Vectors, Lines, and Planes

1. Find a unit vector that has the same direction as $10\mathbf{i} - 11\mathbf{j} + 12\mathbf{k}$.

2. Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{j} + \mathbf{k}$, and $\mathbf{c} = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

3. Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.
$$\begin{cases} L_1 \;:\; x = 1 + 6t, \quad y = 2 - 10t, \quad z = 3 + 4t \\ L_2 \;:\; x = 4 - 21s, \quad y = -5 + 35s, \quad z = 7 - 14s \end{cases}$$

4. Find an equation of the plane through the point $(1, \frac{1}{2}, \frac{1}{3})$ and parallel to the plane $x + y + z = 0$.

5. Find an equation of the plane that passes through the point $(1, -1, 1)$ and contains the line of an equation $x = 2y = 3z$.

6. Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t.
$$\mathbf{r}(t) = \langle t^3 + 3t, t^2 + 1, 3t + 4 \rangle, \qquad t = 1.$$

7. Find an equation of the tangent plane to the given surface at the specified point.

$$z = x^2 + xy + (\ln y)^2, \quad (1, e, e+2).$$

Arc Length and Area

8. Find the area enclosed by the curve $x = t^2 - 3t$, $y = \sqrt{t}$ and the y-axis.

9. Find the exact length of the curve,

$$x = t\sin t, \quad y = t\cos t, \quad 0 \leq t \leq 1.$$

10. Find the area of 4 leaves of the graph of $r = \sin 2\theta$.

11. Find the length of the curve.

$$\mathbf{r}(t) = \sin 3t\mathbf{i} + 4t\mathbf{j} + \cos 3t\mathbf{k}, \quad 1 \leq t \leq 4$$

Limits, the Chain Rule, and Maxima or Minima

11. Find the limit, if it exists, or show that the limit does not exist.

a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy\sin(x+y)}{\sqrt{x^2+y^2}}$ b) $\displaystyle\lim_{(x,y)\to(1,1)} e^{xy}\cos(x-y)$ c) $\displaystyle\lim_{(x,y)\to(0,1)} \frac{x^3+2x^2y}{(x+y)^3}$

12. Find the first partial derivatives of the function.

$$\phi(x,y,z,t) = \frac{x+xy+\ln y}{\cos z + t^2}$$

13. Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$.

$$z = x^2 + xe^{3y}, \quad x = s+t, \quad y = s^2+t^2$$

14. Find the gradient of f, evaluate the gradient at the point P, and find the rate of change of f at P in the direction of the vector $\mathbf{u}$.

$$f(x, y) = x^2y + xy, \quad P = (1, 3), \quad \mathbf{u} = \frac{1}{2}(\mathbf{i} + \sqrt{3}\mathbf{j}).$$

15. Find $\partial z/\partial x$ and $\partial z/\partial y$.

$$x^3 + \ln y + \sin z = 0$$

16. Find the local maximum and minimum values and saddle point(s) of the function.

$$f(x, y) = \sin x \sin y, \quad -\pi < x < \pi, \quad -\pi < y < \pi$$

17. Find the absolute maximum and minimum values of f on the set D.

$$f(x, y) = xy^2, \qquad D = \{(x, y)|x \geq 0,\ y \geq 0,\ x^2 + y^2 \leq 3\}$$