Practice Exam[1] (for Midterm #1)

Math 53, Discussion Section 212 & 213

1. (96') Let C be the curve given by the parametric equations $x = \sin\theta$, $y = 2\cos\theta$, $0 \le \theta \le \pi/2$.

   (a) Describe C by an equation expressing y as a function of x, with restrictions on the values of x if necessary, and sketch the curve.

   (b) Find an equation for the line that is tangent to C at the point having parameter θ.

2. (05')

   (a) Find numbers p, q, r, and s such that the plane $px + qy + rz + s = 0$ goes through the points $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$.

   (b) Now find a DIFFERENT set of numbers p', q', r', and s' such that the plane $p'x+q'y+r'z+s' = 0$ still goes through the points $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$.

[1] This is made by DongGyu Lim exclusively so that these problems might be very different from the Midterm #1

3. (02') Let L be the line $x = 1 + t$, $y = 2 - t$, $z = -1 + 3t$ and let P be the plane $2x - y - z + 1 = 0$.

   (a) Let Q be the point $(-1, -1, 0)$ (which is on P). Find the equation for the plane containing L and Q.

   (b) Show that line L is parallel to plane P.

   (c) Compute the distance from L to P.

4. (03') Let $f(t) = \langle 3t^2, 2t^3 \rangle$. Find the length of $f(t)$ between $0 \leq t \leq 2$.

5. (96') Calculate the arc-length of the space curve

$$\mathbf{r}(t) = (2t^3/3 + 1/3, t + 7, t^2 + 1) \text{ for } -1 \leq t \leq 2.$$

6. (05') Consider the curve described in polar coordinates by $r = 2 + \cos 2\theta$.

   (a) Explain, without doing any computation, why the area enclosed by the curve must be less than 9π.

   (b) Compute the area enclosed by the curve.

   (c) Explain why your result in part (b) implies that the curve cannot be contained within the rectangle with verticles at $(\pm 3, \pm 1)$.

   (d) Sketch the cuvre.

7. (05') Does the limit

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{\sqrt{x^2+y^2}}$$

exist? Explain why or why not.

8. (09') Consider the function

$$f(x,y) = x^2 - 2x + 1 - y^2 + 2y - 1 + xy - y - x + 1.$$

Find the absolute maximum and minimum of f on the rectangle with vertices $(0,0)$, $(0,2)$, $(2,0)$, $(2,2)$ as well as at which points the maximum and minimum are attained.

9. (09') Find the absolute maximum and minimum values of $f(x,y) = x^2 + y^2 - xy - 3x$ on the region $x^2 + y^2 - xy \leq 9$. Also state the points at which these values occur.