IE5600 Project 2:
Time-Dependent Vehicle Routing Problem with Private Fleet and Common Carriers

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1. Problem description

The TDVRPPC problem is defined over a directed graph G=(V,A) with node set V and arc set A. Set $V = \{0, 1, 2, ..., n\}$, where 0 represents the depot, and $V_c = \{1, 2, ..., n\}$ is the customer set. The arc set A is defined as $\{(i, j) : i, j \in V, i \neq j\}$, where each arc (i, j) has an Euclidean distance $d_{i,j}$.

Each customer has demand q_i and must be served by exactly one vehicle from the private fleet or by the common carrier. The homogeneous private fleet is denoted by K, each of which has a maximum capacity Q. The working hour is from e_0 to l_0 and a private vehicle may be activated to start from the depot, serve some customers and return to the depot within the working hour. The route's duration, defined as the arrival time at the returning depot minus the departure time at the starting depot, should not exceed the driver's maximum working hour (T_{max}) as regulated by the government and the company's policy. This is also referred to as the maximum route duration constraint.

Each private vehicle has a fixed cost f if it is activated. A distance-based travel cost $c_{i,j} = \alpha d_{ij}$ will be incurred when a vehicle travels from node i to node j, where α is the rate of variable cost for the vehicle, while a vehicle from the common carrier incurs a charge of e_i if it serves node i. The problem then aims to minimize the sum of both the fixed and variable costs incurred from the private fleet as well as outsourced costs charged by the common carriers.

2. Time dependent travel time function

Pan et al. (2019) characterized the piecewise linear travel time function proposed in (Ichoua et al., 2003), and its main idea is briefly presented below. A typical workday ($[e_0, l_0]$) is divided

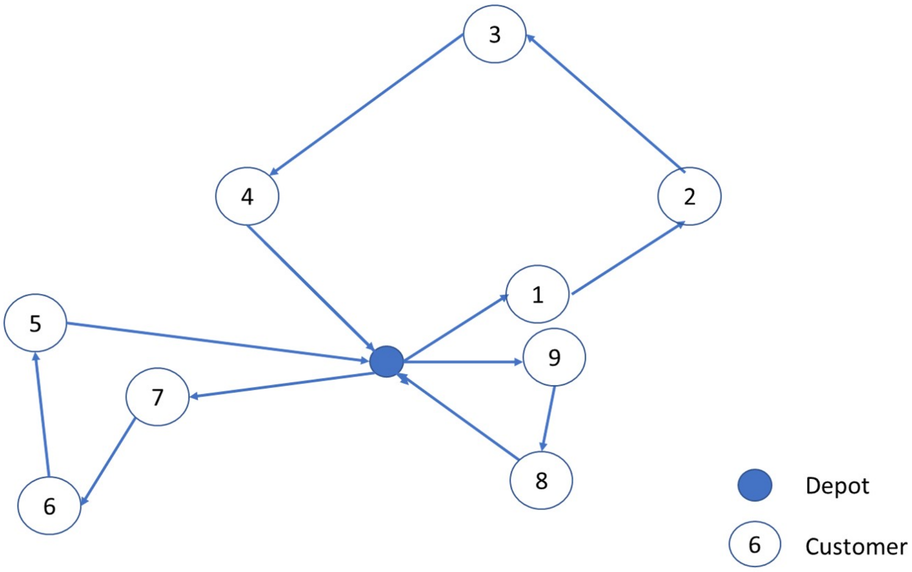

Figure 1: Example of a VRP problem

into non-overlapping time zones T, where the m-th time zone is $[w^{m-1}, w^{m}]$. It is assumed that the travel speed is constant within a time zone and will change only at the end of each time zone.Let $v^{m}_{i,j}$ be the travel speed along (i, j) during the m-th time zone. The actual time-dependent travel time along (i, j) with departure time t_0 (denoted as $\bar{\tau}_{i,j}(t_0)$) depends on the speed profile of (i, j) and d_{ij}, which can be calculated using Algorithm 1. Let the time-dependent travel time along (i, j) with departure time t be denoted as function $\tau_{i,j}(t)$. As an example in Figure 2, this function can be fully determined using the break points ($w^{0}_{i,j}$ to $w^{9}_{i,j}$) of the unique arc time zones $T_{i,j}$ and their actual travel times (e.g, $\bar{\tau}_{i,j}(w^{1}_{i,j})$, $\bar{\tau}_{i,j}(w^{2}_{i,j})$, and etc). Again the arc time zones $T_{i,j}$ is unique to the arc and we denote the m-th arc time zone for (i, j) as $T^{m}_{i,j}$. For ease of calculation, the slope ($\theta^{m}_{i,j}$) and the intersection with y-axis ($\eta^{m}_{i,j}$) of the m-th line segment (representing the m-th arc time zone) can be pre-calculated and used to fully represent the piecewise linear travel time function $\tau_{i,j}(t)$ as below:

$$\tau_{i,j}(t) = \sum_{m \in T_{i,j}} \left(\theta^{m}_{i,j} t + \eta^{m}_{i,j}\right) \mathbf{1}_{T^{m}_{i,j}}(t), \ \forall t \in [e_0, l_0], \tag{1}$$

where the indicator function $\mathbf{1}_{T^{m}_{i,j}}(t)$ indicates whether t belongs to $T^{m}_{i,j}$.

Similarly, the backward travel time function (denoted as $\tau^{-1}_{i,j}(t)$), defined as the travel time

Algorithm 1 Calculation of actual travel time $\bar{\tau}_{i,j}(t_0)$)

Determine the time zone of t_0 as M

$t \leftarrow t_0$, $d \leftarrow d_{ij}$, $t' \leftarrow t + (d/v_{ij}^M)$,

while $t' \geq w^M$ **do**

  $d \leftarrow d - v_{ij}^M \times (w^M - t)$

  $t \leftarrow w^M$

  $t' \leftarrow t + (d/v_{ij}^{M+1})$

  $M \leftarrow M + 1$

end while

return $t' - t_0$

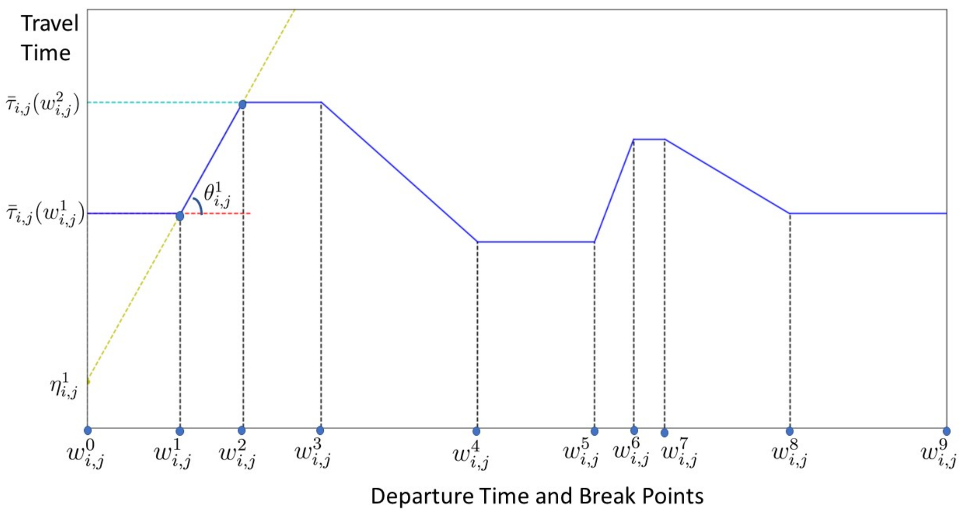

Figure 2: Example of a time-dependent travel time function.

required for the vehicle to definitely arrive at node j at time t along arc (i, j), is also a piecewise linear function and can be determined in a similar way as $\tau_{i,j}(t)$.

3. Test instances

The VRPPC instance in Bolduc et al. (2008) are adopted to derive time-dependent travel time data for a total of 34 instances using the approach in Ichoua et al. (2003). Each arc in the customer network is randomly assigned to one of the three speed profiles, representing roads with slow, medium and fast traffic. The workday is divided into five time zones with two peak hours. The number of customers of the test instances varies from 50 to 483 and up to 31 vehicles can be used.

In order to determine T_{max} for each TDVRPPC instance, we run a preliminary experiment to solve the VRPPC instances. By setting the travel speed as 1, we then set T_{max} to be equal to the average travel time of the routes for the best solution found for the VRPPC.All test instances, detailed routing plans as well as parameter tuning results are available online at https://www.computational-logistics.org/orlib/topic/TDVRPPC/.

References

Bolduc, M.-C., Renaud, J., Boctor, F., & Laporte, G. (2008). A perturbation metaheuristic for the vehicle routing problem with private fleet and common carriers. *Journal of the Operational Research Society*, *59*, 776–787.

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