IE5600 Project 3:
Time-Dependent Pickup and Delivery Problem with Time Windows

1. Problem description

We define an undirected graph $G = (N, A)$, whose node set N consists of pickup nodes $N_P = \{1, 2, \ldots, n\}$, delivery nodes $N_D = \{n + 1, \ldots, 2n\}$, and depots $\{0, 2n + 1\}$. Each request is specified by its pickup node i and delivery node $i + n$. Moreover, all feasible routes need to start from depot 0 and to end at $2n + 1$ (which can be the same node as 0 in reality). For each pickup node $i \in N_P$, a non-negative profit p_i and load q_i is assigned. It must hold that $q_i = q_{n+i}$. Without loss of generality, $q_0 = q_{2n+1} = 0$. A time window$[e_i, l_i]$ is associated with every vertex $i \in N_P \cup N_D$, where e_i and l_i represent the earliest and latest times, respectively, at which service may start at node i. The vehicle waits until time e_i, if arriving at i before e_i; arriving later than l_i is not allowed. The service time of each node $i \in N_P \cup N_D$ is denoted by s_i. The depot nodes also have time windows $[e_0, l_0], [e_{2n+1}, l_{2n+1}]$ representing the earliest and latest times, respectively, at which the vehicle may leave from and return to the depot. Without loss of generality, we assume that $s_0 = s_{2n+1} = 0$. Let K denote the set of vehicles to serve those requests. We assume that vehicles are identical and have capacity Q. Furthermore, a fixed operational cost, Z, is assigned per used vehicle and the cost per unit of route duration is denoted as c_t . The route's duration, defined as the arrival time at the returning depot minus the departure time at the starting depot, should not exceed the driver's maximum working hour (T_{max}) as regulated by the government and the company's policy. This is also referred to as the maximum route duration constraint.

The solution must satisfy the following constraints:

- The route of vehicle k starts from the origin depot and ends at the destination depot, if vehicle k is used.

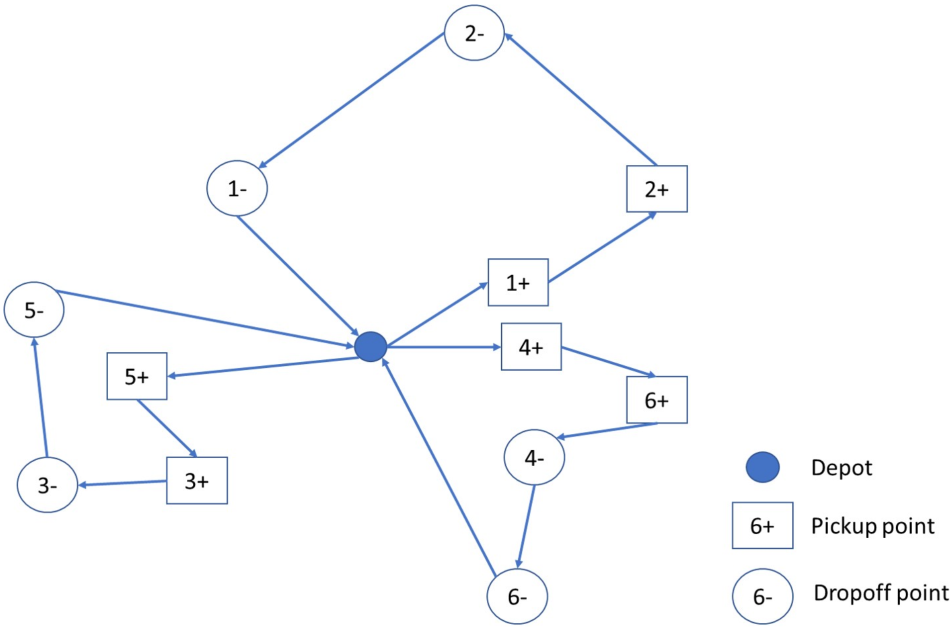

Figure 1: Example of a pickup and delivery problem

- Every request is served at most once and its pickup and delivery nodes are visited by the same vehicle.
- For each request, its pickup node is required to be visited before the delivery node.
- The departure time at each node of the request should be within the given time window (if the request is served).
- Capacity constraints of vehicles.
- The route duration should not exceed the maximum route duration allowed.

The objective function includes three parts: (i) the profit obtained from served requests; (ii) cost related to travel duration; and (iii) total fixed cost for the number of used vehicles.

2. Time dependent travel time function

Pan et al. (2019) characterized the piecewise linear travel time function proposed in (Ichoua et al., 2003), and its main idea is briefly presented below. A typical workday ($[e_0, l_0]$) is divided into non-overlapping time zones T, where the m-th time zone is $[w^{m-1}, w^m]$. It is assumed that the travel speed is constant within a time zone and will change only at the end of each time zone.Let $v^m_{i,j}$ be the travel speed along (i, j) during the m-th time zone. The actual time-dependent travel time along (i, j) with departure time t_0 (denoted as $\bar{\tau}_{i,j}(t_0)$) depends on the speed profile of (i, j) and d_{ij}, which can be calculated using Algorithm 1. Let the time-dependent travel time along (i, j) with departure time t be denoted as function $\tau_{i,j}(t)$. As an example in Figure 2, this function can be fully determined using the break points ($w^0_{i,j}$ to $w^9_{i,j}$) of the unique arc time zones $T_{i,j}$ and their actual travel times (e.g, $\bar{\tau}_{i,j}(w^1_{i,j})$, $\bar{\tau}_{i,j}(w^2_{i,j})$, and etc). Again the arc time zones $T_{i,j}$ is unique to the arc and we denote the m-th arc time zone for (i, j) as $T^m_{i,j}$. For ease of calculation, the slope ($\theta^m_{i,j}$) and the intersection with y-axis ($\eta^m_{i,j}$) of the m-th line segment (representing the m-th arc time zone) can be pre-calculated and used to fully represent the piecewise linear travel time function $\tau_{i,j}(t)$ as below:

$$\tau_{i,j}(t) = \sum_{m \in T_{i,j}} \left(\theta^m_{i,j} t + \eta^m_{i,j} \right) \mathbf{1}_{T^m_{i,j}}(t), \ \forall t \in [e_0, l_0], \tag{1}$$

where the indicator function $\mathbf{1}_{T^m_{i,j}}(t)$ indicates whether t belongs to $T^m_{i,j}$.

Algorithm 1 Calculation of actual travel time $\bar{\tau}_{i,j}(t_0)$)

Determine the time zone of t_0 as M
$t \leftarrow t_0$, $d \leftarrow d_{ij}$, $t' \leftarrow t + (d/v^M_{ij})$,
while $t' \geq w^M$ **do**
    $d \leftarrow d - v^M_{ij} \times (w^M - t)$
    $t \leftarrow w^M$
    $t' \leftarrow t + (d/v^{M+1}_{ij})$
    $M \leftarrow M + 1$
end while
return $t' - t_0$

Similarly, the backward travel time function (denoted as $\tau^{-1}_{i,j}(t)$), defined as the travel time required for the vehicle to definitely arrive at node j at time t along arc (i, j), is also a piecewise linear function and can be determined in a similar way as $\tau_{i,j}(t)$.

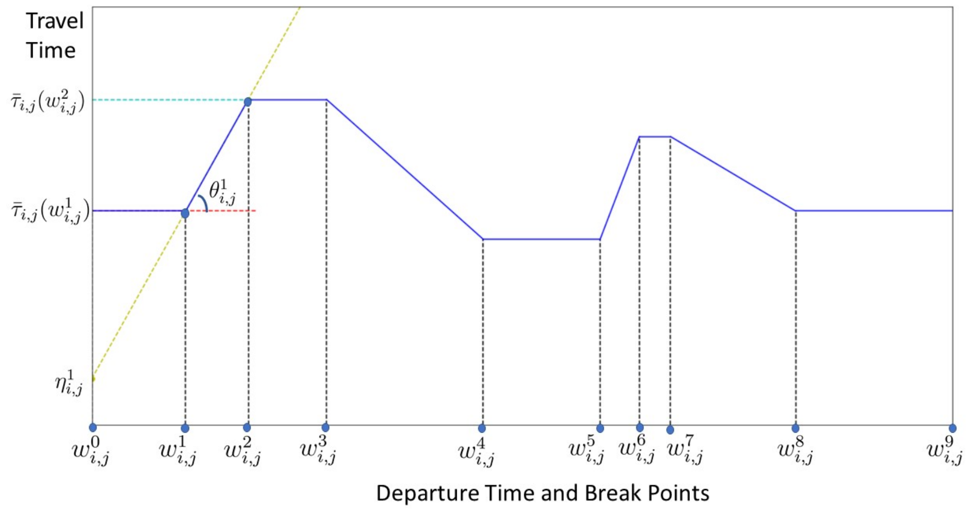

Figure 2: Example of a time-dependent travel time function.

3. Test instances

The data can be retrieved from Sun et al. (2018). Note that no T_{max} is set for the test instance and you need to determine the values yourself for this project.

References

Ichoua, S., Gendreau, M., & Potvin, J.-Y. (2003). Vehicle dispatching with time-dependent travel times. *European Journal of Operational Research*, *144*, 379–396.

Pan, B., Zhang, Z., & Lim, A. (2019). Multi-trip time-dependent vehicle routing problem with time windows. *Submitted*, .

Sun, P., Veelenturf, L. P., Hewitt, M., & Van Woensel, T. (2018). The time-dependent pickup and delivery problem with time windows. *Transportation Research Part B: Methodological*, *116*, 1–24.