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Part - 1

- Compute the gradient vector for a plane in 3D space

$z = f(x,y) = ax + by + c$

$$\nabla f(x,y) = \begin{bmatrix} \frac{\partial f}{x} \\ \frac{\partial f}{y} \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

- Compute the gradient vector for a hyperplane

$$\nabla f(x_1, x_2, \dots x_N) = \begin{bmatrix} \frac{\partial f}{x_1} \\ \vdots \\ \frac{\partial f}{x_N} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{N-1} \\ a_N \end{bmatrix}$$

- Compute the partial derivative of the paraboloid function

$$f_x(x,y) = \left(\frac{\partial f(x,y)}{\partial x}\right)_y = \left(\frac{A(x^2 - 2xx_0 + x_0^2) + B(y-y_0)^2 + C}{\partial x}\right)_y$$

$$= 2Ax - 2Ax_0$$

$$f_y(x,y) = \left(\frac{\partial f(x,y)}{\partial y}\right)_x = \left(\frac{A(x-x_0^2) + B(y-y_0^2) + C}{\partial y}\right)_x$$

$$= 2By - 2By_0$$

- Given the following matrices and vectors compute the following quantities:

$x^T = (3 \quad 1 \quad 4) \quad 1 \times 3$

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$y^T = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$ [3 x 1]

$B^T = \begin{pmatrix} 3 & 5 & 1 \\ 5 & 2 & 4 \end{pmatrix}$ [2 x 3]

$x \cdot x = 3\times3 + 1\times1 + 4\times4 = 26$ [1 x 1]

$x \cdot y^T = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = 3\times2 + 1\times5 + 4\times1 = 15$ [1 x 1]

$x \times y = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \times (2 \quad 5 \quad 1) = \begin{pmatrix} 6 & 15 & 3 \\ 2 & 5 & 1 \\ 8 & 20 & 4 \end{pmatrix}$ [3 x 3]

$y \times X = (2 \quad 5 \quad 1) \times \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = 2\times3 + 5\times1 + 1\times4 = 15$ [1 x 1]

$A \times X = \begin{pmatrix} 4 & 5 & 2 \\ 3 & 1 & 5 \\ 6 & 4 & 3 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4\times3 + 1\times5 + 2\times4 \\ 3\times3 + 1\times1 + 1\times5 \\ 6\times4 + 4\times4 + 3\times4 \end{pmatrix} = \begin{pmatrix} 25 \\ 15 \\ 52 \end{pmatrix}$ [3 x 1]

$A \times B = \begin{pmatrix} 4 & 5 & 2 \\ 3 & 1 & 5 \\ 6 & 4 & 3 \end{pmatrix} \times \begin{pmatrix} 3 & 5 \\ 5 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 4\times3+5\times5+2\times1 & 4\times5+5\times2+2\times4 \\ 3\times3+1\times1+5\times1 & 3\times5+1\times2+5\times4 \\ 6\times3+4\times5+3\times1 & 6\times5+2\times4+3\times4 \end{pmatrix}$

$= \begin{pmatrix} 39 & 38 \\ 19 & 37 \\ 41 & 50 \end{pmatrix}$

B. reshape(1, 6) = (3. 5. 5. 2. 1. 4.)

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Linear least squares (LLS): Single-variable

$$L(p) = \sum_{i=1}^{N} (\hat{y}_i - M(\hat{x}_i, m, b))^2$$

$$= \sum_{i=1}^{N} (\hat{y}_i - m\hat{x}_i - b)^2$$

$$\frac{\partial L(P)}{\partial m} = -2\sum_{i=1}^{N} (\hat{y}_i - m\hat{x}_i - b) \cdot x_i = -2\left(\sum_{i=1}^{N} \hat{x}_i\hat{y}_i + m\sum_{i=1}^{N} \hat{x}_i^2 - b\sum_{i=1}^{N} \hat{x}_i\right)$$

$$\frac{\partial L(P)}{\partial b} = -2\sum_{i=1}^{N} (\hat{y}_i - m\hat{x}_i - b)$$

When fitting parameters, let $\frac{\partial L(P)}{\partial m} = 0$ and $\frac{\partial L(P)}{\partial b} = 0$

then $\sum_{i=1}^{N} \hat{y}_i = m\sum_{i=1}^{N} \hat{x}_i + Nb$

so $b = \frac{\sum_{i=1}^{N} \hat{y}_i}{N} - \frac{m\sum_{i=1}^{N} \hat{x}_i}{N}$

$$= \bar{y} - m\bar{x}$$

take $b = \bar{y} - m\bar{x}$ into $\frac{\partial L(P)}{\partial (m)} = 0$

we can get $\sum_{i=1}^{N} \hat{x}_i\hat{y}_i - m\sum_{i=1}^{N} \hat{x}_i^2 - (\bar{y} - m\bar{x})\sum_{i=1}^{N} \hat{x}_i = 0$

$$\sum_{i=1}^{N} \hat{x}_i\hat{y}_i = m\sum_{i=1}^{N} \hat{x}_i - (\bar{y} - m\bar{x})\sum_{i=1}^{N} \hat{x}_i$$

$$\sum_{i=1}^{N} \hat{x}_i\hat{y}_i = m\sum_{i=1}^{N} \hat{x}_i - \bar{y}\sum_{i=1}^{N} \hat{x}_i + m\bar{x}\sum_{i=1}^{N} \hat{x}_i$$

$$m = \frac{\sum_{i=1}^{N} (\hat{x}_i\hat{y}_i - \bar{y}\hat{x}_i)}{\sum_{i=1}^{N} (\hat{x}_i^2 - \bar{x}\hat{x}_i)}$$

$$m=\frac{\sum_{i=1}^{N}\hat{x}_i\hat{y}_i-\frac{1}{n}\left(\sum_{i=1}^{N}\hat{x}_i\right)\cdot\sum_{i=1}^{N}\hat{y}_i}{\sum_{i=1}^{N}\hat{x}_i^2-\frac{1}{n}\left(\sum_{i=1}^{N}\hat{x}_i\right)\cdot\sum_{i=1}^{N}\hat{x}_i}$$

$$=\frac{\sum_{i=1}^{n}(\hat{x}_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(\hat{x}_i-\bar{x})^2}$$

$$=\frac{cov(x,y)}{var(x,y)}$$

take $m=\frac{cov(x,y)}{var(x,y)}$

we can get $b=\bar{y}-\frac{cov(x,y)}{var(x,y)}\bar{x}$