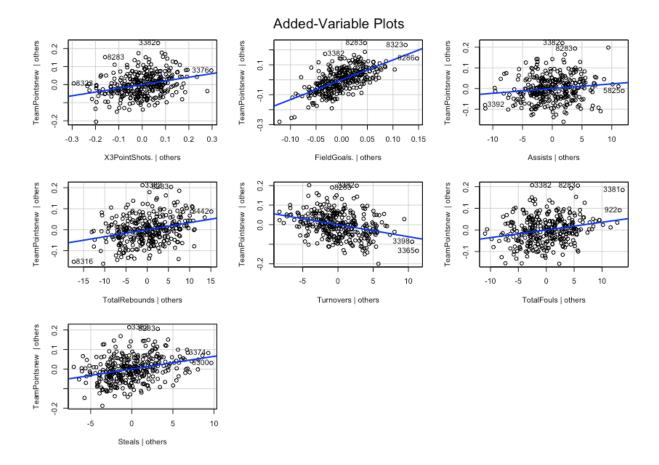




Best Subset Results

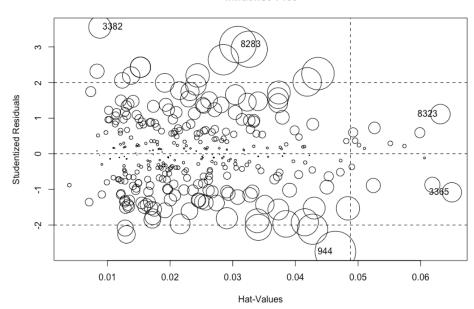
Cross Validation

AV-Plots



Studentized Deleted Residuals





- > # Hati
 > hati = lm.influence(bball.newmod)\$hat
 > bound = 2*mean(hati)
 > bound
 [1] 0.04878049
- > #Cook threshold
 > qf(0.5,8,320) #major
 [1] 0.9199458