Sample T/F problems and Multiple choice problems

1. Mark each statement True or False.

   a. If A is diagonalizable, then $-A$ is also diagonalizable.

   b. Given a subspace W of V, the orthogonal projection from V to W is a one-to-one linear transformation.

   c. The orthogonal complement of the null space of A is the same as the column space of A if A is symmetric.

   d. If the orthogonal complement of the null space of A is the same as the column space of A, then A is symmetric.

   e. Let a vector space $\mathbb{R}^3$ be equipped with a dot product $\cdot$ defined by

   $$(a_1, a_2, a_3) \cdot (b_1, b_2, b_3) = (a_1 + 2a_2)(b_1 + 2b_2) + a_3 b_3.$$

   Then, $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is an orthonormal basis of $\mathbb{R}^3$.

2. Note that matrices below only have one eigenvalue. Which of the following have its (unique) eigenspace of dimension 2?

a) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 7 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ c) $\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ e) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

3. Which of the following are eigenvector bases of A for $\mathbb{R}^4$?

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 2 & 3 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ b) $\left\{ \begin{bmatrix} 1 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \\ 1 \\ 1 \end{bmatrix} \right\}$