# Forecasting Inflation in Brazil with High-Dimensional TVP models

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**Abstract**

Real time inflation forecasting is investigated within a Bayesian framework, with the aim to automatically reduce a high dimensional space with time-varying parameters. This is achieved using shirnkage priors proposed by Bitto and Frühwirth-Schnatter (2019). For the proposed forecast exercise, the shrinkage prior that delivers the best forecast is the normal-gamma with Stochastic Volatility (NG-SV) error specification. Although the NG-SV performs well according to the measure proposed by does not surpass the TOP5 expert forecast, for this time horizon.

*Keywords*: Bayesian inference; Inflation; Real-time Forecasting ; Kalman filter; log predictive density scores; sparsity; state space model.

## 1 Introduction

Forecasting inflation it's a challenge that academics and practitioners face. This paper applies state-of-the-art Bayesian TVP modeling using shrinkage priors proposed by Bitto and Frühwirth-Schnatter (2019) in a real-time and data-rich environment for

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Brazil. This methodology shows a predictive gain in utilizing a stochastic volatility specification to capture heteroscedasticity in the error variances. As far as I know this is the first paper to use this methodology. However, the experts forecast surveyed by the central bank still provides the best one-step-ahead forecast.

Emerging markets usually display higher volatility and inflation persistence compared to Developed Countries, which tends to shorten the investment horizon for entrepreneurs and international investors. Consequently, forecasting of short-term inflation is more important in this scenario than in advanced economies, which incentives market players to delve resources into this task. This motivates us to construct a model that better supply this needs.

Following this introduction this paper contemplates the Methodology ; The MCMC scheme ; Data and Main Results. A more detailed description of the dataset is included in the Appendix.

# 2 Methodology

## 2.1 Sparse Time-Varying Model

We present the univariate time series model from Bitto and Frühwirth-Schnatter (2019). Let $\pi_t$ be observed inflation for $T$ time points $t = 1, \ldots, T$. In a state space model, the distribution of $\pi_t$ is driven by an unobservable $d$-dimensional state vector $\boldsymbol{\beta}_t$. In the time-varying parameter (TVP) model the state space form can be regarded as a regression model with time-varying regression coefficients $\boldsymbol{\beta}_t$ following a random walk:

$$\begin{aligned} \boldsymbol{\beta}_t &= \boldsymbol{\beta}_{t-1} + \boldsymbol{\omega}_t, \quad \boldsymbol{\omega}_t \sim \mathcal{N}_d\left(\mathbf{0}, \mathbf{Q}\right), \qquad (1) \\ \pi_t &= \mathbf{x}_t \boldsymbol{\beta}_t + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}\left(0, \sigma_t^2\right), \qquad (2) \end{aligned}$$

where $\mathbf{x}_t = (x_{t1}, x_{t2}, \ldots, x_{td})$ is a $d$-dimensional row vector, containing the regressors of the model and $x_{t1}$ . It is assumed that the initial value $\boldsymbol{\beta}_0$ follow a normal distribution i.e $\boldsymbol{\beta}_0 | \boldsymbol{\beta}, \mathbf{Q} \sim \mathcal{N}_d\left(\boldsymbol{\beta}, \mathbf{P}_0 \mathbf{Q}\right)$, with $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_d)'$ being unknown fixed regression coefficients and $\mathbf{P}_0 = \text{Diag}\,(P_{0,11}, \ldots, P_{0,dd})$ being a diagonal matrix. In addition, $\boldsymbol{\beta}_0$ is independent of the innovations $(\varepsilon_t)$ and $(\boldsymbol{\omega}_t)$, which are independent Gaussian white noise processes. For simplicity, Bitto and Frühwirth-Schnatter (2019) assume that $\mathbf{Q} = \text{Diag}\,(\theta_1, \ldots, \theta_d)$ is a diagonal matrix, hence each element $\beta_{jt}$ of $\boldsymbol{\beta}_t = (\beta_{1t}, \ldots, \beta_{dt})'$ follows a random walk for $j = 1, \ldots, d$:

$$\beta_{jt} = \beta_{j,t-1} + \omega_{jt}, \quad \omega_{jt} \sim \mathcal{N}\left(0, \theta_j\right), \tag{3}$$

with initial value $\beta_{j0}|\beta_j, \theta_j, P_{0,jj} \sim \mathcal{N}\left(\beta_j, \theta_j P_{0,jj}\right)$. Thus, $\theta_j$ is the process variance governing the dynamics of the time-varying coefficient $\beta_{jt}$. The error variances in the observation equation (2) it's specified in two diferrent ways. First, it's consider the homocedastic case, i.e $\sigma_t^2 \equiv \sigma^2$ for all $t = 1, \ldots, T$. Second, to capture heteroscedasticity we use a stochastic volatility(SV) specification as in Jacquier et al. (2002) where $\sigma_t^2 = \mathrm{e}^{h_t}$ and the log volatility $h_t$ follows an AR(1) process:

$$h_t|h_{t-1}, \mu, \phi, \sigma_\eta^2 \sim \mathcal{N}\left(\mu + \phi(h_{t-1} - \mu), \sigma_\eta^2\right). \tag{4}$$

The initial state $h_0$ it is assumed to follow a stationary distribution of the autoregressive process defined as $h_0|\mu, \phi, \sigma_\eta^2 \sim \mathcal{N}\left(\mu, \sigma_\eta^2/(1-\phi^2)\right)$.

The model presented in (1) and (2) can be reparameterized in the non-centered form as

$$\begin{aligned} \beta_{jt} &= \beta_j + \sqrt{\theta_j}\tilde{\beta}_{jt}, \qquad t = 0, \ldots, T, & (5)\\ \pi_t &= \mathbf{x}_t\boldsymbol{\beta} + \mathbf{x}_t\text{Diag}(\sqrt{\theta_1}, \ldots, \sqrt{\theta_d})\tilde{\boldsymbol{\beta}}_t + \varepsilon_t. & (6) \end{aligned}$$

where $\tilde{\boldsymbol{\beta}}_t = (\tilde{\beta}_{1t}, \ldots, \tilde{\beta}_{dt})'$ is defined as $\tilde{\beta}_{jt} = \tilde{\beta}_{j,t-1} + \tilde{\omega}_{jt}, \quad \tilde{\omega}_{jt} \sim \mathcal{N}\left(0, 1\right)$ and initial value $\tilde{\beta}_{j0}|P_{0,jj} \sim \mathcal{N}\left(0, P_{0,jj}\right)$.

## 2.2 Prior Specification

This analysis perform Bayesian inference for the TVP models utilizing a new family of shrinkage priors[1] for the unknown model parameters $\boldsymbol{\beta}$ and $\sqrt{\boldsymbol{\theta}}$. We place conditionally independent normal-gamma priors on both unknown parameters. This prior is represented as a conditionally normal distribution, with the component specific variance following a gamma distribution, that is

$$\sqrt{\theta}_j|\xi_j^2 \sim \mathcal{N}\left(0, \xi_j^2\right), \qquad \xi_j^2|a^{\xi}, \kappa^2 \sim \mathcal{G}\left(a^{\xi}, a^{\xi}\kappa^2/2\right). \tag{7}$$

$$\beta_j|\tau_j^2 \sim \mathcal{N}\left(0, \tau_j^2\right), \qquad \tau_j^2|a^{\tau}, \lambda^2 \sim \mathcal{G}\left(a^{\tau}, a^{\tau}\lambda^2/2\right). \tag{8}$$

The strength with which each individual parameter $\sqrt{\theta}_j$ and $\tau_j^2$ is pulled toward zero is determined by the prior variances, which are referred to as local shrinkage parameters. The parameters $\kappa^2$ and $\lambda^2$ are dubbed global shrinkage parameters, as they determine how strongly all parameters are pulled to zero[2]. At last, we refer to $a^{\epsilon}$ and $a^{\tau}$ as shrinkage adaptation parameters. As $a^{\xi}$ and $a^{\tau}$ decrease, more mass is marginally placed around zero and jointly more mass is put on sparse specifications of the model.

To infer appropriate values from the data, the parameters $\kappa^2, \lambda^2, a^{\xi}, a^{\tau}$ have specific priors. As priors for the global shrinkage parameters, it's specified

$$\kappa^2 \sim \mathcal{G}\left(d_1, d_2\right), \qquad \lambda^2 \sim \mathcal{G}\left(e_1, e_2\right).$$

For the shrinkage adaption parameters, the priors specified are

$$\begin{aligned} a^{\xi} &\sim \mathcal{E}(b^{\xi}), \qquad b^{\xi} \geq 1. \\ a^{\tau} &\sim \mathcal{E}(b^{\tau}), \qquad b^{\tau} \geq 1. \end{aligned}$$

[1] Bitto and Frühwirth-Schnatter (2019), Griffin et al. (2010)

[2] $\mathrm{E}(\theta_j|a^{\xi}, \kappa^2) = \frac{2}{\kappa^2}$ and $\mathrm{E}(\beta_j^2|a^{\tau}, \lambda^2) = \frac{2}{\lambda^2}$, the larger $\kappa^2$ and $\lambda^2$, the greatest the effect.

### 2.3 Prior on the volatility parameter

The specification for the error distribution in (2) for the homoscedacity case is a common hierarchical prior,

$$\sigma^2|C_0 \sim \mathcal{G}^{-1}(c_0, C_0), \qquad C_0 \sim \mathcal{G}(g_0, G_0)$$

with hyperparameters $c_0 = 2.5$, $g_0 = 5$ and $G_0 = \frac{g_0}{\mathrm{E}(\sigma^2)(c_0-1)}$, with $\mathrm{E}(\sigma^2)$ beign a prior guess of $\sigma^2$. In the SV scheme 4, the unknown parameters are the following : $\mu, \phi$ and $\sigma_\eta^2$. The priors are choosen as in Kastner and Frühwirth-Schnatter (2014), that is

$$\mu \sim \mathcal{N}(b_\mu, B_\mu), \quad \frac{\phi+1}{2} \sim \mathcal{B}(a_\phi, b_\phi), \quad \sigma_\eta^2 \sim \mathcal{G}(1/2, 1/2B_\sigma)$$

with hyperparameters defined as $b_\mu = 0, B_\mu = 100, a_0 = 20, b_0 = 1.5$, and $B_\sigma = 1$.

# 3 MCMC algorithm

This paper apply the implementation of the package **shrinkTVP** by Bitto-Nemling et al. (2019) to perform the draws from the posterior distribution of the model parameters. In this section is presented a simple version of the algorithm[3].

**Algorithm 1.** *Gibbs Sampling Algorithm*

1. *Sample the latent states* $\tilde{\boldsymbol{\beta}} = (\tilde{\boldsymbol{\beta}}_0, \ldots, \boldsymbol{\beta}_T)$ *in the non-centered parametrization from a mutivarite normal distribution;*
2. *Sample jointly* $\beta_1, \ldots, \beta_d$ *and* $\sqrt{\theta_1}, \ldots, \sqrt{\theta_d}$ *in the non-centered parameterization from a multivariate normal distribution;*
3. *Perform an ancillarity-sufficiency interweaving step and redraw each* $\beta_1, \ldots, \beta_d$ *from a normal distribution and each* $\theta_1, \ldots, \theta_d$ *from a generalized inverse Gaussian distribution*

[3]For further details: Bitto and Frühwirth-Schnatter (2019) and its Appendix.

*using **GIGrvg**, Leydold et al. (2017);*

4. *(a) Sample the variance shrinkage adaption parameter $a^{\xi}$ using a random walk Metropolis-Hastings step;*

*(b) Sample the parameter shrinkage adaption parameter $a^{\tau}$ using a random walk Metropolis-Hastings step;*

5. *(a) Sample the local shrinkage parameters $\xi_j^2$ for $j = 1, \ldots, d$ from conditionally independent generalized inverse Gaussian distributions;*

*(b) Sample the error variance $\sigma^2$ from an inverse gamma distribution in the homoscedastic case or, in the SV case, sample the level $\mu$, the persistence $\phi$, the volatility of the volatility $\sigma_\eta^2$ and the log-volatilities $\boldsymbol{h} = (h_0, \ldots, h_T)$ using **stochvol**, Kastner (2016).*

As detailed described in Bitto and Frühwirth-Schnatter (2019), in step 3 it's used the ancillarity-sufficiency interweaving strategy (ASIS) introduced by Yu and Meng (2011). ASIS is an acknowledged to improve mixing by sampling certain parameters both in the centered and non-centered parameterization.

# 4 Shrinkage priors specifications and Comparison through log predictive density scores

In the present paper we specify two different shrinkage priors on $\beta_j$ and $\sqrt{\theta_j}$ with different volatility specifications. The first shrinkage prior specification is a hierarchical double gamma prior, introduced in 7, with $a^{\tau} \sim \mathcal{E}(10)$ and $a^{\xi} \sim \mathcal{E}(10)$ under hyperparameter setting $d_1 = d_2 = e_1 = e_2 = 0.001$. The second shrinkage prior specification with hierarchical Bayesian Lasso prior, that is $a^{\tau} = a^{\xi} = 1$ applied by Belmonte et al. (2014). For shrinkage prior we specify a Homoscedastic volatility and a Stochastic Volatility. For each prior, MCMC inference is based on Algorithm 1 with $M = 100,000$ draws after a burn-in of the size of $50,000$.

To compare the two different shrinkage priors we estimated the log predictive density scores as proposed in Bitto and Frühwirth-Schnatter (2019). Log predictive density scores (LPDS) are often used scoring rule to compare models in the context of bayesian predictive analysis. As common in this framework, the first $t_0$ time series observations $\mathbf{y}^{\text{tr}} = (y_1, \ldots, y_{t_0})$ are used as a "training sample", while evaluation is performed for the remaining time series observations $y_{t_0+1}, \ldots, y_T$, based on the log predictive density:

$$\text{LPDS} = \log p(y_{t_0+1}, \ldots, y_T | \mathbf{y}^{\text{tr}}) = \sum_{t=t_0+1}^{T} \log p(y_t | \mathbf{y}^{t-1}) = \sum_{t=t_0+1}^{T} \text{LPDS}_t^{\star}. \tag{9}$$

LPDS can be interpreted as a log marginal likelihood based on the training sample prior $p(\boldsymbol{\vartheta} | \mathbf{y}^{\text{tr}})$, since

$$p(y_{t_0+1}, \ldots, y_T | \mathbf{y}^{\text{tr}}) = \int p(y_{t_0+1}, \ldots, y_T | \mathbf{y}^{\text{tr}}, \boldsymbol{\vartheta}) p(\boldsymbol{\vartheta} | \mathbf{y}^{\text{tr}}) d\, \boldsymbol{\vartheta}, \tag{10}$$

where $\boldsymbol{\vartheta}$ summarises the unknown model parameters, e.g. $\boldsymbol{\vartheta} = (\beta_1, \ldots, \beta_d, \sqrt{\theta_1}, \ldots, \sqrt{\theta_d}, \sigma^2)$ for the homoscedastic state space model, whereas for a TVP model with SV errors, $\boldsymbol{\vartheta} = (\tilde{\boldsymbol{\beta}}_0, \ldots, \tilde{\beta}_{t_0+1}, \sqrt{\theta_1}, \ldots, \sqrt{\theta_d}, \beta_1, \ldots, \beta_d, \sigma_1^2, \ldots, \sigma_{t_0+1}^2)$. Given M samples from the posterior distribution of the parameters and latent variables, $p(\boldsymbol{\vartheta} | y_1, \ldots, y_{t_0})$, Monte Carlo integration could be applied immediately to (1). However, Bitto and Frühwirth-Schnatter (2019) propose a more efficient approximation of the predictive density, the so-called conditionally optimal Kalman mixture apporixmation which is obtained analytically integrating out $\tilde{\beta}_{t_0+1}$ from the likelihood at time $t_0 + 1$. In the homoscedastic error case, given M samples from the posterior distribution of the parameters and the latent variables up to $t_0$, Monte Carlo integration of the resulting predictive density yields following mixture approximation,

$$
\begin{aligned}
p(y_0+1|y_1,\dots,y_{t_0},\mathbf{x}_{t_0+1}) &\approx \frac{1}{M}\sum_{m=1}^{M} f_N\left(y_t;\hat{y}^{(m)}_{t_0+1},S^{(m)}_{t_0+1}\right), &(11)\\
\hat{y}^{(m)}_{t_0+1} &= \mathbf{x}_{t_0+1}\boldsymbol{\beta}^{(m)}+\mathbf{F}^{(m)}_{t_0+1}\mathbf{m}^{(m)}_{t_0}, &(12)\\
S^{(m)}_{t_0+1} &= F^{(m)}_{t_0+1}\left(\Sigma^{(m)}_{t_0}+I_d\right)\left(F^{(m)}_{t_0+1}\right)^{\top}+\left(\sigma^2\right)^{(m)} &(13)
\end{aligned}
$$

where the conditional predictive densities are Gaussian and the conditional moments depend on the MCMC draws. The mean $\hat{y}^{(m)}_{t_0+1}$ and the variance $S^{(m)}_{t_0+1}$ are computed for the $m_{t_h}$ MCMC iteration from $F_{t_0+1}=x_{t_0+1}\,\mathrm{Diag}\left(\sqrt{\theta_1},\dots,\sqrt{\theta_d}\right)$ and the mean $m_{t_0}$ and the covariance matrix $\Sigma_t$ and $\mathbf{m}_t$ up to time $t_0$:

$$
\begin{aligned}
\Sigma_1 &= (\Omega_{11})^{-1}, & m_1 &= \Sigma_1 c_1\\
\Sigma_t &= \left(\Omega_{tt}-\Omega^{\top}_{t-1,t}\Sigma_{t-1}\Omega_{t-1,t}\right)^{-1}, & m_t &= \Sigma_t\left(c_t-\Omega^{\top}_{t-1,t}m_{t-1}\right)
\end{aligned}
$$

The quantities $c_t, \Omega_{tt}$ and $\Omega_{t-1,t}$ for $t=1,\dots,t_0$ are given in the Appendix . For the SV case, it is still possible to analytically integrate out $\tilde{\beta}_{t_0+1}$ from the likelihood at time $t_0+1$ conditional on a known value of $\sigma^2_{t_0+1}$, however it is not possible to integrate the likelihood with respect to both latent variables $\tilde{\beta}_{t_0+1}$ and $\sigma^2_{t_0+1}$. Hence, at each MCMC iteration a draw is taken from the predictive distribution of $\sigma^2_{t_0+1}=\exp\left(h_{t_0+1}\right)$, derived from Equation (4), and used to calculate the conditional predictive density of $y_{t_0+1}$. The approximation of the one-step ahead predictive density can then be obtained through the following steps:

1. for each MCMC draw of $\left(\mu,\phi,\sigma^2_\eta\right)^{(m)}$ and $h^{(m)}_{t_0}$, obtain a draw of $\left(\sigma^2_{t_0+1}\right)^{(m)}$
2. calculate the conditionally optimal Kalman mixture approximation as in (11) with following slightly different values $S^{(m)}_{t_0+1}$ :

$$
S^{(m)}_{t_0+1}=F^{(m)}_{t_0+1}\left(\Sigma^{(m)}_{t_0}+I_d\right)\left(F^{(m)}_{t_0+1}\right)^{\top}+\left(\sigma^2_{t_0+1}\right)^{(m)}
$$

where $F_{t_0+1}$ and $\Sigma_{t_0}$ are the same as defined above.

# 5 The data

Inflation data is measured by the Brazilian consumer price index (IPCA), which is the official inflation index in Brazil. One particularity of Brazilian security market is a sizeable number of inflation-linked bonds use the IPCA as their reference. The dataset is taken from Garcia et al. (2017), where the data source is from Bloomberg and the Central Bank of Brazil, and covers the period from January 2003 to December 2015, a total of 156 observations. The dataset contains 59 macroeconomic variables and 34 variables linked to specialist forecasts. Keep in mind that this dataset is using only variables that were available in the period when the forecast was computed. The composition of this variables are divided in Macroeconomic variables and *FOCUS* expectation variables. The FOCUS variables include the median of the h-period-ahead specialist forecasts; the median of the top five experts; mean and standard deviation of the Top5. The macroeconomic variables cover several inflation and industry indexes, unemployment and other variables related to labour, energy consumption, exchange rates, stock markets, government accounts, expenditure and debt, taxes, monetary variables and exchange of goods and services[4].

# 6 Main Results

## 6.1 Forecasting Real-time Brazilian Inflation using Shrinkage Priors

As in Garcia et al. (2017), consider a direct forecasting approach where the inflation $h$ periods ahead, $\pi_{t+h}$, is modeled as a function of a set of predictors measured at time $t$, such as:

$$\pi_{t+h} = \boldsymbol{\beta}'\mathbf{x}_t + u_{t+h} \tag{14}$$

[4]Details in the appendix.

where $\boldsymbol{\beta} \in \mathbb{R}^q$ is a vector of unknown parameters and $\mathbf{x} = (x_{1t}, \ldots, x_{qt})' \in \mathbb{X} \subseteq \mathbb{R}^q$. In the context of real-time forecasts means that $\mathbf{x}_t$ contains only variables that are observer and available to the statistician at time $t$.

In this present paper, all models described in detail in 4 are estimated for h = 1[5]. Recent studies have focused their efforts to implement machine learning algorithms to forecasting macroeconomic variables. Although there is a predictive gain using this methodologies, there is no effort to specify different varying parameter specifications and volatilities. My contribution to the literature introduces TVP models to forecasting real-time inflation in Brazil with different shrinkage priors and volatility specifications. This section discusses the results and compare the forecasting errors of all models and random walk forecasts. All models are estimated in a 12 last-months rolling-window scheme, with the last forecast being for December 2015.

Figure (1) shows the comparison using the log predictive scores (LPDS) to evaluate different shrinkage priors, using the conditionally optimal Kalman mixture approximation derived in (11). Undoubtedly for inflation forecasting in Brazil, the hierarchical double gamma prior with a stochastic volatility is clearly preferable to the other alternatives. Summary statistics of $p(\beta_j|\mathbf{y})$ and $p(\sqrt{\theta_j}|\mathbf{y})$ for the hierarchical double gamma prior with SV in the observation error are given in the Appendix B and a sample of MCMC paths obtained with the interweaving scheme. In the Figure (3), we compare the two best shrinkage priors and the realized inflation. When comparing with the market top 5 the model performs poorly for $t + 1$.

[5] h =1 is five days before the inflation index is published

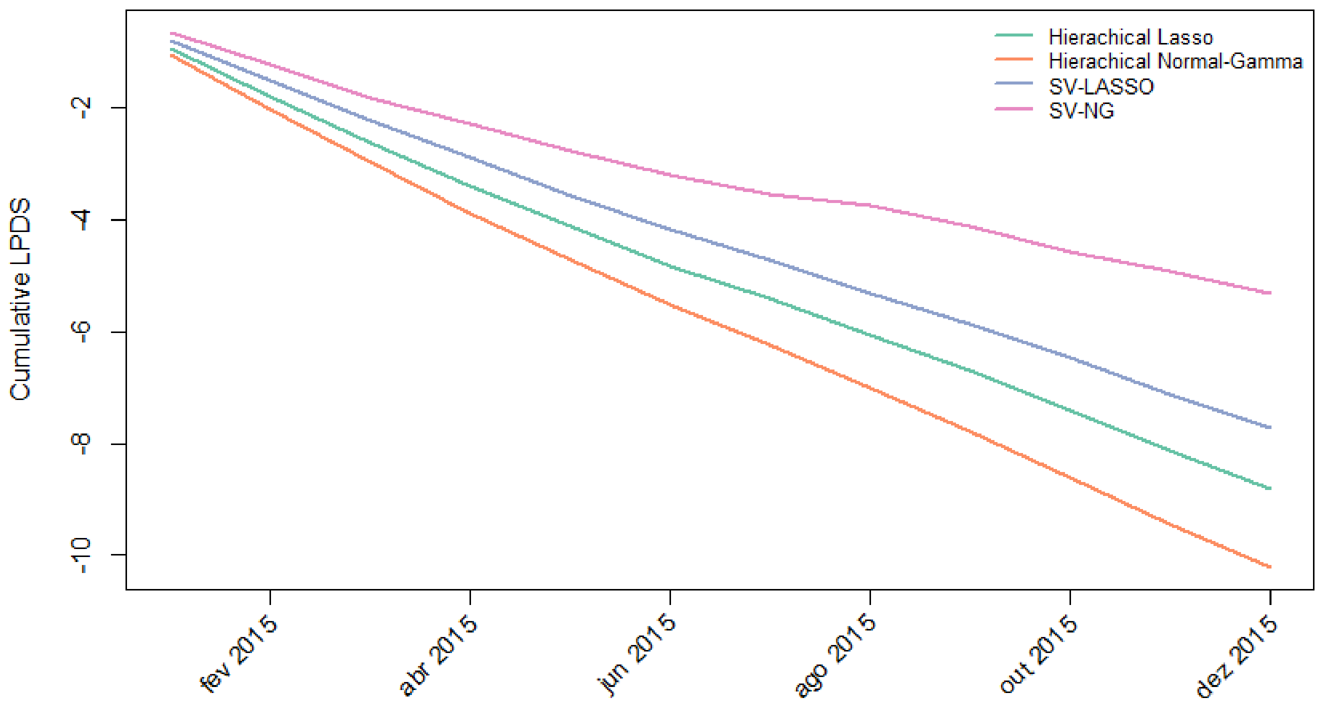


Figure 1: Cumulative log predictive scores for the last 12 time point

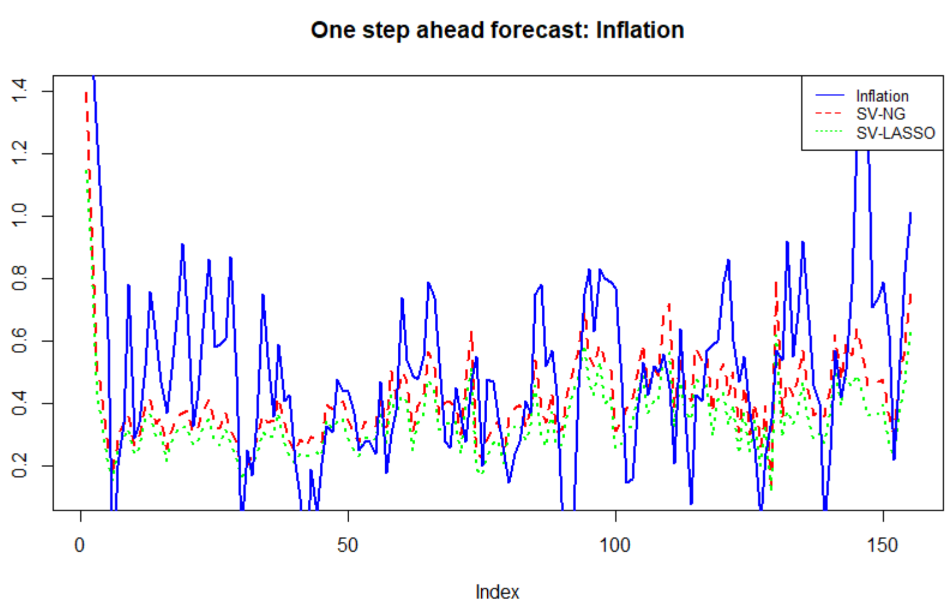


Figure 2

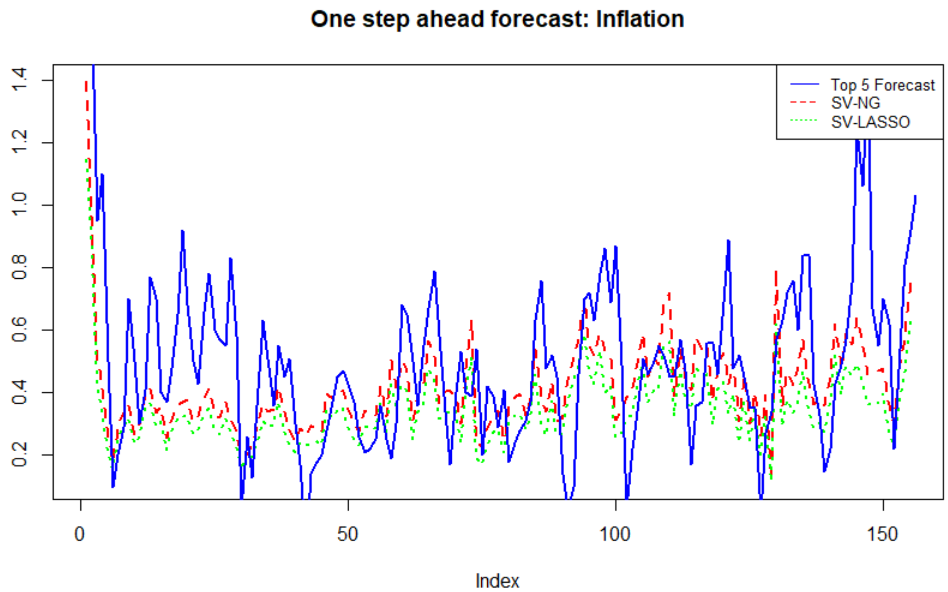


Figure 3

# 7 Conclusion

This paper tested different shrinkage priors and observation errors for forecasting inflation in real-time and using a large number of predictors. The results can be summarised as follows: For $t+1$(five days ahead) forecast the shrinkage prior that delivers the best forecast is the normal-gamma with Stochastic Volatility (NG-SV) error specification. There is evidence of a possible change in the underlying structure of the economy, or the SV is preventing the detection of spurious variations in the TVP coefficients. Although the NG-SV performs well according to the measure proposed by Bitto and Frühwirth-Schnatter (2019), yet does not surpass the TOP5 expert forecast.

# Appendix A Data appendix

Macroeconomic variables.

| | | | |
|---|---|---|---|
| | **Prices and money** | | **Government and international transactions** |
| 1 | Brazil CPI IPCA | 32 | Brazil National Treasury Revenue Total |
| 2 | FGV Brazil General Prices IGP-M | 33 | Brazil Social Contribution over Net Profit Tax Income |
| 3 | FGV Brazil General Prices IGP-DI | 34 | Brazil PIS & PASEP Tax Income |
| 4 | FGV Brazil General Prices IGP-10 | 35 | Brazil Central Government Net Revenue |
| 5 | Brazil CPI IPCA-15 | 36 | Brazil Central Government Revenue from the Central Bank |
| 55 | Brazil Monetary Base | 37 | Brazil Central Government Total Expenditures |
| 56 | Brazil Money Supply M1 Brazil | 38 | Brazil National Treasury Gross Revenue |
| 57 | Brazil Money Supply M2 Brazil | 39 | Brazil Importing Tax Income |
| 58 | Brazil Money Supply M3 Brazil | 40 | BNDES Brazil Income Taxes |
| 59 | Brazil Money Supply M4 Brazil | 41 | Brazil National Treasury Revenue from Industrialized Products |
| | | 42 | Brazil National Treasury Revenues from Other Taxes |
| | **Employment** | 43 | Brazil Central Government Revenue from the Social Security |
| 14 | IBGE Brazil Unemployment Rate | 44 | Brazil National Treasury Revenue from Import Tax |
| 15 | Brazil Unemployment Statistic Male | 45 | Brazil Current Account |
| 16 | Brazil Unemployment Statistic Total | 46 | Brazil Trade Balance FOB |
| 17 | IMF Brazil Unemployment Rate | 47 | Brazil Public Net Fiscal Debt as a percentage of GDP |
| 18 | CNI Brazil Manufacturing Industry Employment | 48 | Brazil Public Net Fiscal Debt |
| 19 | Brazil Industry Working Hours | 49 | Brazilian Federal Government Domestic Debt |
| | | 50 | Brazil Public Net Government & Central Bank Domestic Debt |
| | **Exchange rates and finance** | 51 | Brazilian States Debt Total Consolidated Net Debt |
| 22 | USD-BRL X-RATE | 52 | Brazilian States Debt to Foreigners |
| 23 | USD-BRL X-RATE Tourism | 53 | Brazilian Cities Debt |
| 24 | EUR-BRL X-RATE | 54 | Brazilian Cities Debt to Foreigners |
| 25 | BRAZIL IBOVESPA INDEX | | |
| 26 | Brazil Savings Accounts Deposits | | |
| 27 | Brazil Total Savings Deposits | | |
| 28 | Brazil BNDES Long Term Interest Rate | | |
| 29 | Brazil Selic Target Rate | | |
| 30 | Brazil Cetip DI Interbank Deposits | | |

Focus expectation variables.

| | | | |
|---|---|---|---|
| 60 | $t+1$ median | 77 | Top5 $t+5$ median |
| 61 | $t+2$ median | 78 | Top5 $t+6$ median |
| 62 | $t+3$ median | 79 | Top5 $t+7$ median |
| 63 | $t+4$ median | 80 | Top5 $t+8$ median |
| 64 | $t+5$ median | 81 | Top5 $t+9$ median |
| 65 | $t+6$ median | 82 | Top5 $t+10$ median |
| 66 | $t+7$ median | 83 | Top5 $t+11$ median |
| 67 | $t+8$ median | 84 | Top5 $t+12$ median |
| 68 | $t+9$ median | 85 | Top5 $t+13$ median |
| 69 | $t+10$ median | 86 | $t+1$ median^2 |
| 70 | $t+11$ median | 87 | $t+1$ mean |
| 71 | $t+12$ median | 88 | $t+1$ mean^2 |
| 72 | $t+13$ median | 89 | $t+1$ Std |
| 73 | Top5 $t+1$ median | 90 | $t+12$ median^2 |
| 74 | Top5 $t+2$ median | 91 | $t+2$ mean |
| 75 | Top5 $t+3$ median | 92 | $t+2$ mean^2 |
| 76 | Top5 $t+4$ median | 93 | $t+2$ Std |

# Appendix B    Summary Statistics

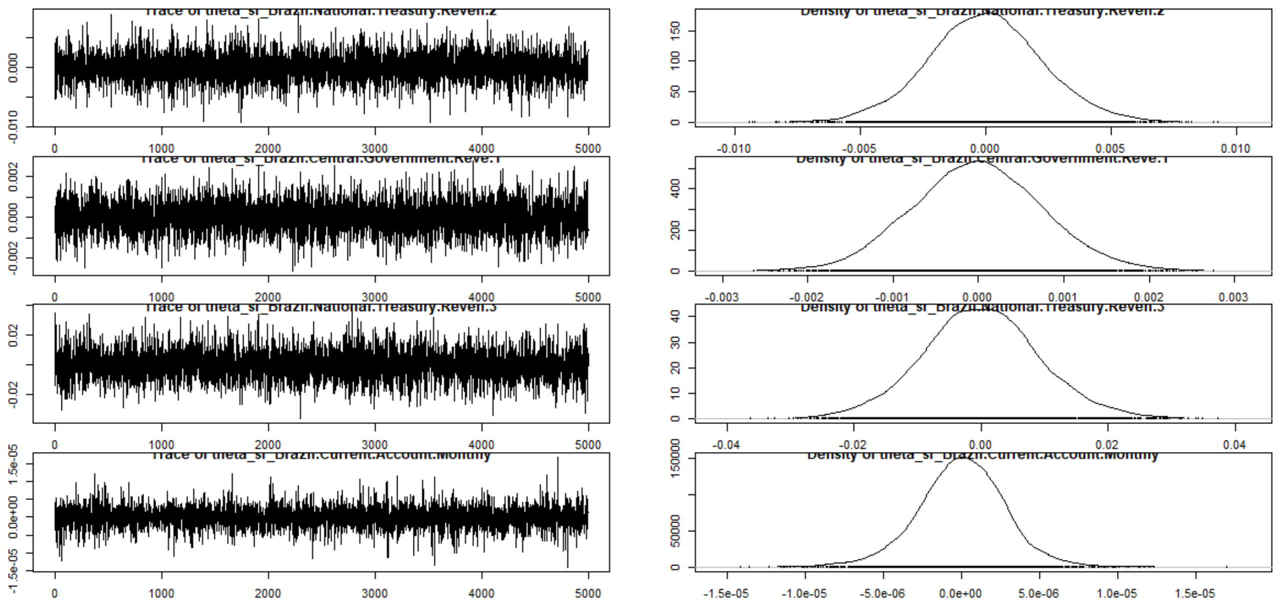


Figure 4: Sample of the MCMC

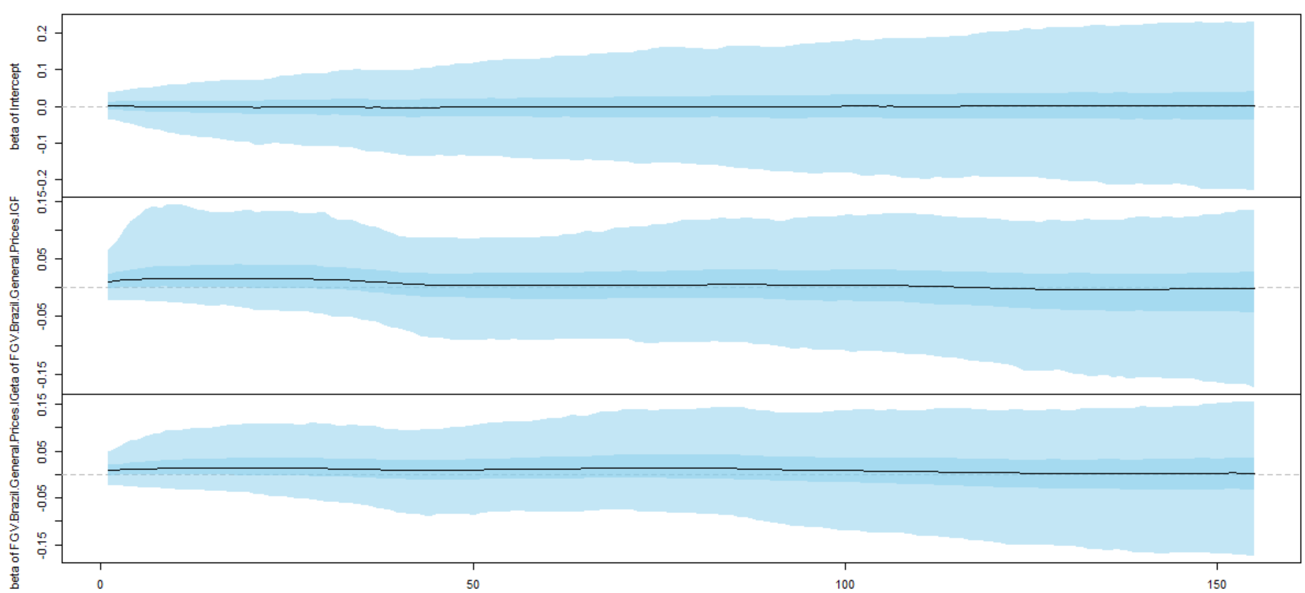


Figure 5: Sample of the $\beta$

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