3. a) Eigenvalues are 0, 1, -2. For each of them, eigenvectors are

$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

They are linearly independent since they are associated with distinct eigenvalues. So, they form a 3-dimensional subspace of $\mathbb{R}^3$ which should be $\mathbb{R}^3$ eventually.

b) Let the eigenvectors be v_1, v_2, v_3. If

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = a_1 v_1 + a_2 v_2 + a_3 v_3$$

, then (by multiplying A) we get

$$\begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix} = a_2 v_2 - 2 a_3 v_3$$

, and then

$$\begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} = a_2 v_2 + 4 a_3 v_3$$

. By subtracting the last two equations, we get $6 a_3 v_3 = \begin{bmatrix} 0 \\ 6 \\ 6 \end{bmatrix}$, so $a_3 = 1$ and, as a result, $a_2 = 1$. Now, $a_1 v_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ so that $a_1 = -1$.

Assuming that we have $\mathcal{B} = \{v_1, v_2, v_3\}$ in this order, the $\mathcal{B}$-coordinate of v is

$$\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}.$$